Lecture 17: Shadows, Mirrors and Shiny Surfaces

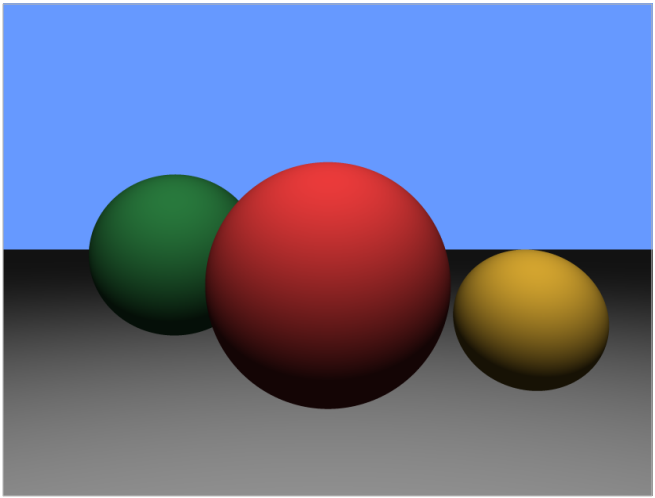
Is the Lamp Blocked, and Where Does the Ray Go Next?

CS 453 — Computer Graphics

Fakhir Shaheen

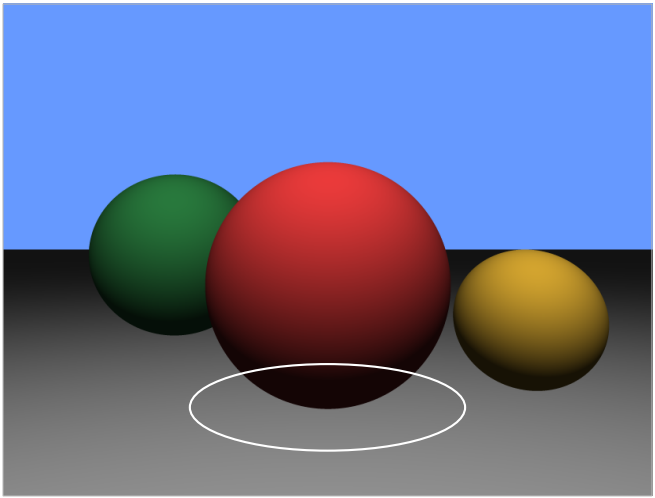
Forman Christian University

Today: three things the picture lacks



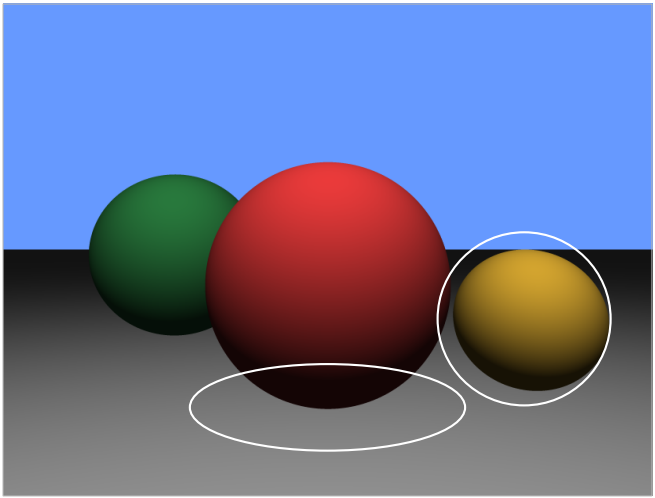
- Lecture 16: lit, but floating

Today: three things the picture lacks



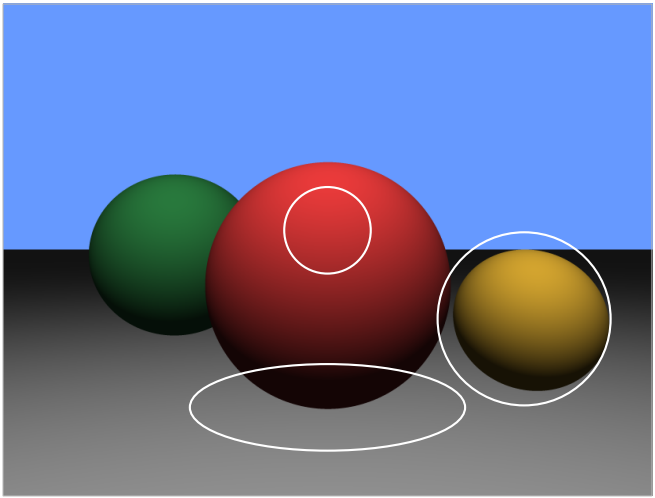
- Lecture 16: lit, but floating
- No shadow under the ball

Today: three things the picture lacks



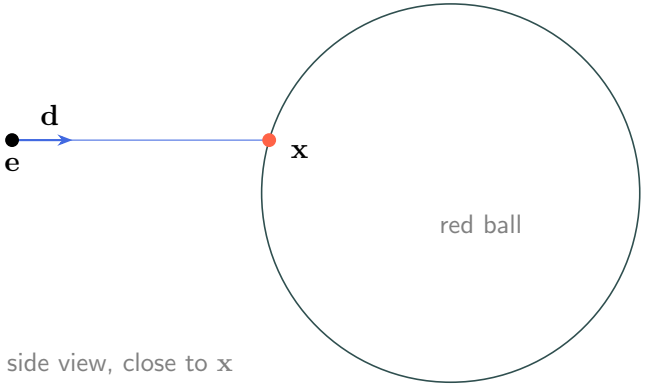
- Lecture 16: lit, but floating
- No shadow under the ball
- No reflection in the gold ball

Today: three things the picture lacks



- Lecture 16: lit, but floating
- No shadow under the ball
- No reflection in the gold ball
- No bright spot on the ball

Our conventions in one picture



- $\mathbf{d}$ points in, toward the hit $\mathbf{x}$

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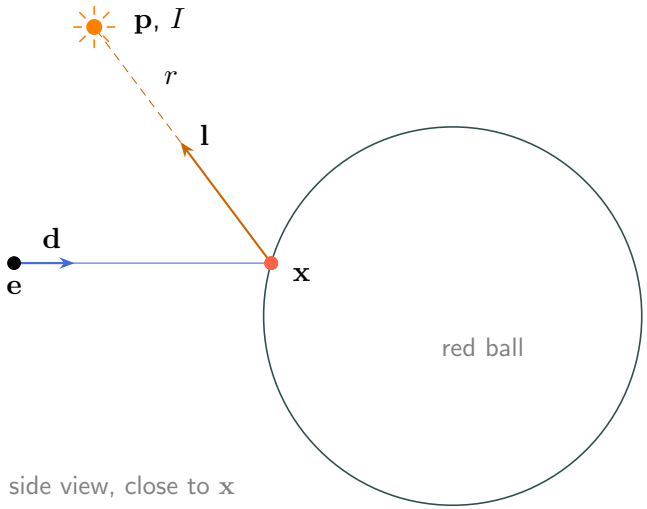

- $\mathbf{d}$ points in, toward the hit $\mathbf{x}$
- $\mathbf{p}$, I: the lamp; plain r: its distance

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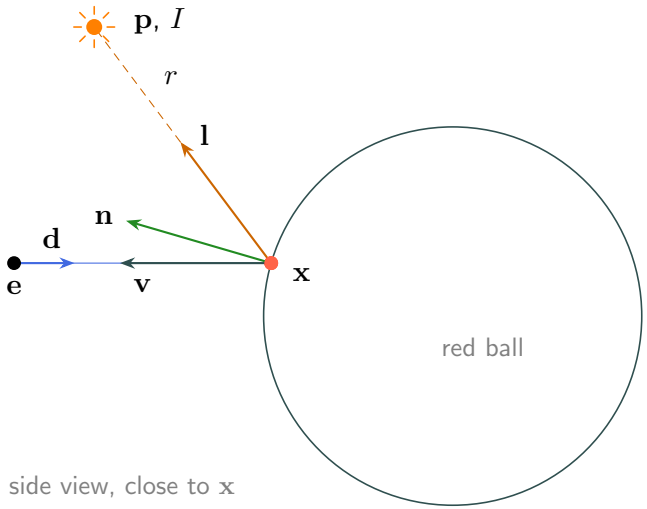
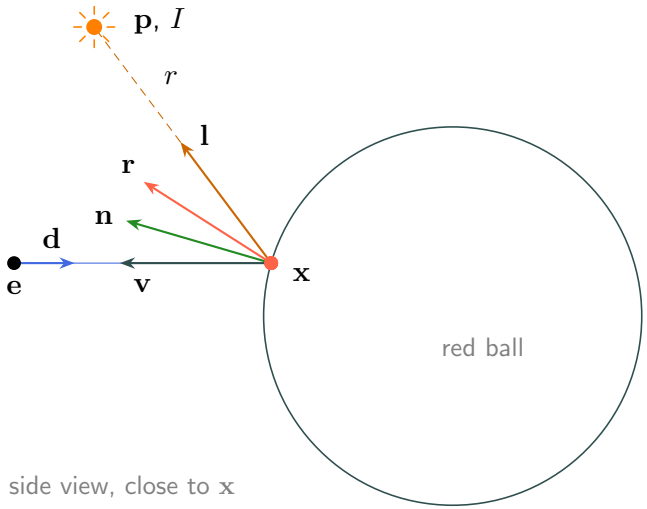


- **d** points in, toward the hit **x**
- **p**, I: the lamp; plain r: its distance
- **n**, **l**, **v**: length 1; **v** points back

Our conventions in one picture

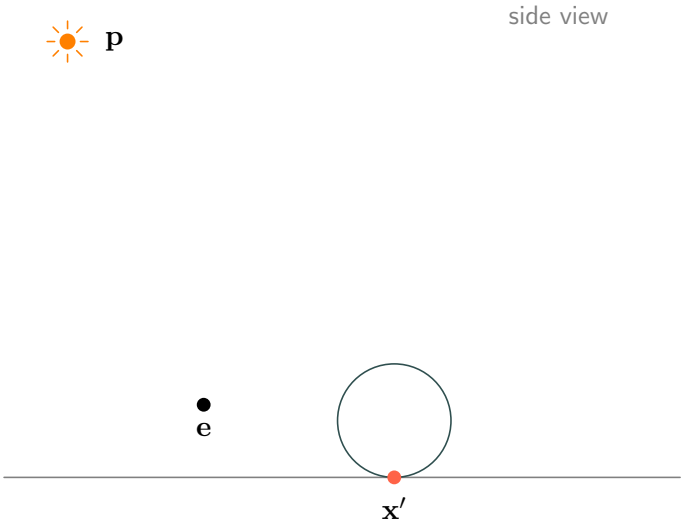


- **d** points in, toward the hit **x**
- **p**, I: the lamp; plain r: its distance
- **n**, **l**, **v**: length 1; **v** points back
- Bold **r**: the mirror direction

Shadows

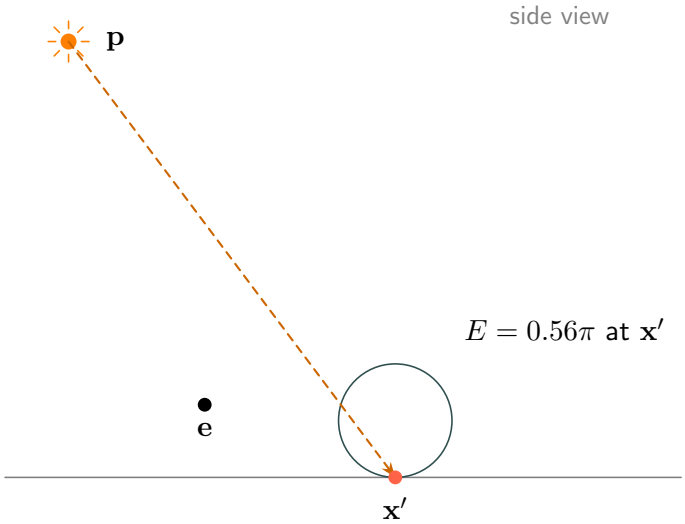
One more ray: is the lamp blocked?

