Lecture 5: Recursion Foundations

Base Cases & Call Stack Visualization

Comp 111 — Programming 2

Forman Christian University

What is Recursion?

The Infinite Mirror

The Infinite Mirror

Try Googling `"recursion"` ...

"Did you mean: recursion?"

Recursion is Everywhere!

File Folders Folders contain folders contain folders...

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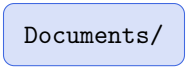
Recursion is everywhere. Today we learn to harness it in code.

Example: File Folders



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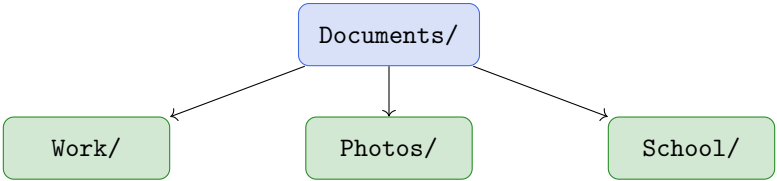

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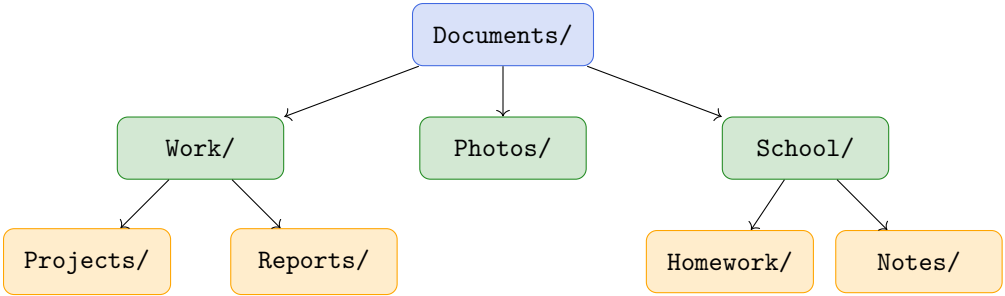

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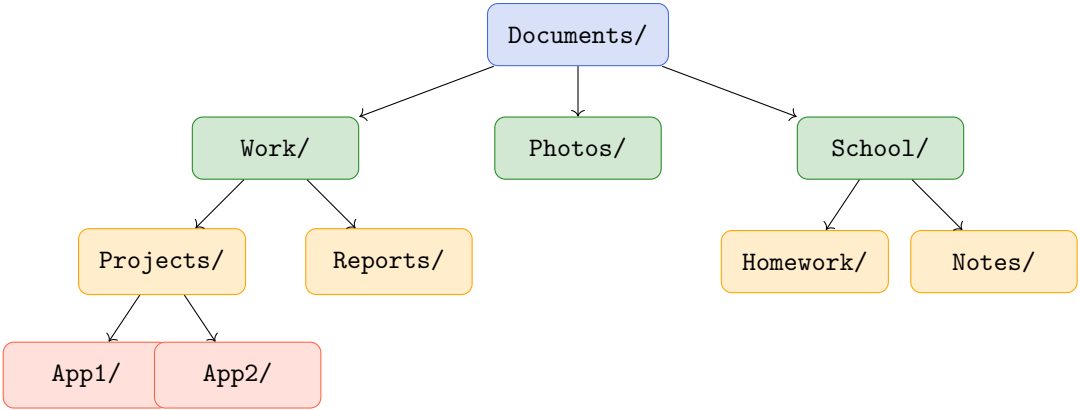

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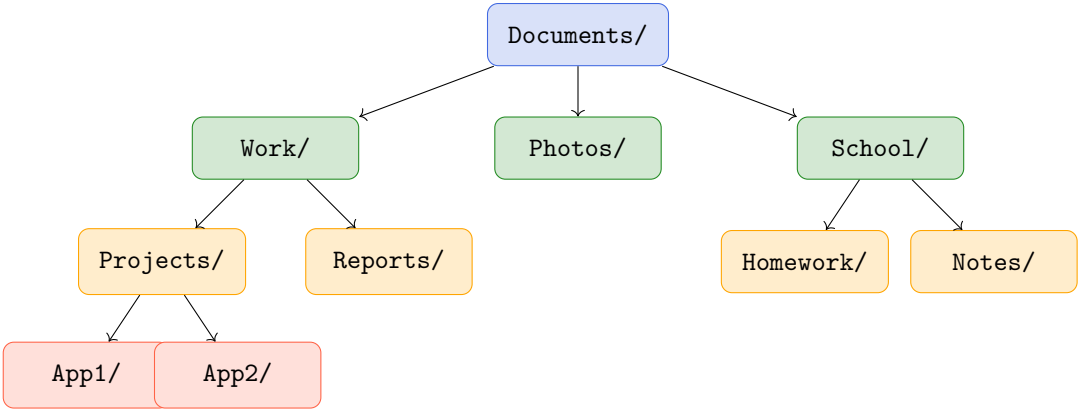

How do you count all files?

Count files here + count files in *each subfolder*

Example: Counting Files

Task: Count all files in "Documents/"

To count files in Documents/:

Example: Counting Files

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Example: Counting Files

Task: Count all files in "Documents/"

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5. Add them all up!

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Base case: Empty folder → return 0

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Base case: Empty folder → return 0

The problem "looks like itself" at each level!

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Recursion is when a function **calls itself** to solve a **smaller version** of the same problem.

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Key Insight: A problem that *"looks like itself"*

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1. BASE CASE

When to **STOP**

(The answer is obvious)

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Every recursive solution needs **both**!

The Lazy Helper Analogy

"Count 100 sheep for me!"

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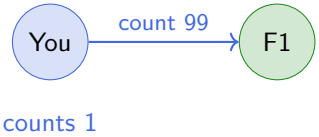

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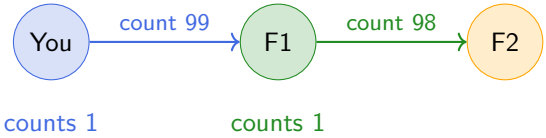

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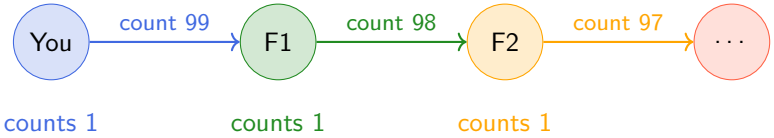

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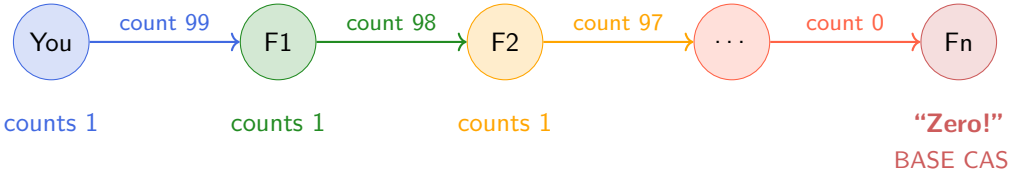

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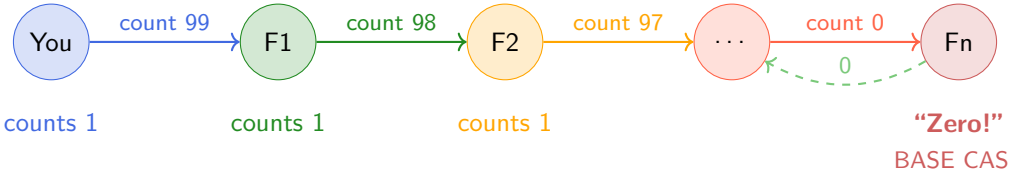

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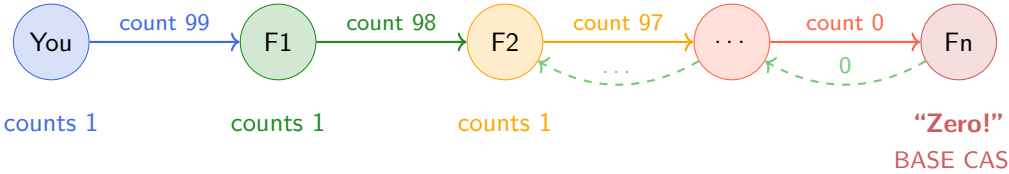