Lecture 16's light ignores the ball



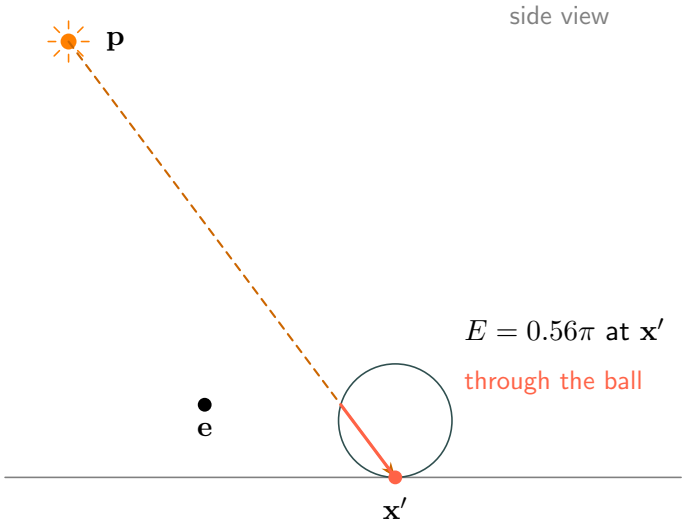
- $\mathbf{x}'$: the floor under the ball

Lecture 16's light ignores the ball



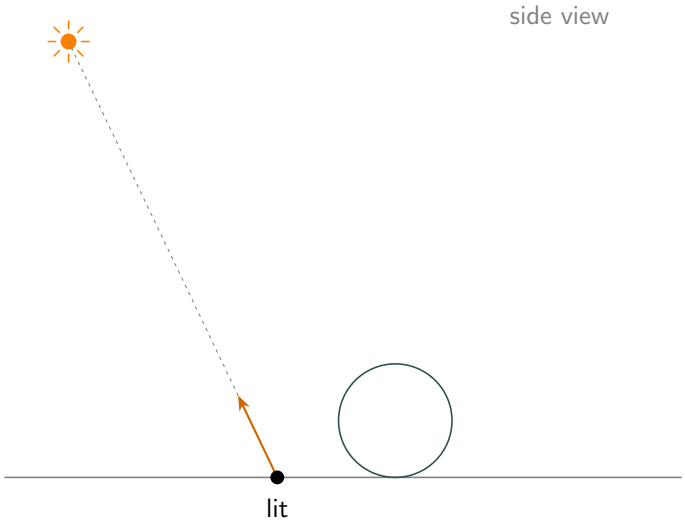
- $\mathbf{x}'$: the floor under the ball
- Lecture 16 lights it fully

Lecture 16's light ignores the ball



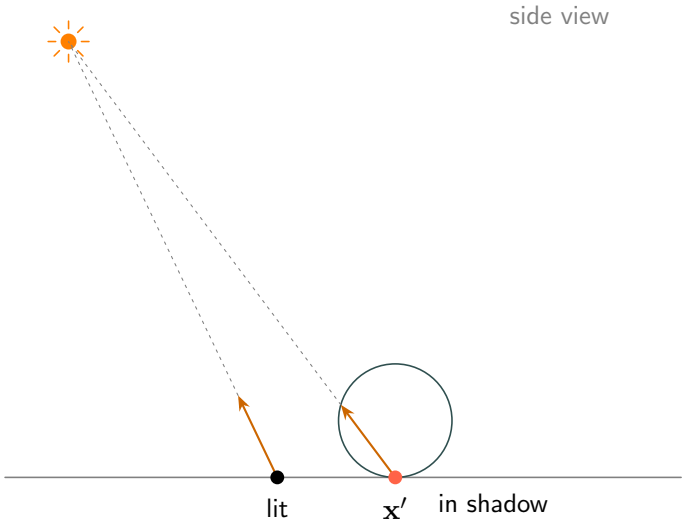
- $\mathbf{x}'$: the floor under the ball
- Lecture 16 lights it fully
- The formula never asks what is between

In shadow: an object between point and lamp



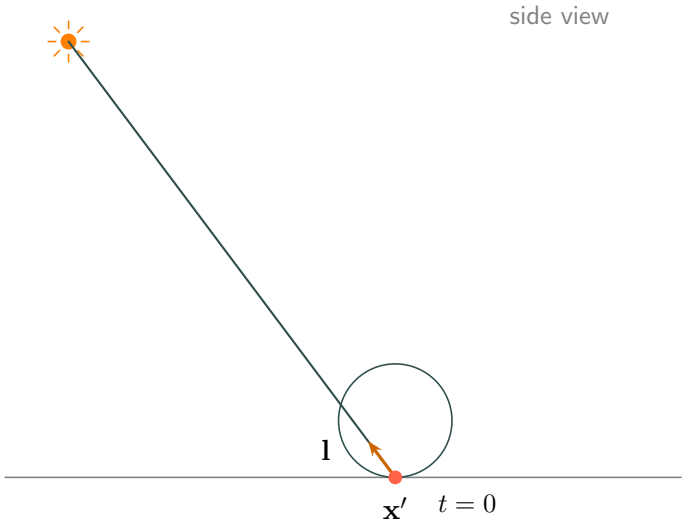
- From the point, look toward the lamp
- Nothing in between: lit

In shadow: an object between point and lamp



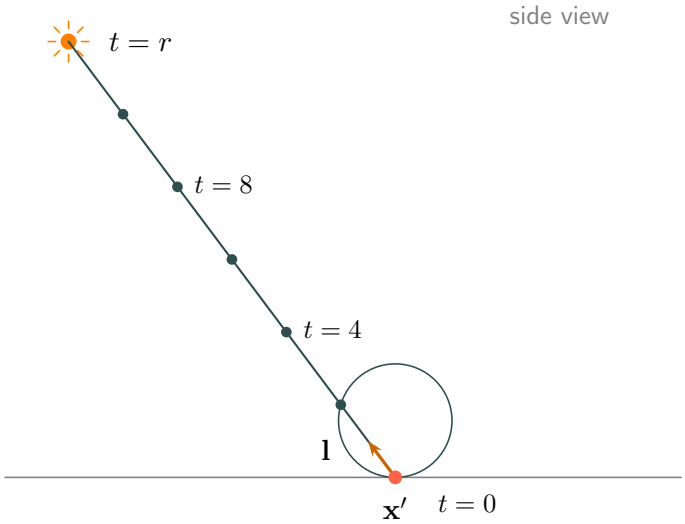
- From the point, look toward the lamp
- Nothing in between: lit
- An object in between: in shadow

A shadow ray: from the point toward the lamp



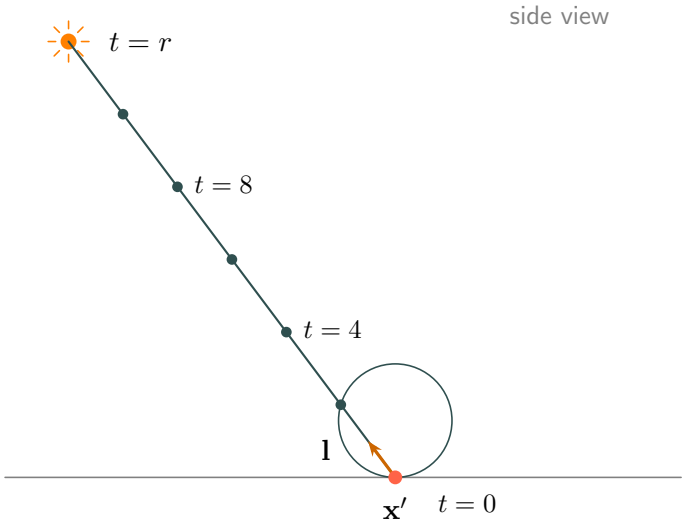
- Shadow ray: $\mathbf{x} + t\,\mathbf{l}$

A shadow ray: from the point toward the lamp



- Shadow ray: $\mathbf{x} + t\,\mathbf{l}$
- $\mathbf{l}$ has length 1: t is distance

A shadow ray: from the point toward the lamp



- Shadow ray: $\mathbf{x} + t\,\mathbf{l}$
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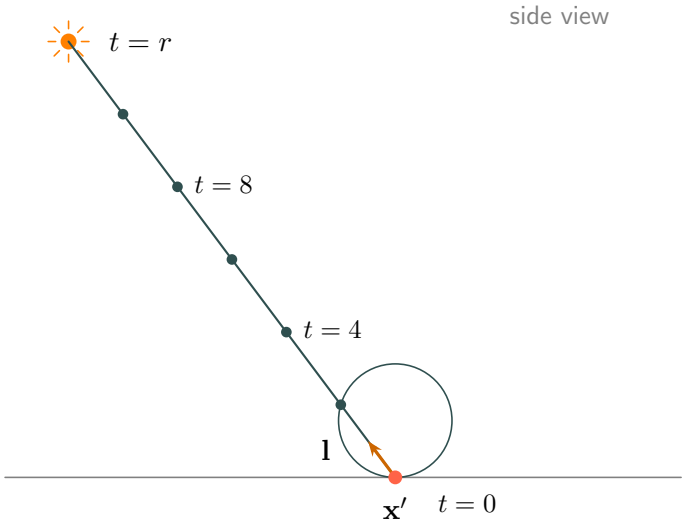
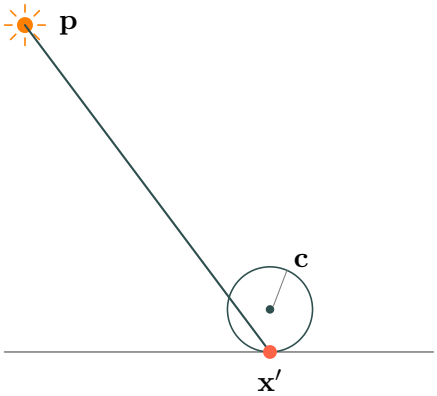
- A hit with t in $[0, r]$: in shadow

A shadow ray: from the point toward the lamp



- Shadow ray: $\mathbf{x} + t\,\mathbf{l}$
- $\mathbf{l}$ has length 1: t is distance
- A hit with t in $[0, r]$: in shadow
- Beyond the lamp: does not count

Worked: is the ball's foot in shadow?



- Try first: in shadow, or lit?

$\mathbf{x}' = (0, -1.6, -4.2)$, $\mathbf{p} = (0, 8, 3)$

ball: $\mathbf{c} = (0, -0.35, -4.2)$, radius 1.25

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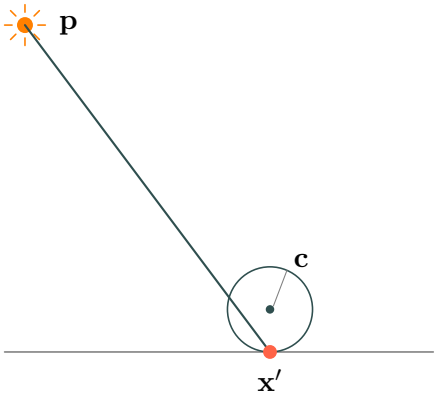

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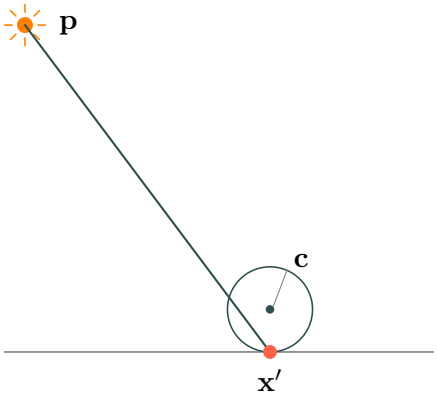

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$\mathbf{l} = (0, 0.8, 0.6)$, $\mathbf{x}' - \mathbf{c} = (0, -1.25, 0)$

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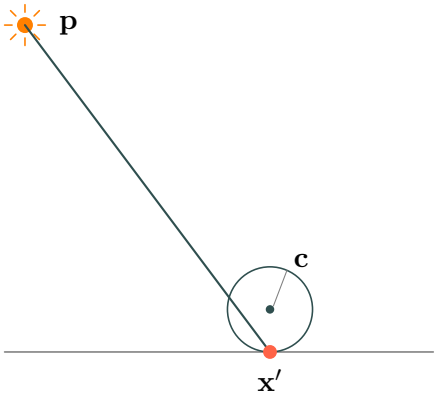

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$A = \mathbf{l} \cdot \mathbf{l} = 1$

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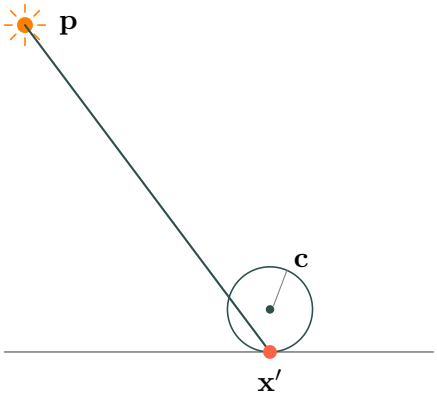

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$A = \mathbf{l} \cdot \mathbf{l} = 1$

$B = 2\mathbf{l} \cdot (\mathbf{x}' - \mathbf{c}) = 2(0.8)(-1.25) = -2$

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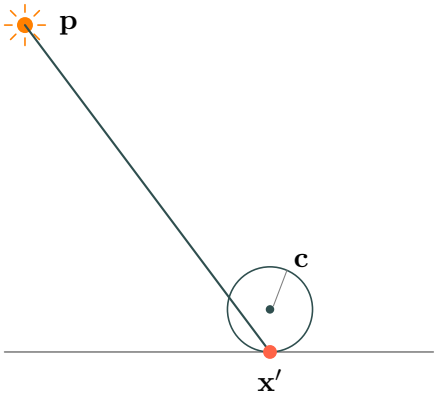

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$C = 1.25^2 - 1.25^2 = 0$

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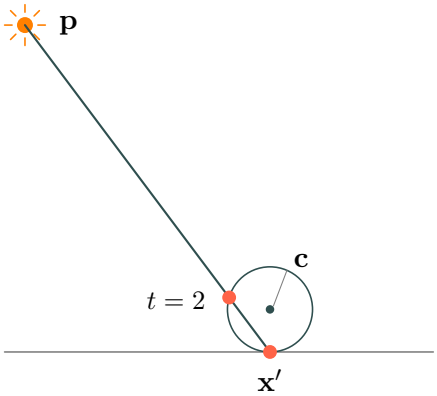

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$C = 1.25^2 - 1.25^2 = 0$

$t^2 - 2t = 0$: $t = 0$ or $t = 2$

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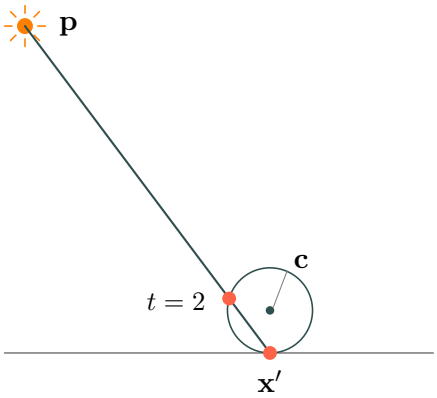

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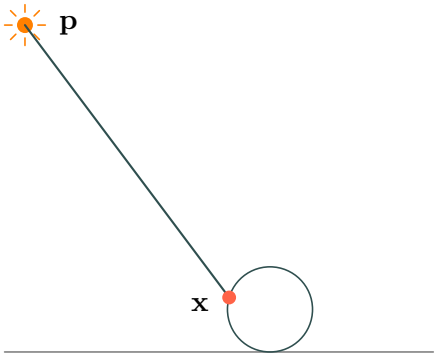
$t^2 - 2t = 0$: $t = 0$ or $t = 2$