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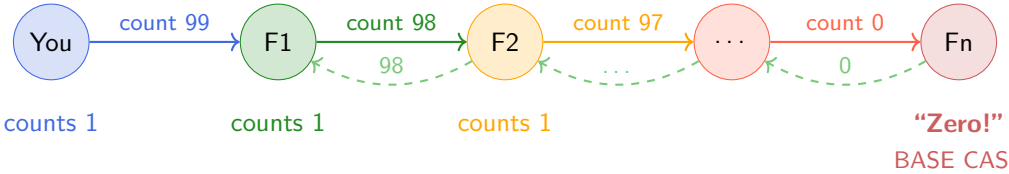

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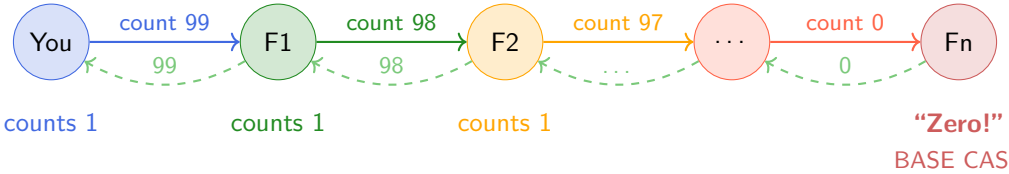

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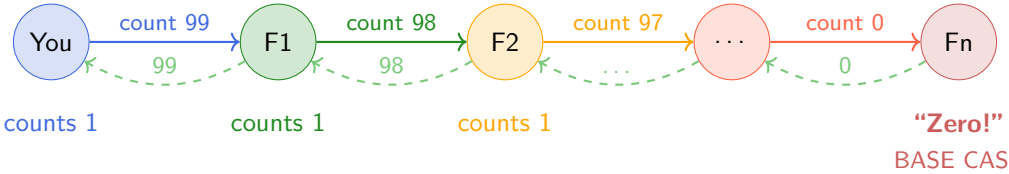

Answers bubble back up!

First Example: Countdown

```
1  def countdown(n):
2      # Base case
3      if n <= 0:
4          print("Blastoff!")
5          return
6
7      # Recursive case
8      print(n)
9      countdown(n - 1)
10
11 countdown(5)
```