$t = 2$ lies in $[0, 12]$: in shadow

Worked: the same test at our hit $\mathbf{x}$



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$$\mathbf{x} = (0, 0, -3), \quad r = 10, \quad \mathbf{l} = (0, 0.8, 0.6)$$

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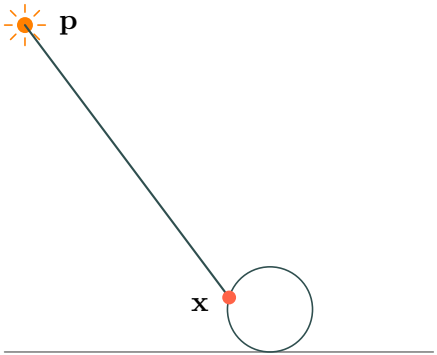

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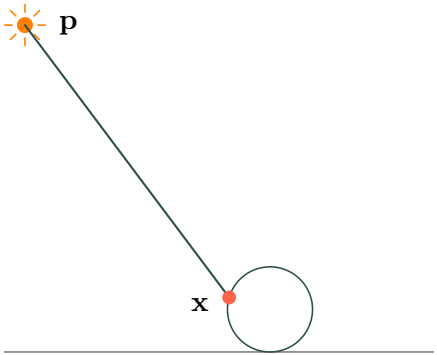

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$$A = 1$$

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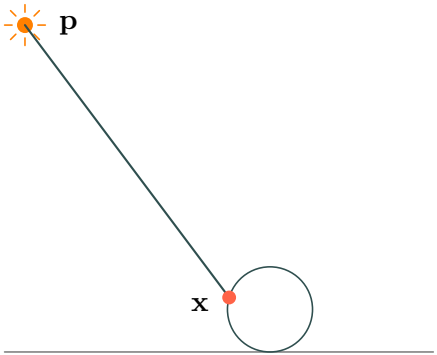

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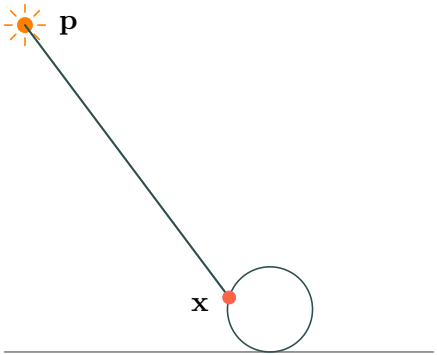
$$\mathbf{x} - \mathbf{c} = (0, 0.35, 1.2)$$

$$A = 1$$

$$B = 2\,(0.8 \times 0.35 + 0.6 \times 1.2) = 2$$

Worked: the same test at our hit $\mathbf{x}$



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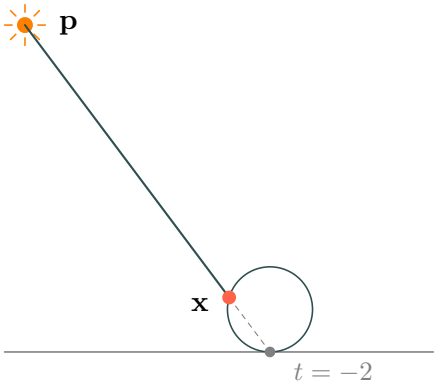
$$\mathbf{x} - \mathbf{c} = (0, 0.35, 1.2)$$

$$A = 1$$

$$B = 2\,(0.8 \times 0.35 + 0.6 \times 1.2) = 2$$

$$C = 0.1225 + 1.44 - 1.5625 = 0$$

Worked: the same test at our hit $\mathbf{x}$



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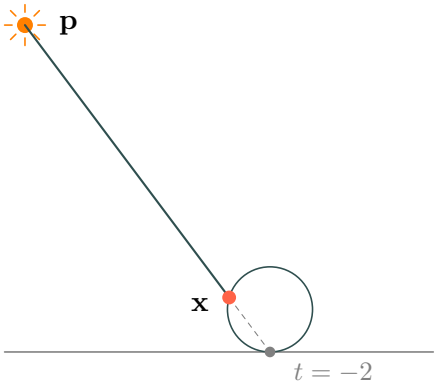

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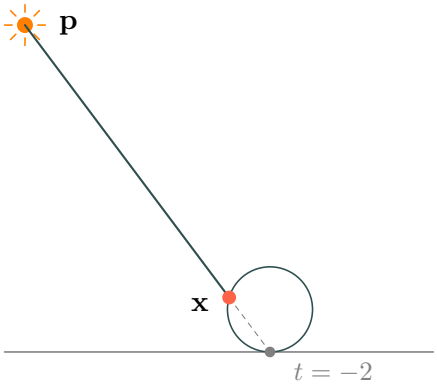
$t = 0$ lies in $[0, 10]$: in shadow?

Worked: the same test at our hit $\mathbf{x}$



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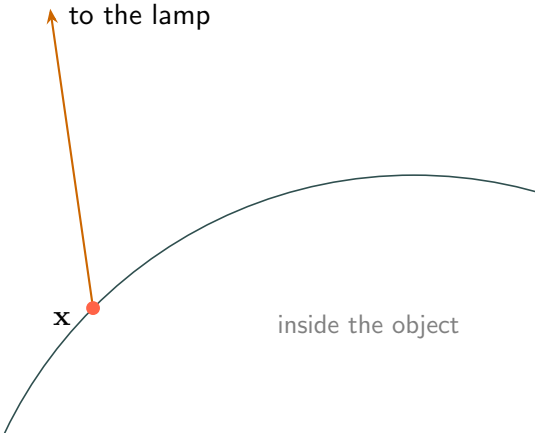
$t = 0$ lies in $[0, 10]$: in shadow?

- The ray hits the ball it starts on

Start the test a little way along

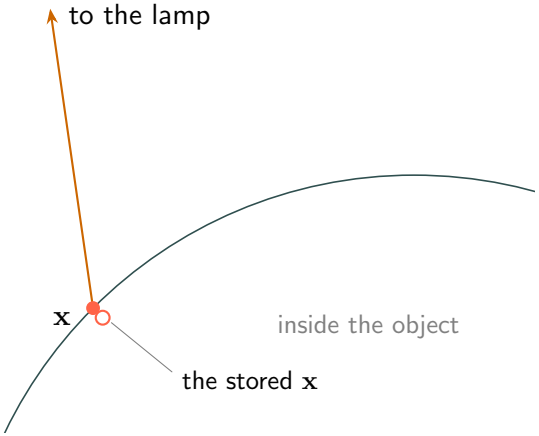
much enlarged



- On paper: a root at $t = 0$

Start the test a little way along

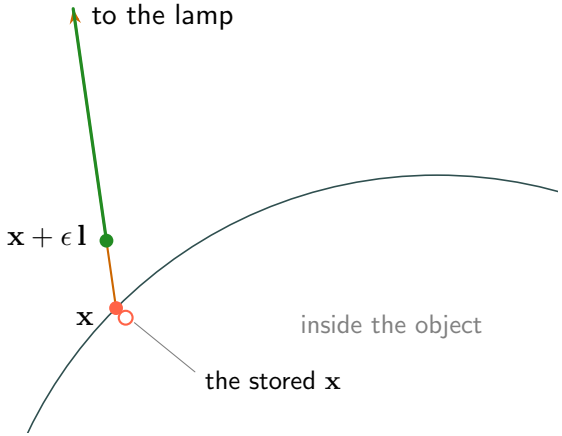
much enlarged



- On paper: a root at $t = 0$
- Rounding: $\mathbf{x}$ lands a hair off

Start the test a little way along

much enlarged



- On paper: a root at $t = 0$
- Rounding: $\mathbf{x}$ lands a hair off
- Fix: test t in $[\epsilon, r]$
- ϵ: a small positive number

With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

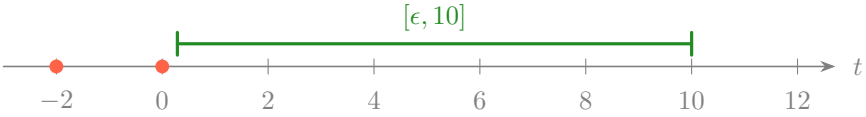
from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$



- The root at $t = 0$: skipped

With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

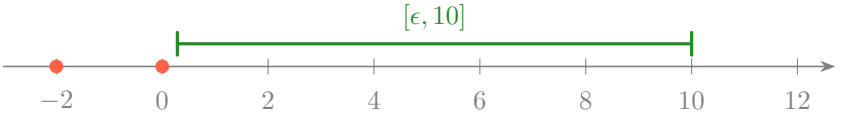
from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$



- The root at $t = 0$: skipped

With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$

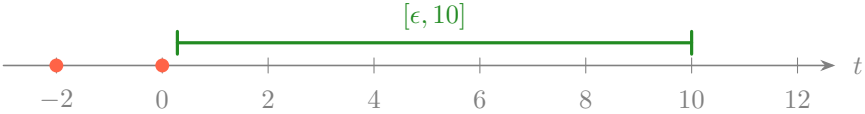


no root inside: lit

- The root at $t = 0$: skipped

With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$



no root inside: lit

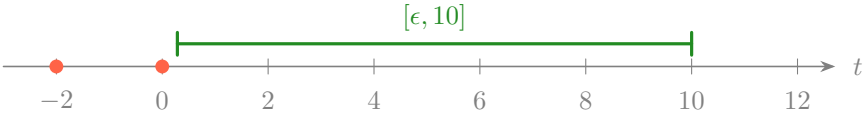
from $\mathbf{x}'$: roots $t = 0$ and $t = 2$; $r = 12$



- The root at $t = 0$: skipped

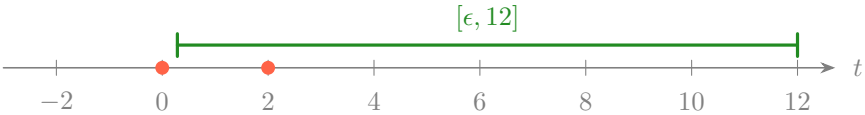
With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$



no root inside: lit

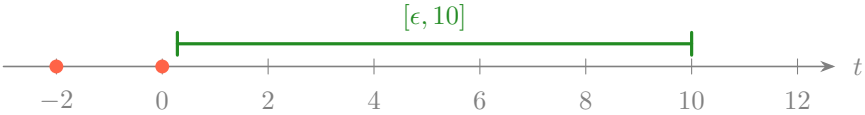
from $\mathbf{x}'$: roots $t = 0$ and $t = 2$; $r = 12$



- The root at $t = 0$: skipped

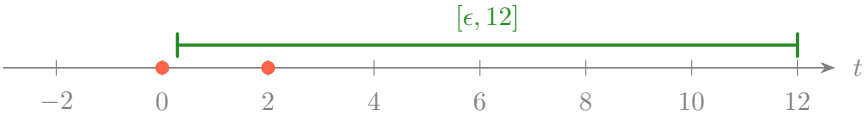
With ϵ: $\mathbf{x}$ is lit, $\mathbf{x}'$ is in shadow

from $\mathbf{x}$: roots $t = -2$ and $t = 0$; $r = 10$



no root inside: lit

from $\mathbf{x}'$: roots $t = 0$ and $t = 2$; $r = 12$



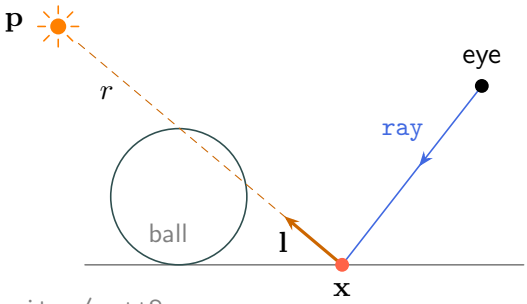
$t = 2$ inside: in shadow

- The root at $t = 0$: skipped
- Only a root inside $[\epsilon, r]$ blocks

The shadow test, in Python

```
class PointLight(Light):
    def illuminate(self, ray, rec):
        x = ray.evaluate(rec.t)
        r = (self.position - x).length()
        l = (self.position - x) / r


        n = rec.normal
        v = -ray.d / ray.d.length()
        E = max(0, n.dot(l)) * self.intensity / r**2
        k = rec.surface.material.evaluate(l, v, n)
        return k * E
```



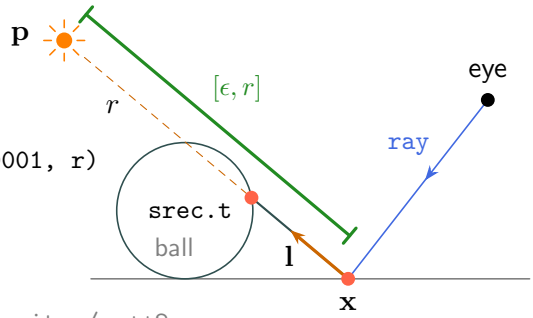
- Grey lines: lecture 16, unchanged
- $\mathbf{x}$, $\mathbf{l}$, r: already computed

The shadow test, in Python

```
class PointLight(Light):
    def illuminate(self, ray, rec):
        x = ray.evaluate(rec.t)
        r = (self.position - x).length()
        l = (self.position - x) / r
        srec = scene.hit(Ray(x, l), 0.00001, r)



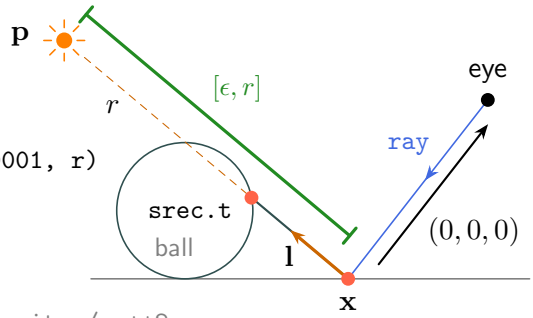
        n = rec.normal
        v = -ray.d / ray.d.length()
        E = max(0, n.dot(l)) * self.intensity / r**2
        k = rec.surface.material.evaluate(l, v, n)
        return k * E
```