Output:

```
5
4
3
2
1
Blastoff!
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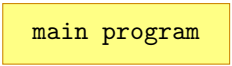
The Call Stack

What is a Call Stack?

Analogy: Stack of Sticky Notes

- Each function call = new sticky note



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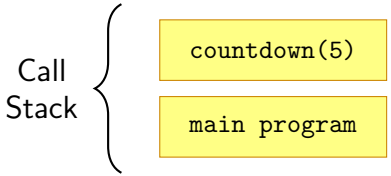

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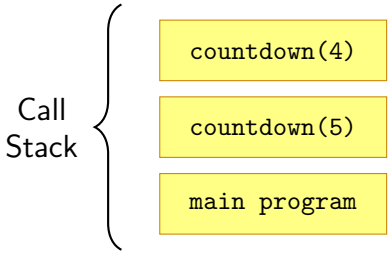

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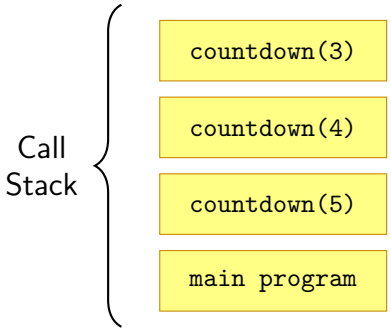

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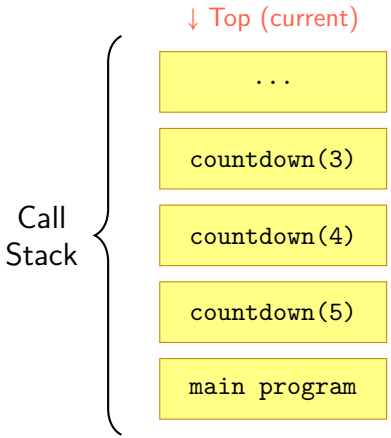
LIFO: Last In, First Out



Tracing: Sum of Numbers

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9  result = sum_to(4)
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sum_to(4)

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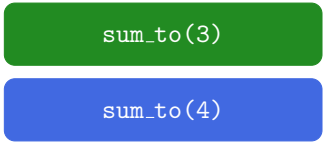

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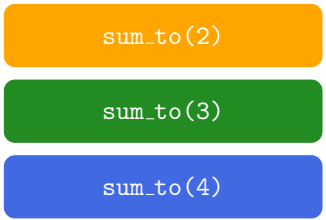

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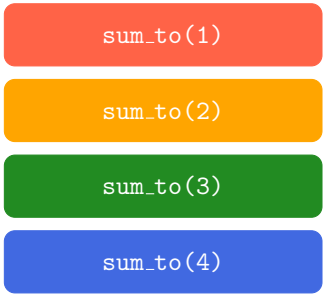

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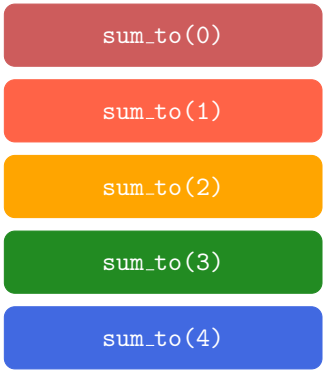

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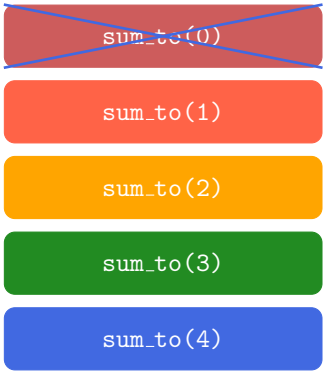

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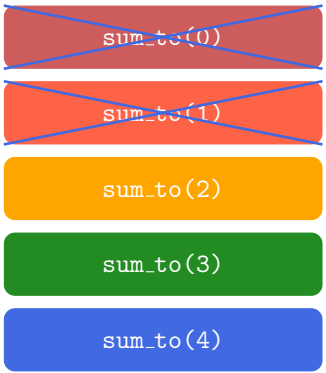

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returns 1+0=1

returns 2+1=3

Mathematical:

$sum\_to(n) = n + (n-1) + \cdots + 1 + 0$

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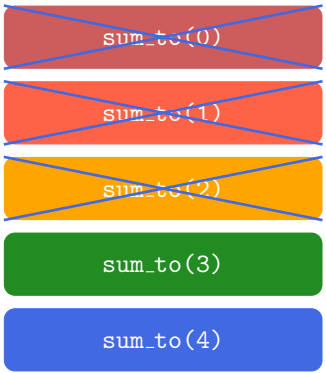

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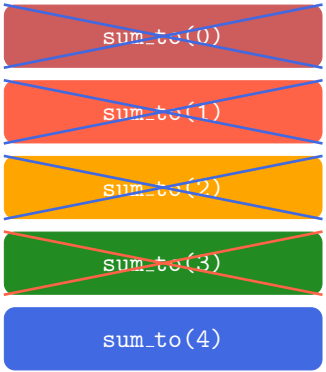

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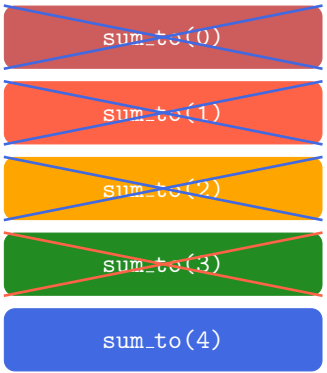


BASE!
returns 0

returns 1+0=1

returns 2+1=3

returns 3+3=6

returns 4+6=10

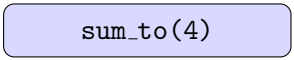
Mathematical:

$sum\_to(n) = n + (n-1) + \cdots + 1 + 0$

result = 10

Visualizing the Trace

Calls



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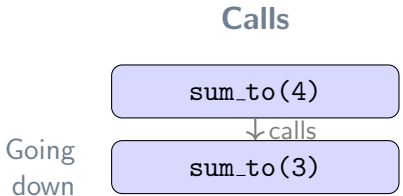