- Grey lines: lecture 16, unchanged
- Interval: ϵ up to the lamp
- $\mathbf{x}$, $\mathbf{l}$, r: already computed

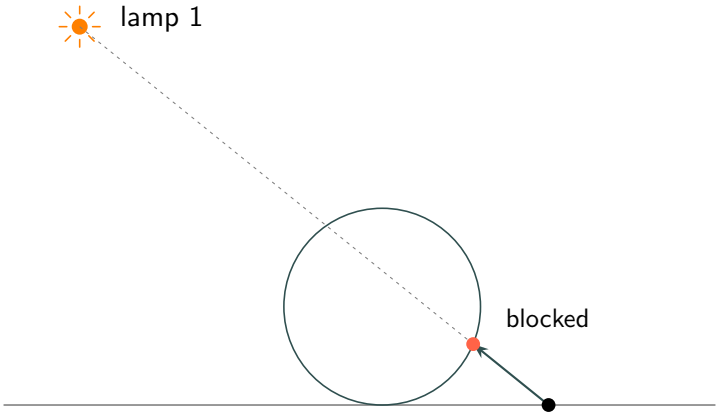
The shadow test, in Python

```
class PointLight(Light):
    def illuminate(self, ray, rec):
        x = ray.evaluate(rec.t)
        r = (self.position - x).length()
        l = (self.position - x) / r
        srec = scene.hit(Ray(x, l), 0.00001, r)
        if srec.t < math.inf:
            return Vec3(0,0,0)
        n = rec.normal
        v = -ray.d / ray.d.length()
        E = max(0, n.dot(l)) * self.intensity / r**2
        k = rec.surface.material.evaluate(l, v, n)
        return k * E
```



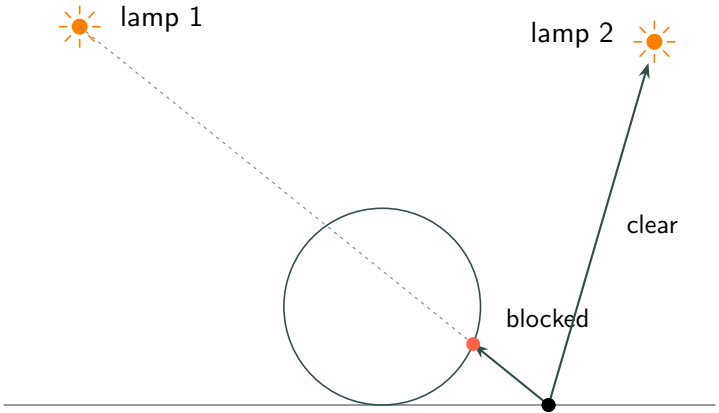
- Grey lines: lecture 16, unchanged
- Interval: ϵ up to the lamp
- $\mathbf{x}$, $\mathbf{l}$, r: already computed
- In shadow: this lamp adds black

Each lamp gets its own shadow ray



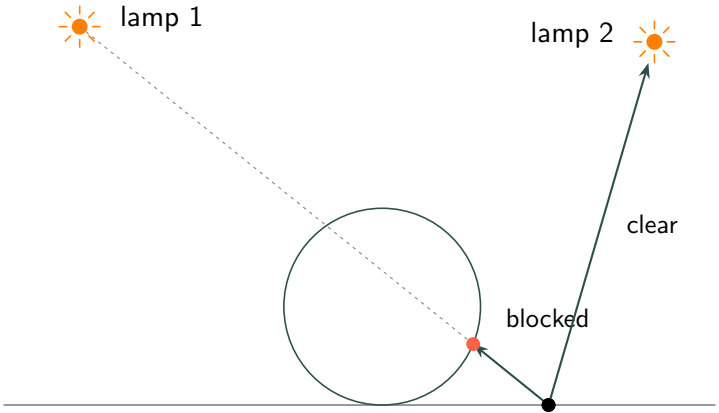
- Lamp 1 is blocked: it adds 0

Each lamp gets its own shadow ray



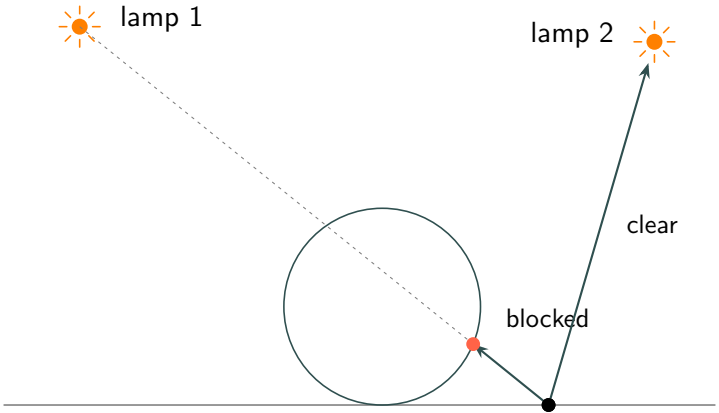
- Lamp 1 is blocked: it adds 0
- Lamp 2 still adds its share

Each lamp gets its own shadow ray



- Lamp 1 is blocked: it adds 0
- Lamp 2 still adds its share
- One shadow ray for each lamp

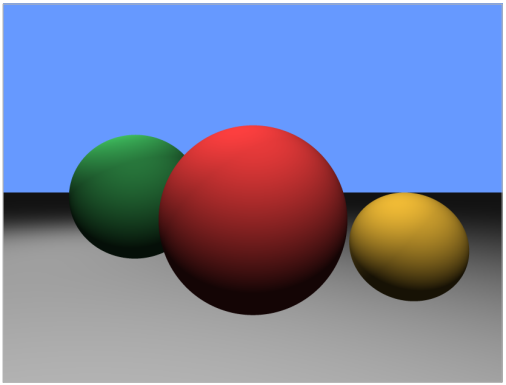
Each lamp gets its own shadow ray



- Lamp 1 is blocked: it adds 0
- Lamp 2 still adds its share
- One shadow ray for each lamp
- Ambient light: no test at all

a directional light: test t in $[\epsilon, \infty)$

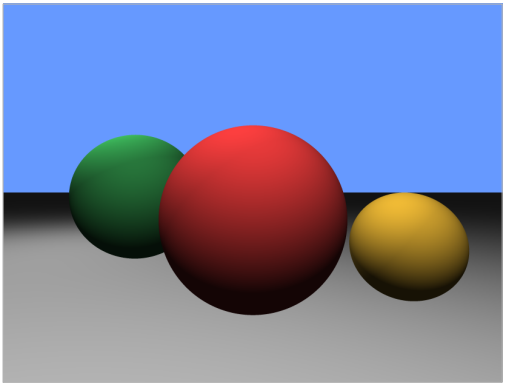
Shadows put the balls on the floor



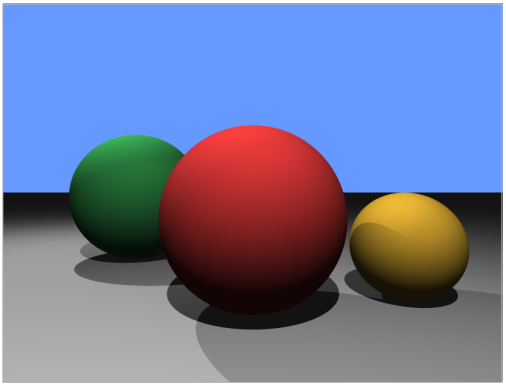
two lamps, no shadow test

- Without the test: floating

Shadows put the balls on the floor



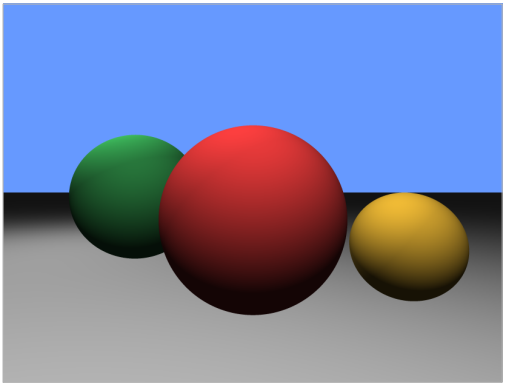
two lamps, no shadow test



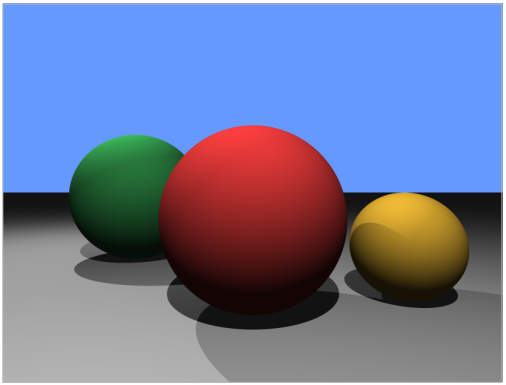
with shadow rays

- Without the test: floating
- With it: standing

Shadows put the balls on the floor



two lamps, no shadow test

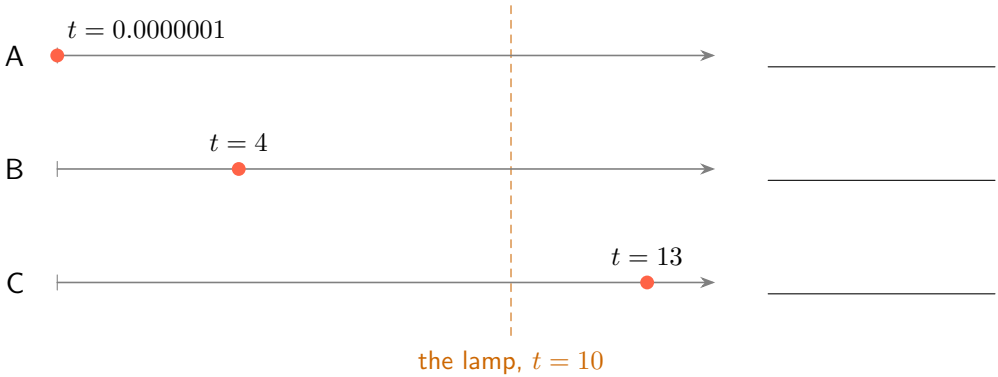


with shadow rays

- Without the test: floating
- With it: standing
- Shadows: grey, lit by ambient

Quick check 1: which points are lit?

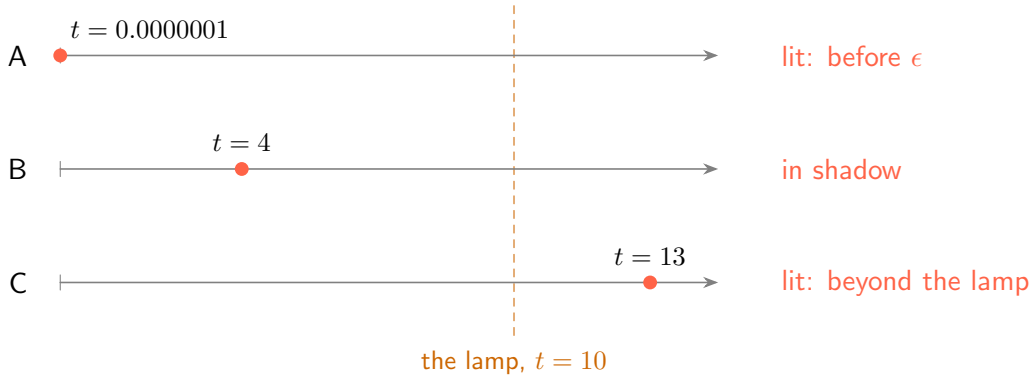
The lamp is $r = 10$ away; $\epsilon = 0.001$. Dots: each shadow ray's only hit.



- Lit, or in shadow?

Quick check 1: which points are lit?

The lamp is $r = 10$ away; $\epsilon = 0.001$. Dots: each shadow ray's only hit.