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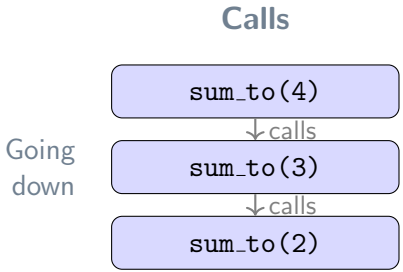

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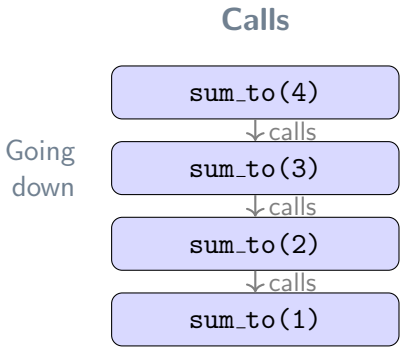

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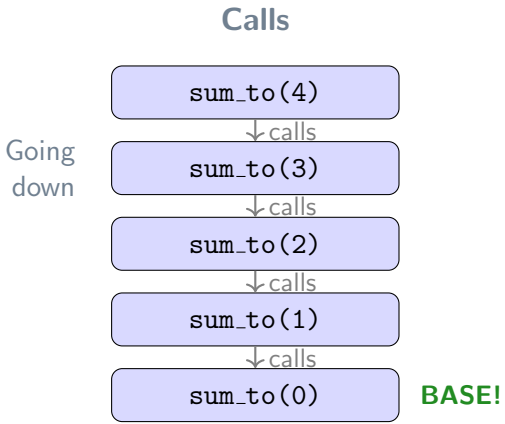

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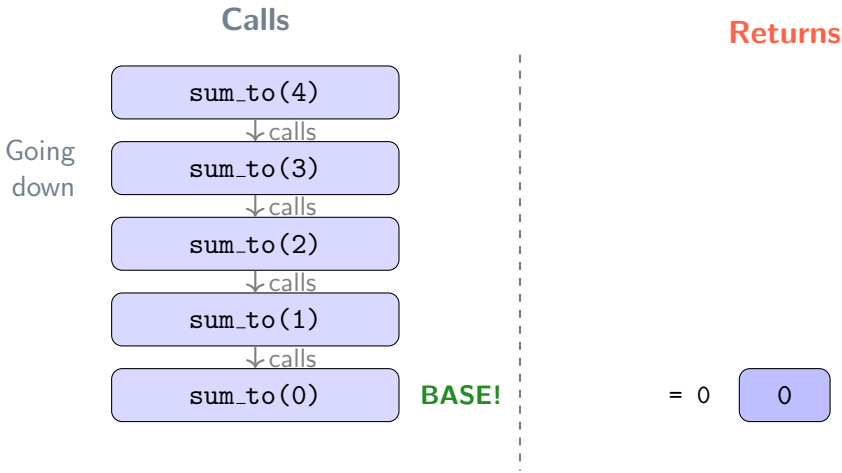

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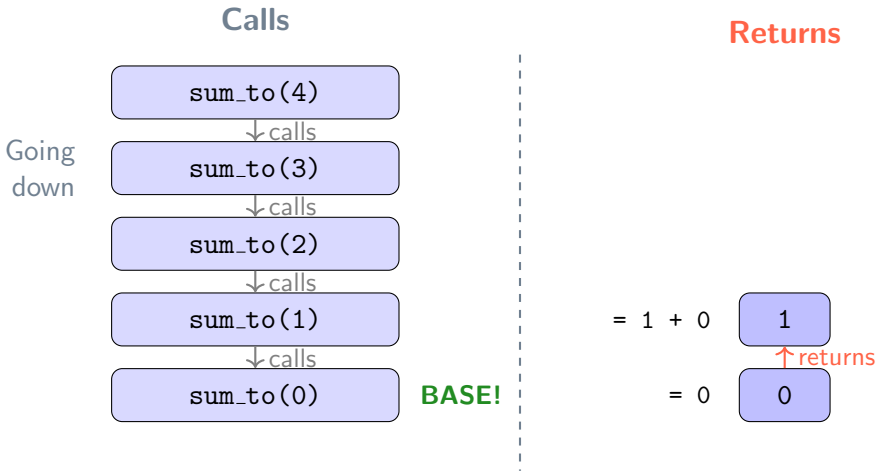

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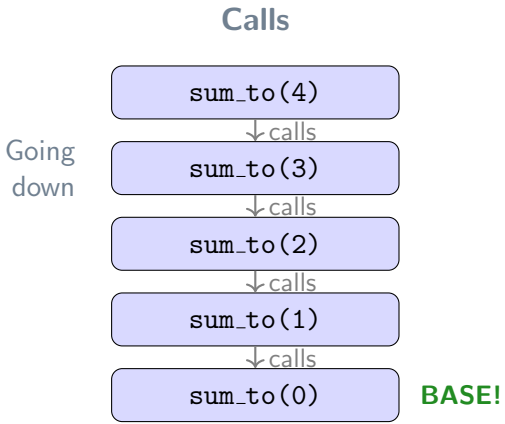

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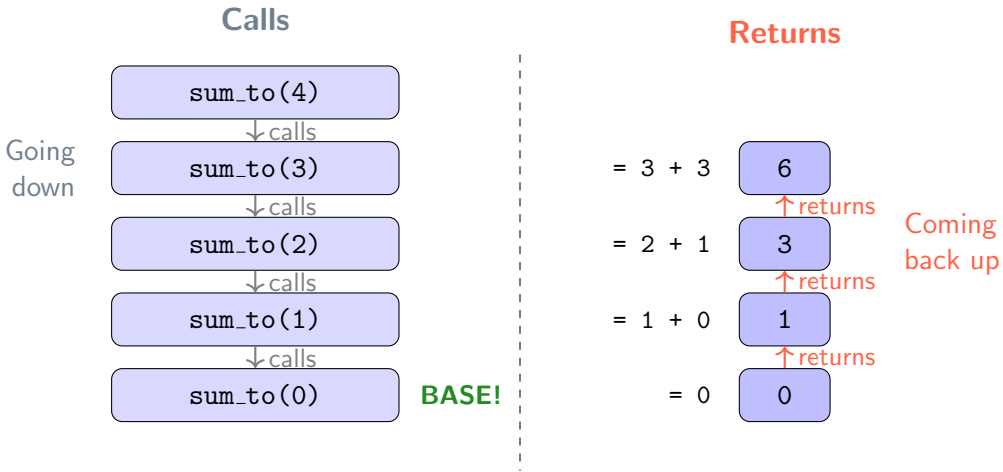
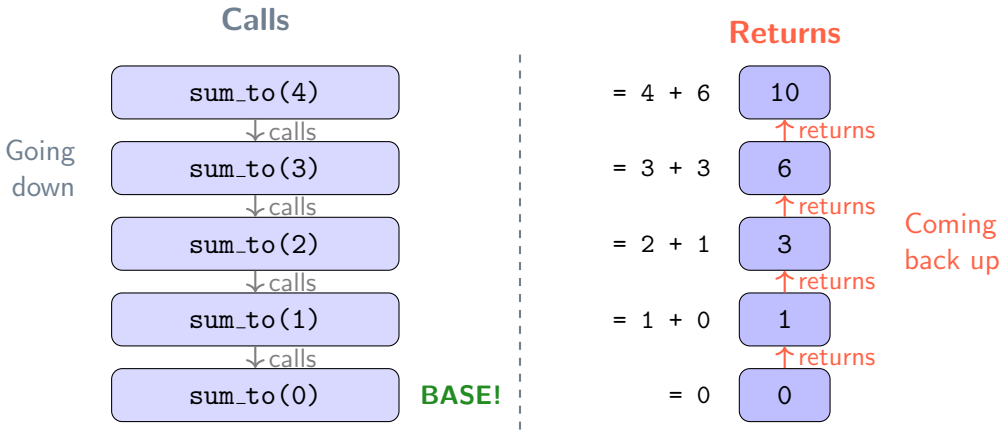


Visualizing the Trace



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Always ask: "What weird inputs could someone pass to my function?"

Python Tutor Demo

Let's visualize this live!

`https://pythontutor.com`

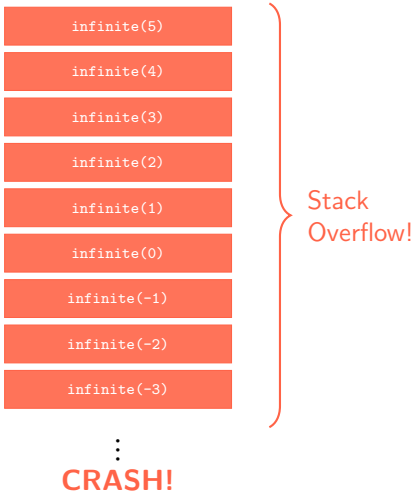
The Critical Base Case

What If We Forget the Base Case?