- Lit, or in shadow?
- Only a hit inside $[\epsilon, r]$ blocks

Recap: the shadow test

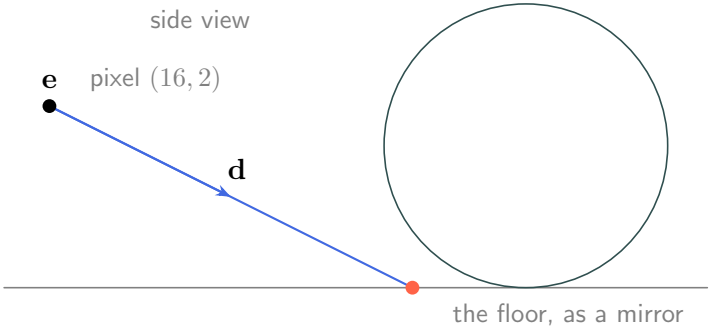
$$\mathbf{x} \text{ is in shadow } \iff \mathbf{x} + t\,\mathbf{l} \text{ hits something with } t \text{ in } [\epsilon,\ r]$$

one test for every hit and every lamp; none for ambient light

Mirror reflection

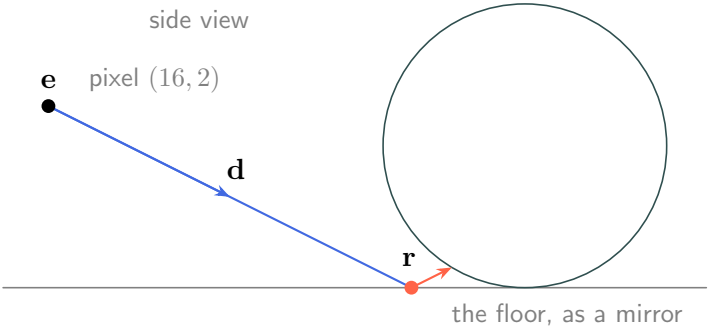
The ray does not stop: it bounces on

A mirror shows what lies along $\mathbf{r}$



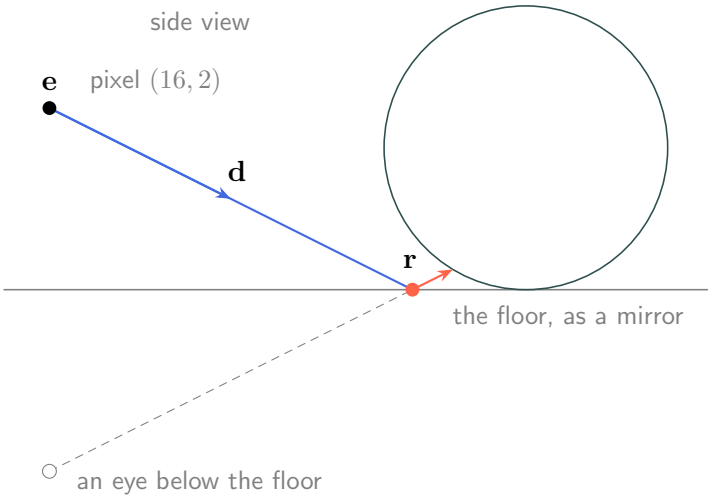
- The viewing ray meets a mirror

A mirror shows what lies along $\mathbf{r}$



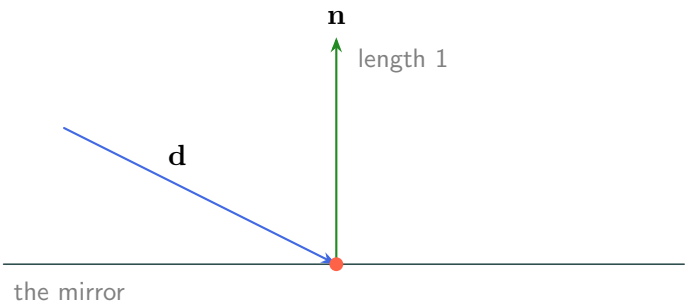
- The viewing ray meets a mirror
- The pixel shows what $\mathbf{r}$ meets

A mirror shows what lies along $\mathbf{r}$



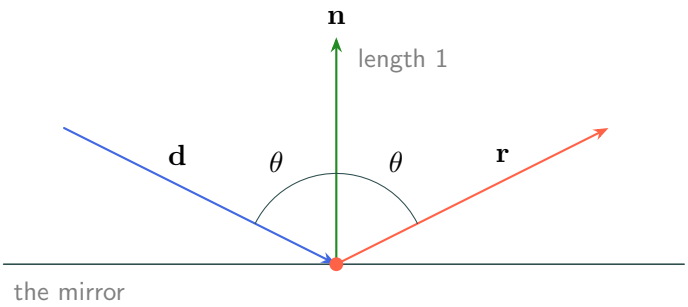
- The viewing ray meets a mirror
- The pixel shows what $\mathbf{r}$ meets
- The same view, from below

The mirror direction: equal angles, one formula



- $\mathbf{d}$ arrives, pointing into the surface

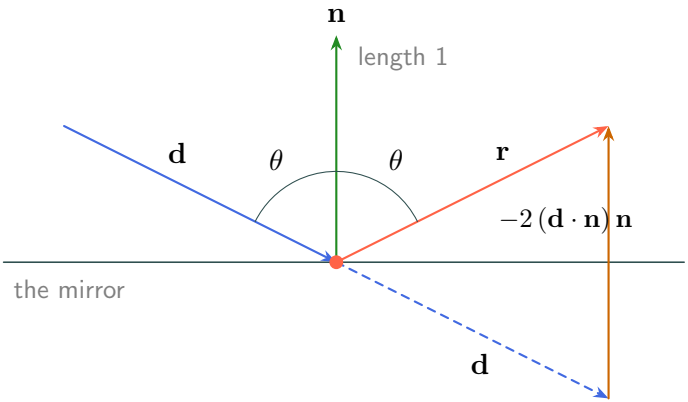
The mirror direction: equal angles, one formula



- $\mathbf{d}$ arrives, pointing into the surface
- $\mathbf{r}$: same angle, other side

The mirror direction: equal angles, one formula

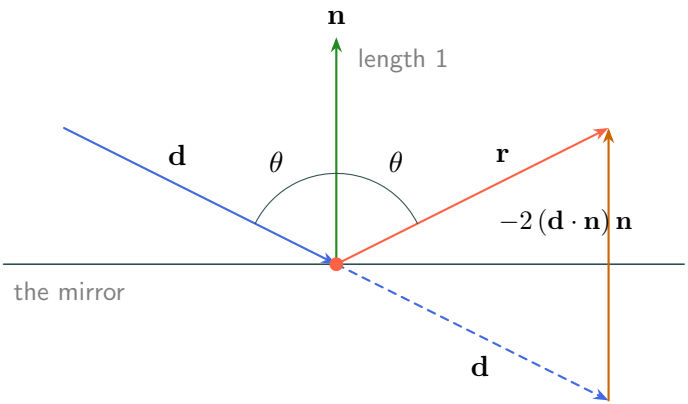
$$\mathbf{r} = \mathbf{d} - 2\,(\mathbf{d} \cdot \mathbf{n})\,\mathbf{n}$$



- $\mathbf{d}$ arrives, pointing into the surface
- $\mathbf{r}$: same angle, other side
- Remove $\mathbf{d}$'s part along $\mathbf{n}$, twice

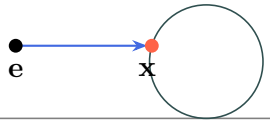
The mirror direction: equal angles, one formula

$$\mathbf{r} = \mathbf{d} - 2\,(\mathbf{d} \cdot \mathbf{n})\,\mathbf{n}$$



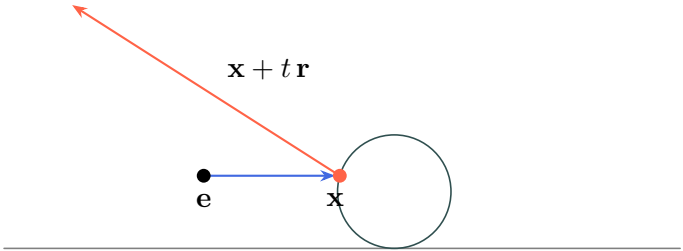
- $\mathbf{d}$ arrives, pointing into the surface
- $\mathbf{r}$: same angle, other side
- Remove $\mathbf{d}$'s part along $\mathbf{n}$, twice
- Lecture 4's formula: `d.reflect(n)`

A mirror's colour: trace one more ray



side view

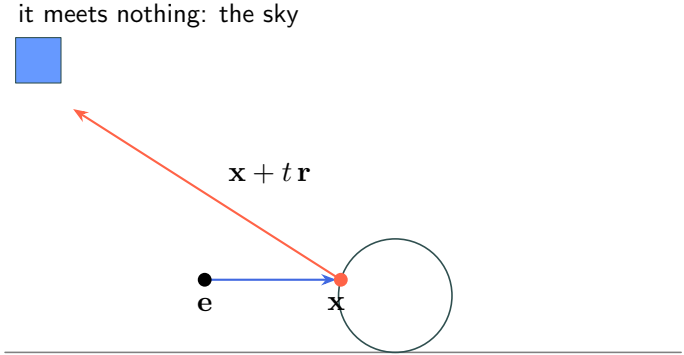
A mirror's colour: trace one more ray



side view

- A new ray starts at $\mathbf{x}$

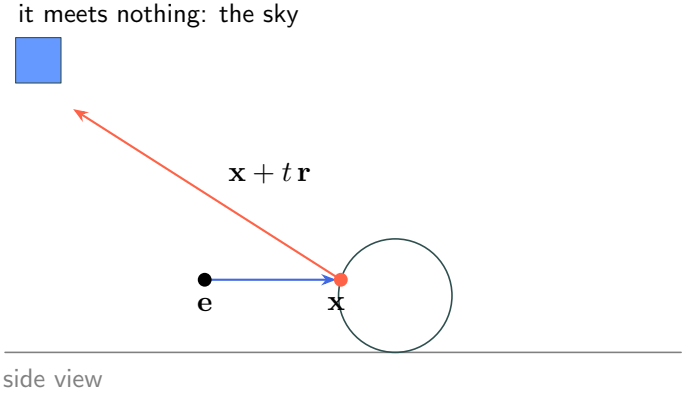
A mirror's colour: trace one more ray



- A new ray starts at $\mathbf{x}$

A mirror's colour: trace one more ray

$$c = k_m \text{ trace}\big(\text{Ray}(\mathbf{x}, \mathbf{r})\big)$$



- A new ray starts at $\mathbf{x}$
- Its colour comes back, times k_m
- ϵ again: skip the mirror itself
- No lamp to stop at: up to ∞

k_m is a colour: gold tints what it shows



$(0.4,\ 0.6,\ 1.0)$ $\times$



$(0.9,\ 0.7,\ 0.2)$

what the ray saw

k_m of polished gold

- k_m: one number per channel

k_m is a colour: gold tints what it shows



$(0.4,\ 0.6,\ 1.0)$

what the ray saw

$\times$



$(0.9,\ 0.7,\ 0.2)$

k_m of polished gold

$=$



$(0.36,\ 0.42,\ 0.20)$

what the mirror shows

- k_m: one number per channel
- Gold: little blue

k_m is a colour: gold tints what it shows



$(0.4,\ 0.6,\ 1.0)$ $\times$



$(0.9,\ 0.7,\ 0.2)$ $=$



$(0.36,\ 0.42,\ 0.20)$

what the ray saw | k_m of polished gold | what the mirror shows



$(0.4,\ 0.6,\ 1.0)$ $\times$



$(0.25,\ 0.25,\ 0.25)$ $=$



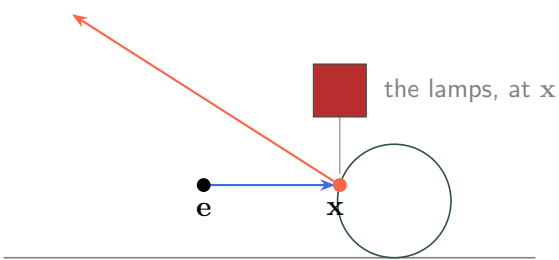
$(0.10,\ 0.15,\ 0.25)$

a grey k_m

- k_m: one number per channel
- Gold: little blue
- Grey: dimmer, same colour

Worked: pixel $(16, 12)$ on a mirror ball

- Try first: the pixel, if this ball is a mirror with $k_m = (0.9,\ 0.7,\ 0.2)$

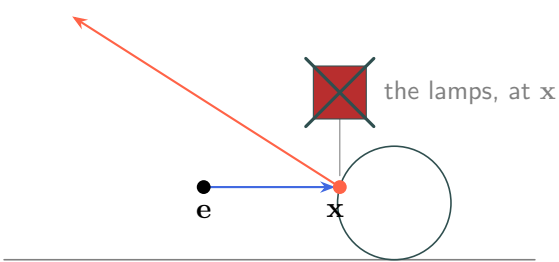


side view

$(0.72,\ 0.18,\ 0.18)$
the lamps' shares at $\mathbf{x}$ (lecture 16)

Worked: pixel $(16, 12)$ on a mirror ball

- Try first: the pixel, if this ball is a mirror with $k_m = (0.9,\ 0.7,\ 0.2)$



side view



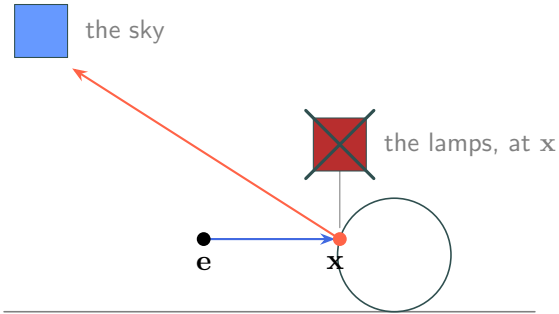
$(0.72,\ 0.18,\ 0.18)$ not used

the lamps' shares at $\mathbf{x}$ (lecture 16)

- A mirror hit adds up no lamps.

Worked: pixel $(16, 12)$ on a mirror ball

- Try first: the pixel, if this ball is a mirror with $k_m = (0.9,\ 0.7,\ 0.2)$





$(0.72,\ 0.18,\ 0.18)$ not used

the lamps' shares at $\mathbf{x}$ (lecture 16)



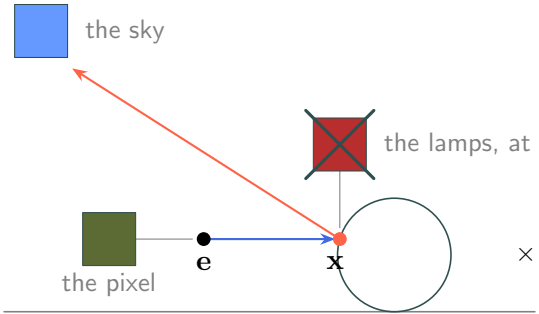
$(0.4,\ 0.6,\ 1.0)$

what the new ray sees: the sky

- A mirror hit adds up no lamps.

Worked: pixel $(16, 12)$ on a mirror ball

- Try first: the pixel, if this ball is a mirror with $k_m = (0.9,\ 0.7,\ 0.2)$



$(0.72,\ 0.18,\ 0.18)$ not used
the lamps' shares at $\mathbf{x}$ (lecture 16)

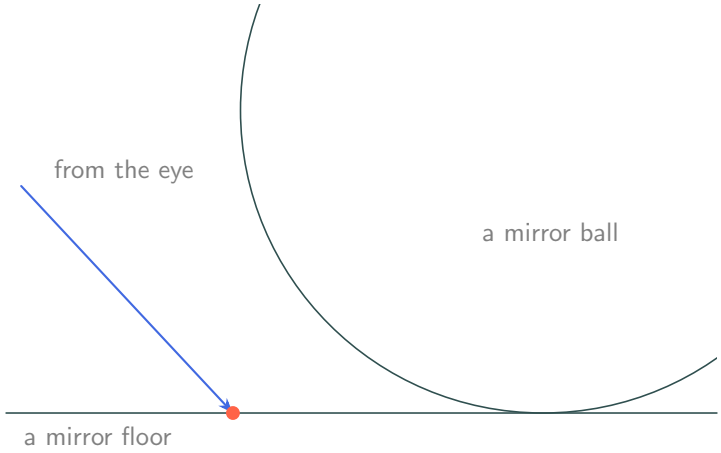
$(0.4,\ 0.6,\ 1.0)$
what the new ray sees: the sky

$\times\ k_m =$

$(0.36,\ 0.42,\ 0.20)$
the pixel: the last slide's product

- A mirror hit adds up no lamps.
- The pixel shows the sky, tinted gold.

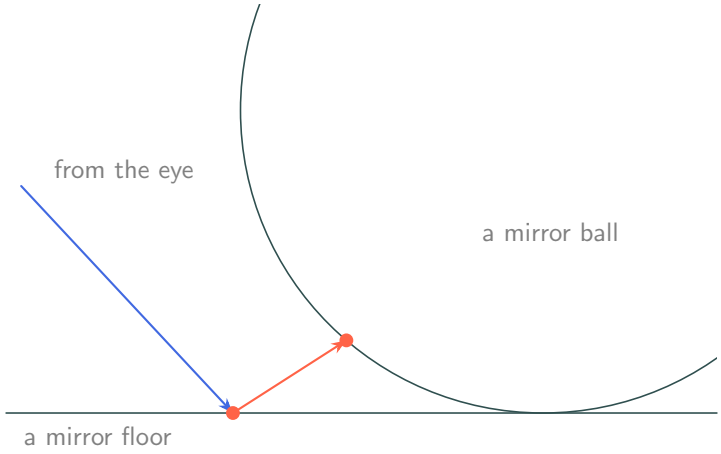
Between two mirrors, a ray keeps bouncing



a sketch, not to scale: a mirror ball on a mirror floor

- The ray meets the floor, then the ball

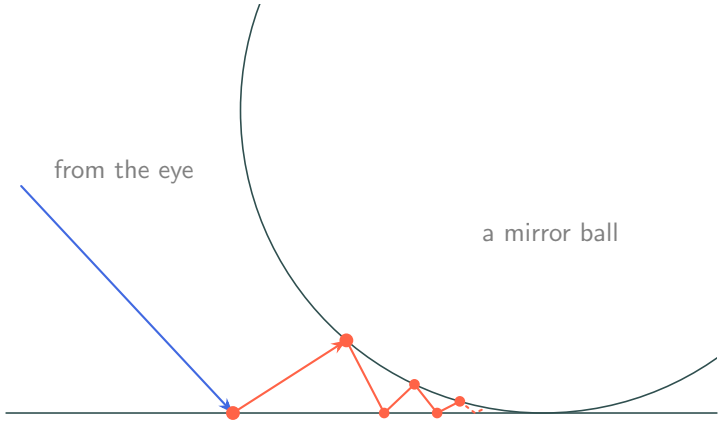
Between two mirrors, a ray keeps bouncing



a sketch, not to scale: a mirror ball on a mirror floor

- The ray meets the floor, then the ball

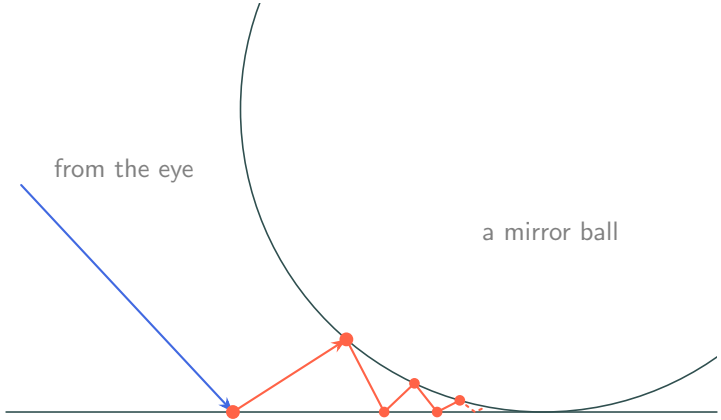
Between two mirrors, a ray keeps bouncing



a sketch, not to scale: a mirror ball on a mirror floor

- The ray meets the floor, then the ball
- Then floor, ball, floor, ...

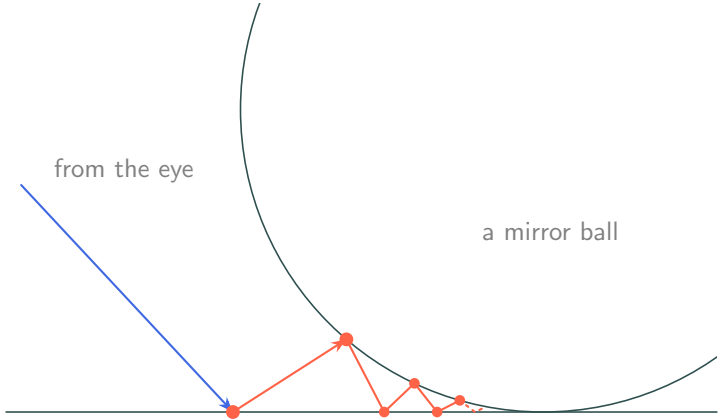
Between two mirrors, a ray keeps bouncing



a sketch, not to scale: a mirror ball on a mirror floor

- The ray meets the floor, then the ball
- Then floor, ball, floor, ...
- Every bounce: one more call

Between two mirrors, a ray keeps bouncing

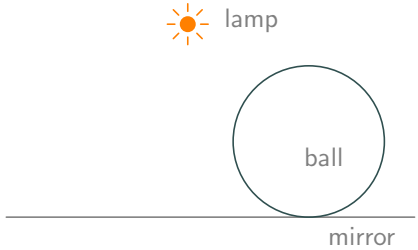


a sketch, not to scale: a mirror ball on a mirror floor

- The ray meets the floor, then the ball
- Then floor, ball, floor, ...
- Every bounce: one more call
- Fix: a maximum number of bounces

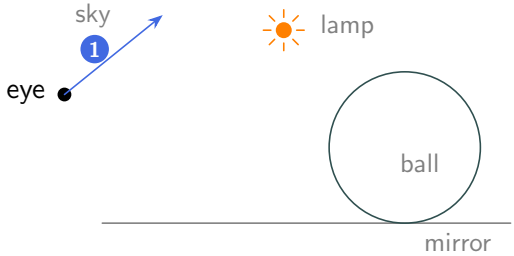
`trace` in outline: the sky, the lamps, the mirror

```
def trace(ray, depth):
    rec = scene.hit(ray)
```



trace in outline: the sky, the lamps, the mirror

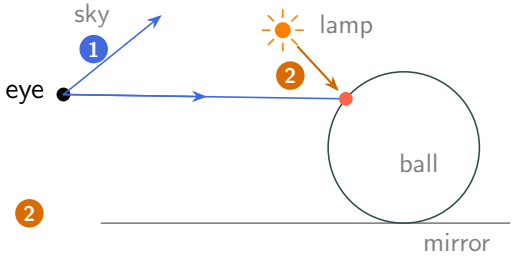
```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour    (1)
```



1. The ray hits nothing. Return the sky's colour.

`trace` in outline: the sky, the lamps, the mirror

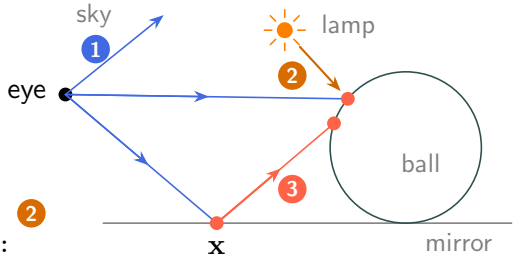
```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour          1
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up   2
```



1. The ray hits nothing. Return the sky's colour.
2. The surface is `Lambertian`. Add up the share of every lamp, as in lecture 16.

trace in outline: the sky, the lamps, the mirror

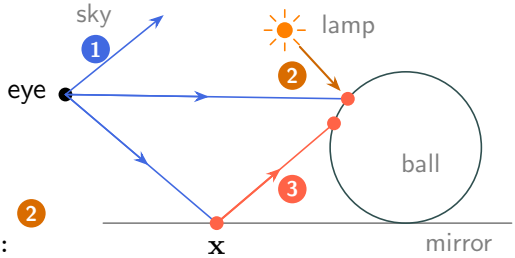
```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour          1
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up   2
    elif isinstance(material, Reflective):
        return the mirror's colour       3
```



1. The ray hits nothing. Return the sky's colour.
2. The surface is `Lambertian`. Add up the share of every lamp, as in lecture 16.
3. The surface is `Reflective`, a mirror. Return k_m times the colour of one more ray.

trace in outline: the sky, the lamps, the mirror

```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour          1
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up   2
    elif isinstance(material, Reflective):
        return the mirror's colour       3
    lecture 18: one more elif goes here, for glass
```

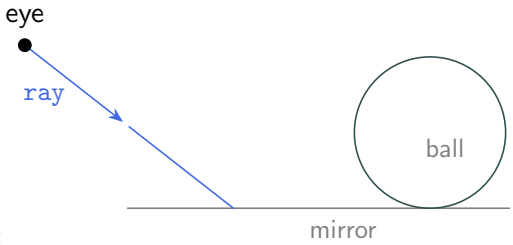


1. The ray hits nothing. Return the sky's colour.
2. The surface is `Lambertian`. Add up the share of every lamp, as in lecture 16.
3. The surface is `Reflective`, a mirror. Return k_m times the colour of one more ray.

The camera's call for a viewing ray is `trace(ray, 4)`. The 4 allows four bounces.

A mirror hit, in Python

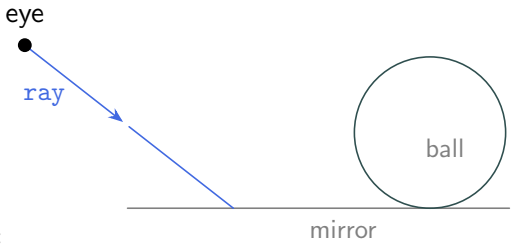
```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up
    elif isinstance(material, Reflective):
```



- Grey lines: the last slide's outline.

A mirror hit, in Python

```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up
    elif isinstance(material, Reflective):
        if depth == 0:
            return Vec3(0,0,0)
```



- Grey lines: the last slide's outline.
- At depth 0 a mirror hit is black.

A mirror hit, in Python

```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
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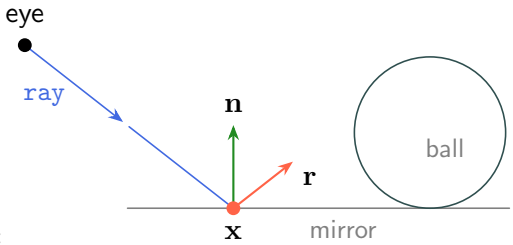
    if isinstance(material, Lambertian):
        return the lamps' shares, added up
    elif isinstance(material, Reflective):
        if depth == 0:
            return Vec3(0,0,0)
        x = ray.evaluate(rec.t)
        r = ray.d - 2 * ray.d.dot(rec.normal) * rec.normal
```



- Grey lines: the last slide's outline.
- At depth 0 a mirror hit is black.

A mirror hit, in Python

```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
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    if isinstance(material, Lambertian):
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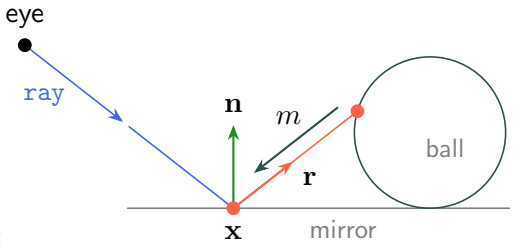
        if depth == 0:
            return Vec3(0,0,0)
        x = ray.evaluate(rec.t)
        r = ray.d - 2 * ray.d.dot(rec.normal) * rec.normal
        m = trace(Ray(x, r), depth - 1)
```

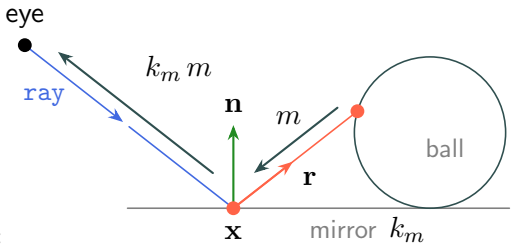


- Grey lines: the last slide's outline.
- At depth 0 a mirror hit is black.

- The function calls itself, with one bounce fewer.

A mirror hit, in Python

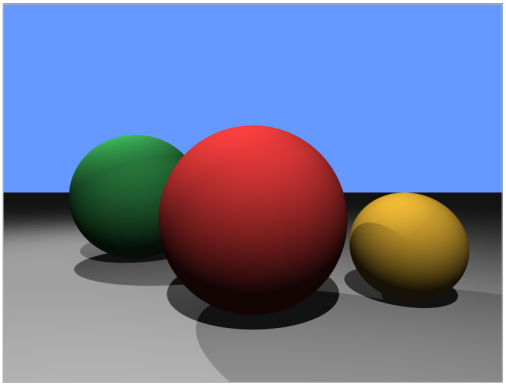
```
def trace(ray, depth):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        return the sky's colour
    material = rec.surface.material
    if isinstance(material, Lambertian):
        return the lamps' shares, added up
    elif isinstance(material, Reflective):
        if depth == 0:
            return Vec3(0,0,0)
        x = ray.evaluate(rec.t)
        r = ray.d - 2 * ray.d.dot(rec.normal) * rec.normal
        m = trace(Ray(x, r), depth - 1)
        return material.albedo * m
```



- Grey lines: the last slide's outline.
- At depth 0 a mirror hit is black.