```
1 def infinite(n):
2     print(n)
3     infinite(n - 1)
4     # No base case!
5
6 infinite(5)
```

Output:

```
5
4
3
...
-993
-994
RecursionError!
```



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- Every function call uses **memory**
- Python has a **recursion limit** (~1000 calls)
- This **protects** your computer from crashing!
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Lesson: Always have a base case that will be reached!

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Lesson: Always have a base case that will be reached!

Practical tip: If your solution is correct but the input is too large, that's a signal to rewrite using a loop. We'll compare next lecture.

Common Base Case Mistakes

Forgetting base case
No `if` statement

→ Infinite recursion

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if n == 1:  print("done")
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→ Returns `None`

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```
if n == 1:  print("done")
```

→ Returns `None`

Not getting smaller
```
return n + f(n)
```

→ Same size forever

You Try: Spot the Bug!

What's wrong with each function?

Problem 1:

```
1  def factorial(n):
2      return n * factorial(n-1)
```

Problem 2: What does `countdown(3)` print?

```
1  def countdown(n):
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Surprise: Prints `1 2 3` (ascending!)
Why? `print` happens *after* the recursive call returns.

Guided Practice

Example: Factorial

Mathematical Definition:

- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- $n! = n \times (n-1)!$
- $0! = 1$ and $1! = 1$ (base cases)

```
1  def factorial(n):
2      # Base case
3      if n <= 1:
4          return 1
5      # Recursive case
6      return n * factorial(n-1)
7
8  print(factorial(5))  # 120
9  print(factorial(0))  # 1
```

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8  print(factorial(5))  # 120
9  print(factorial(0))  # 1
```

```
factorial(5)
  = 5 * factorial(4)
    = 5 * 4 * factorial(3)
      = 5 * 4 * 3 * factorial(2)
        = 5 * 4 * 3 * 2 * 1

= 120
```

Example: String Length

How would you count characters **without** `len()`?

Recursive Thinking:

- Empty string `""` has length **0** (base case)

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- Empty string `""` has length **0** (base case)
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```
1  def string_length(s):
2      # Base case: empty string
3      if s == "":
4          return 0
5      # Recursive case
6      return 1 + string_length(s[1:])
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```
string_length("hello")
 = 1+ str_len("ello")
  = 1+1 + str_len("llo")
   = 1+1+1 + str_len("lo")
    = 1+1+1+1 + str_len("o")
     = 1+1+1+1+1 + 0

= 5
```

You Try: Multiply Using Addition

Write a recursive function to multiply two positive integers **using only addition**.

Hint: $3 \times 4 = 3 + 3 + 3 + 3 = 3 + (3 \times 3)$

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```
1  def multiply(a, b):
2      # Base case: anything times 0 is 0
3      if b == 0:
4          return 0
5      # Recursive case: a * b = a + a * (b-1)
6      return a + multiply(a, b - 1)
7
8  print(multiply(3, 4))  # 12
```

Summary

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Recursion Checklist

- ✓ Define your **BASE CASE** (when to stop)
- ✓ Define your **RECURSIVE CASE** (smaller subproblem)
- ✓ Make sure you're moving **TOWARD** the base case
- ✓ Handle **edge cases** (0, negatives, empty inputs)
- ✓ **Trust** that the recursive call will work!

The Recursive Leap of Faith

When writing recursive code:

Trust that your function will work for smaller inputs.

If your base case is correct and you're making the problem smaller, **it WILL work**.

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Trust that your function will work for smaller inputs.

If your base case is correct and you're making the problem smaller, **it WILL work**.

Don't try to trace every call in your head!

Do trust the recursive structure.

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- Two essential parts: **Base case** + **Recursive case**
- **Call stack** tracks each function call
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- Without base case → `RecursionError`
- Use **Python Tutor** to visualize!

Next Lecture Preview

Recursion Practice & Debugging

- More practice problems
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Homework:

1. Trace `factorial(4)` on paper (draw the call stack!)
2. Write recursive `power(base, exp)` $\rightarrow 2^3 = 8$
3. Write recursive `count_down_to_zero(n)` that prints `n` down to 0
4. **Bonus:** Write recursive `reverse_string(s)`

Questions?

Practice on `pythontutor.com`