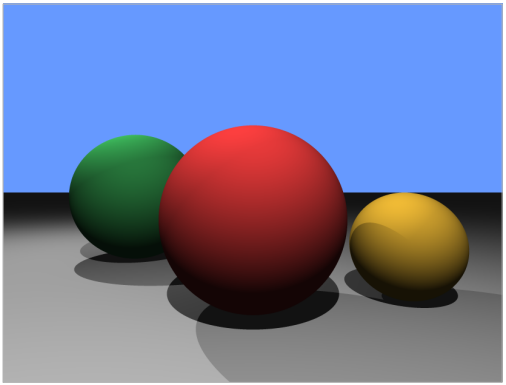
- The function calls itself, with one bounce fewer.
- A `Reflective` material's `albedo` is the colour k_m. No lamp is asked.

Mirrors show the scene inside the scene

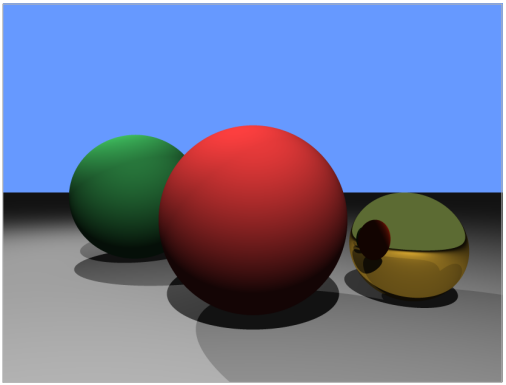


shadows only

Mirrors show the scene inside the scene



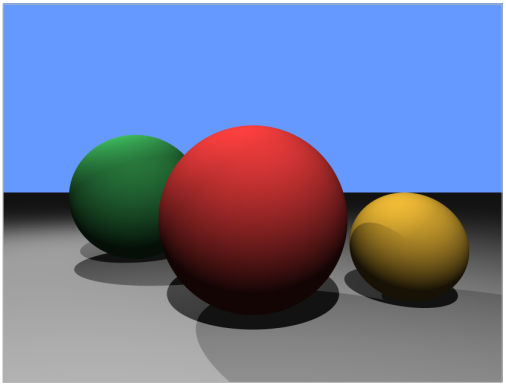
shadows only



the gold ball as a mirror

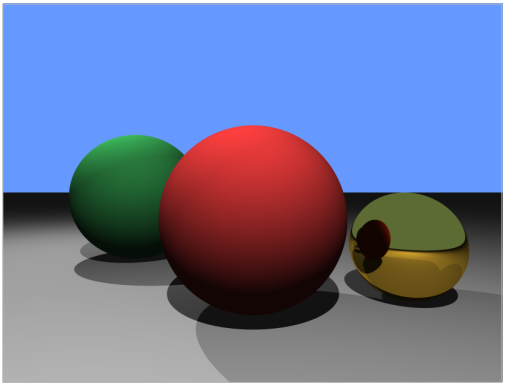
- The gold ball is a mirror now.

Mirrors show the scene inside the scene



shadows only

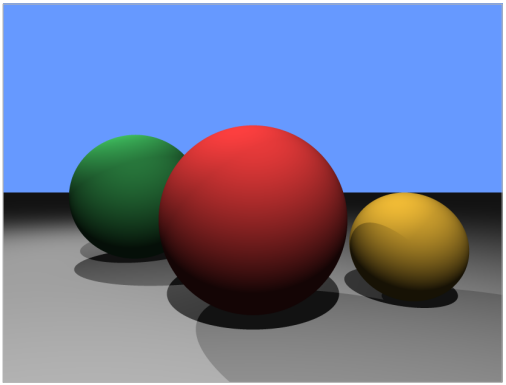
- The gold ball is a mirror now.



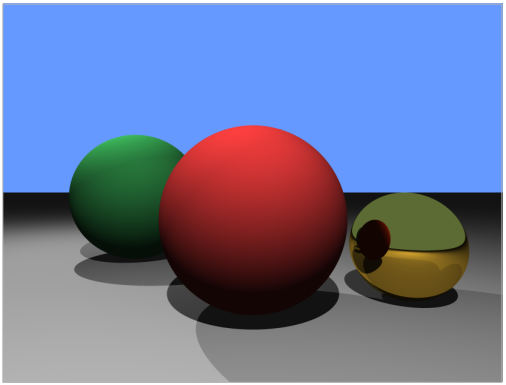
the gold ball as a mirror

- It shows the scene, tinted gold.

Mirrors show the scene inside the scene



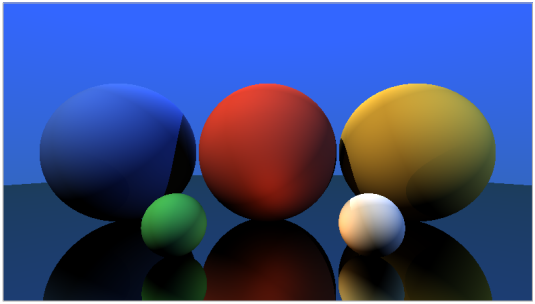
shadows only



the gold ball as a mirror

- The gold ball is a mirror now.
- It shows the scene, tinted gold.
- The floor and the other balls are not mirrors, so they look as before.

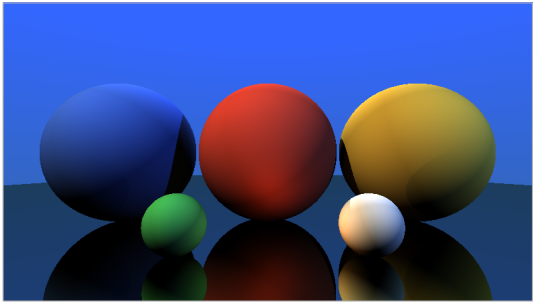
A mirror hit asks no lamp: two pictures



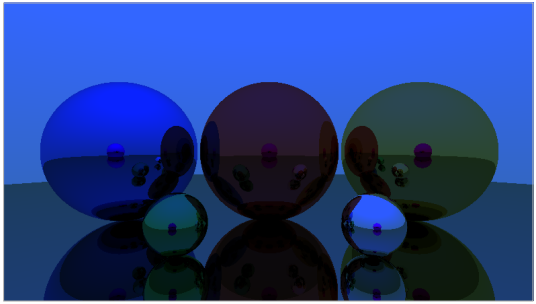
chalk balls on a mirror floor

- Left: the lamps light the balls. No shadow falls on the mirror floor.

A mirror hit asks no lamp: two pictures



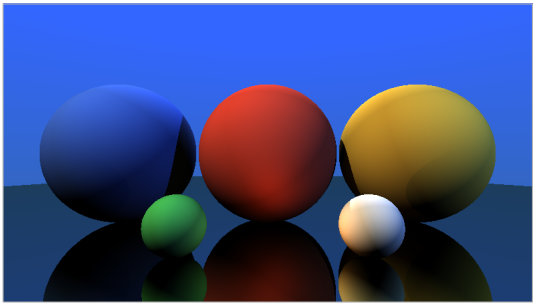
chalk balls on a mirror floor



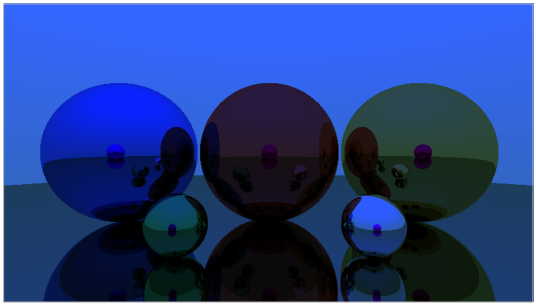
no lamps, every surface a mirror

- Left: the lamps light the balls. No shadow falls on the mirror floor.
- Right: the scene has no lamps, and the picture is not black.

A mirror hit asks no lamp: two pictures



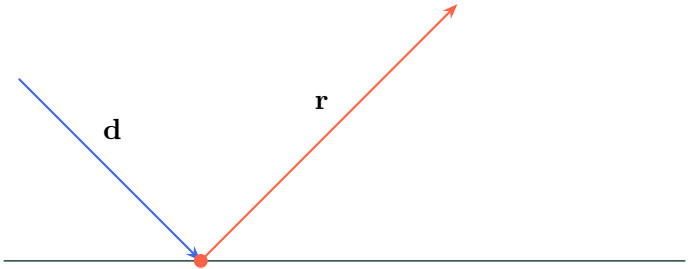
chalk balls on a mirror floor



no lamps, every surface a mirror

- Left: the lamps light the balls. No shadow falls on the mirror floor.
- Right: the scene has no lamps, and the picture is not black.
- A mirror's colour comes from the ray it sends on, never from a lamp.

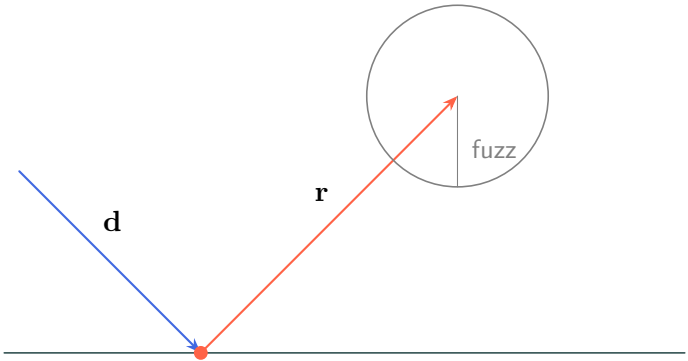
Brushed metal: nudge $\mathbf{r}$ by a random arrow



- Make $\mathbf{r}$ one unit long

Ray Tracing in One Weekend, chapter 10

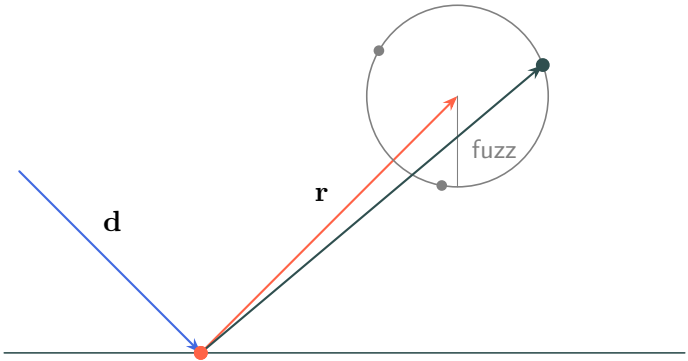
Brushed metal: nudge $\mathbf{r}$ by a random arrow



- Make $\mathbf{r}$ one unit long
- A small sphere at its tip

Ray Tracing in One Weekend, chapter 10

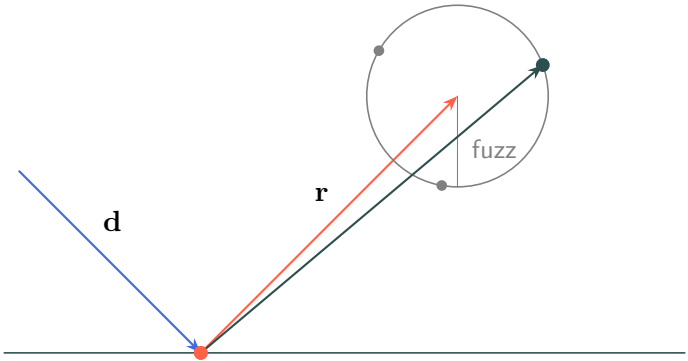
Brushed metal: nudge $\mathbf{r}$ by a random arrow



- Make $\mathbf{r}$ one unit long
- A small sphere at its tip
- Aim at a random point on it

Ray Tracing in One Weekend, chapter 10

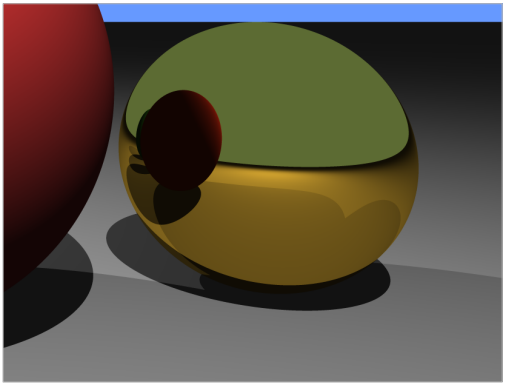
Brushed metal: nudge $\mathbf{r}$ by a random arrow



- Make $\mathbf{r}$ one unit long
- A small sphere at its tip
- Aim at a random point on it
- Fuzz 0: a perfect mirror

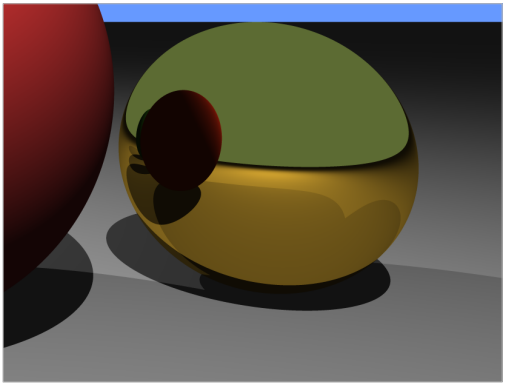
Ray Tracing in One Weekend, chapter 10

Brushed metal: the reflections blur

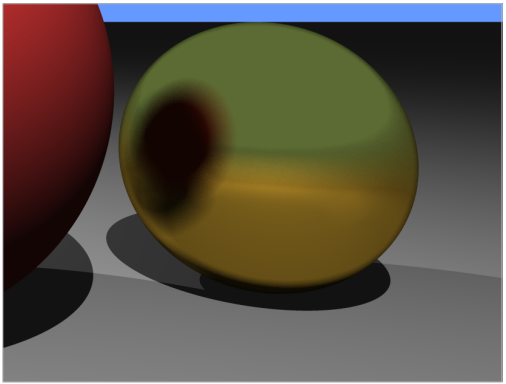


the gold ball, close up: fuzz 0

Brushed metal: the reflections blur



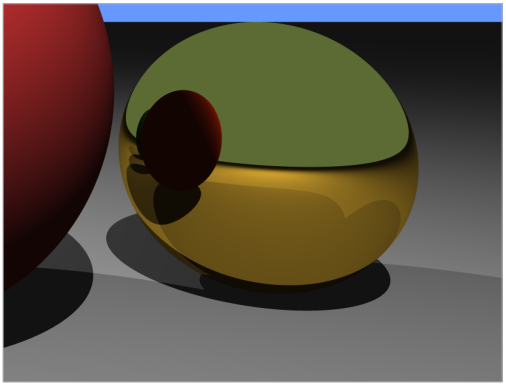
the gold ball, close up: fuzz 0



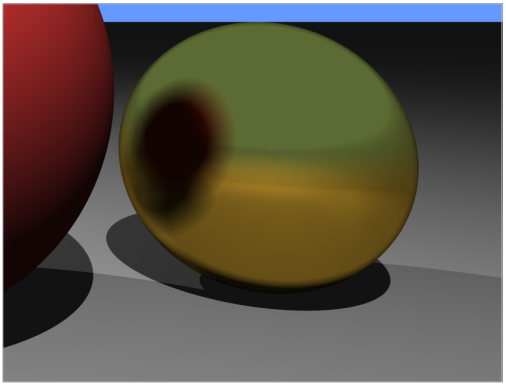
the same ball: fuzz 0.25

- The red ball's image blurs

Brushed metal: the reflections blur



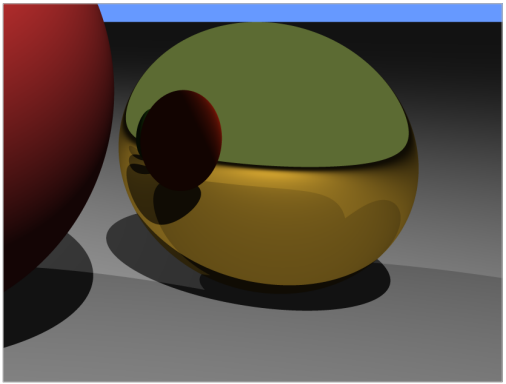
the gold ball, close up: fuzz 0



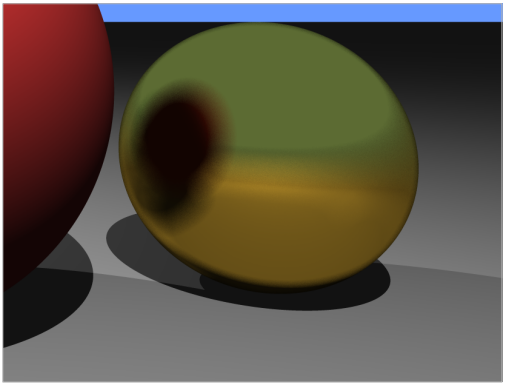
the same ball: fuzz 0.25

- The red ball's image blurs
- The ball's outline stays sharp

Brushed metal: the reflections blur



the gold ball, close up: fuzz 0



the same ball: fuzz 0.25

- The red ball's image blurs
- The blur: many random rays, averaged
- The ball's outline stays sharp

Recap: the mirror direction and a mirror's colour

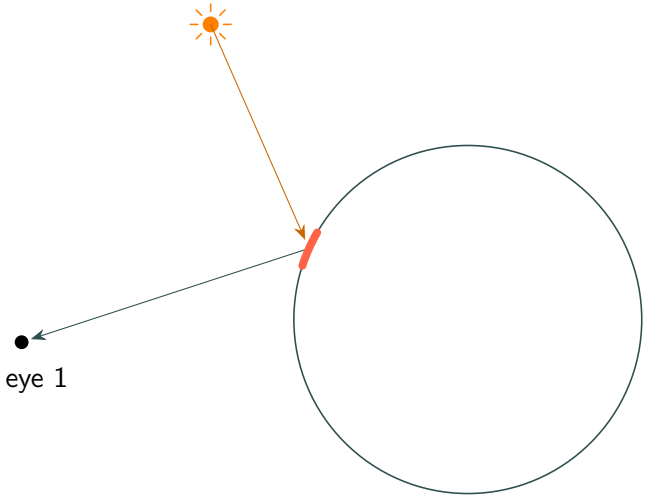
$$\mathbf{r} = \mathbf{d} - 2\,(\mathbf{d} \cdot \mathbf{n})\,\mathbf{n}$$

$$c = k_m \ \mathsf{trace}\big(\mathsf{Ray}(\mathbf{x}, \mathbf{r})\big)$$

Shiny surfaces

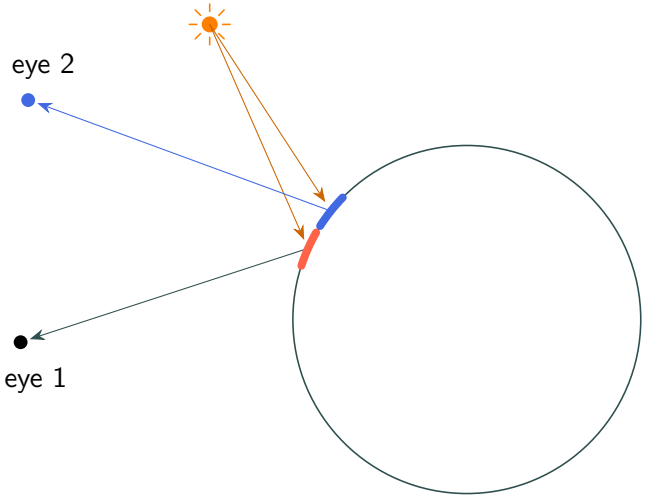
The bright spot where the lamp is almost mirrored

A shiny spot moves when the eye moves



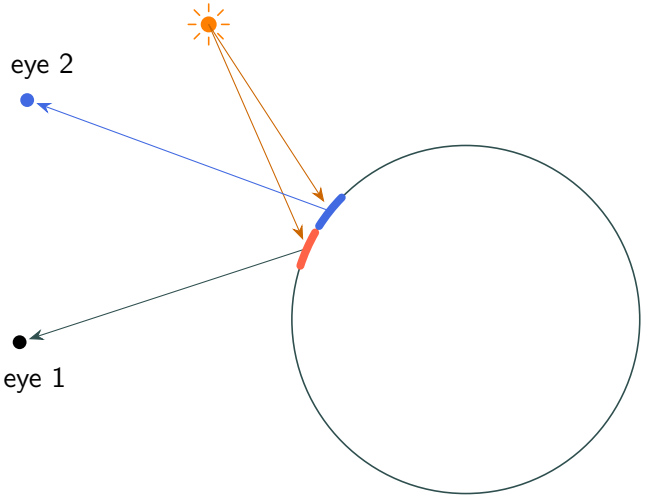
- Eye 1 sees the spot here

A shiny spot moves when the eye moves



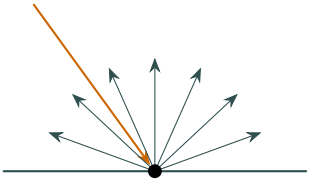
- Eye 1 sees the spot here
- Eye 2 sees it higher up

A shiny spot moves when the eye moves



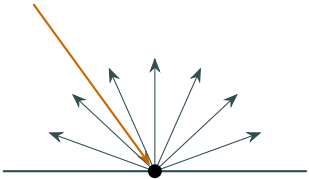
- Eye 1 sees the spot here
- Eye 2 sees it higher up
- Shiny: the colour depends on the eye
- Chalk: the same from everywhere

Three ways a surface sends light back

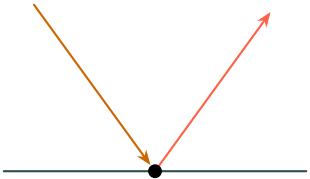


- Diffuse: all directions (lecture 16)

Three ways a surface sends light back

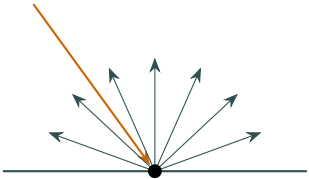


- Diffuse: all directions (lecture 16)

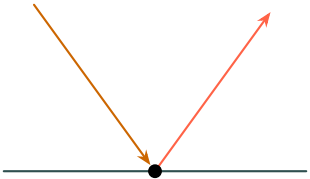


- Mirror: exactly one direction

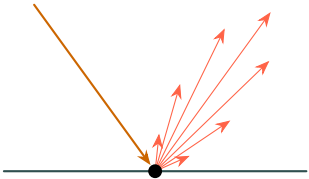
Three ways a surface sends light back



- Diffuse: all directions (lecture 16)

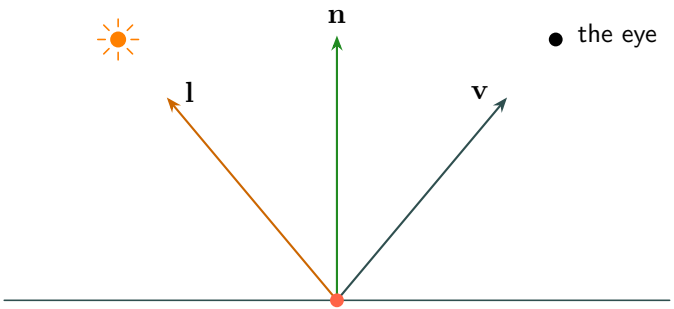


- Mirror: exactly one direction



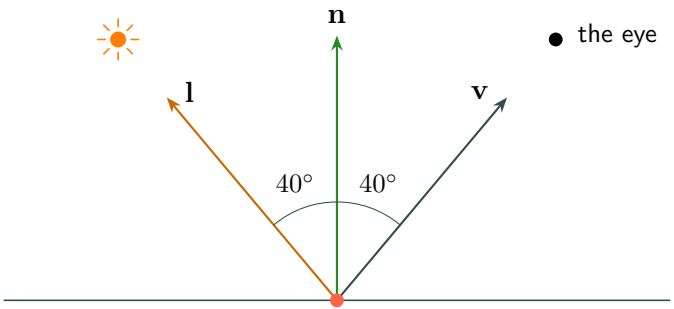
- Glossy: a spread around it

Brightest when $\mathbf{l}$ and $\mathbf{v}$ mirror each other



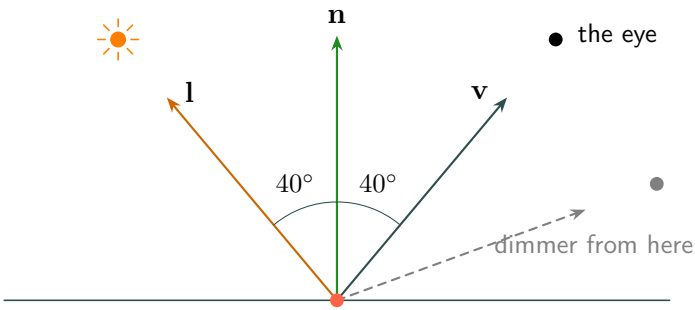
- $\mathbf{l}$: to the lamp; $\mathbf{v}$: to the eye

Brightest when $\mathbf{l}$ and $\mathbf{v}$ mirror each other



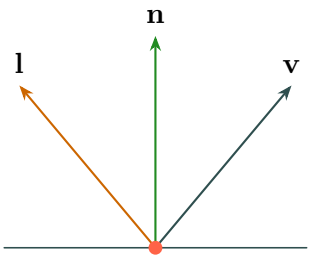
- $\mathbf{l}$: to the lamp; $\mathbf{v}$: to the eye
- Equal angles: the brightest view

Brightest when $\mathbf{l}$ and $\mathbf{v}$ mirror each other



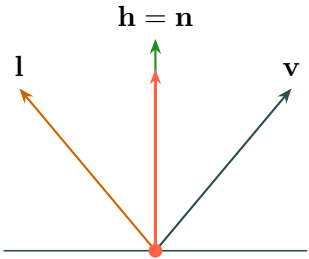
- $\mathbf{l}$: to the lamp; $\mathbf{v}$: to the eye
- Equal angles: the brightest view
- Move away: it fades smoothly

The half vector $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$



mirror configuration

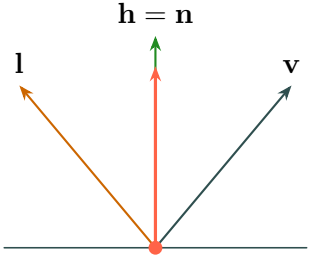
The half vector $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$



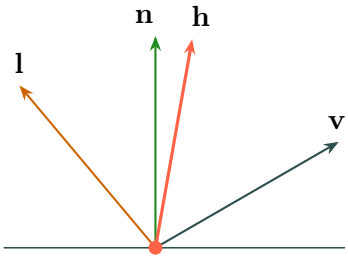
mirror configuration

- $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$
- Mirror configuration: $\mathbf{h}$ lies on $\mathbf{n}$

The half vector $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$



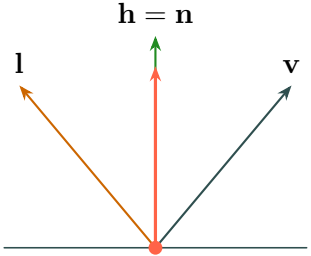
mirror configuration



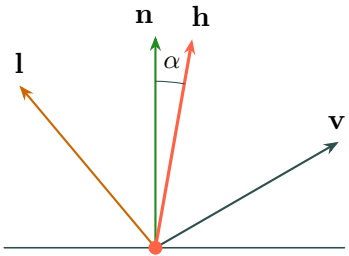
the eye moved

- $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$
- Mirror configuration: $\mathbf{h}$ lies on $\mathbf{n}$

The half vector $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$



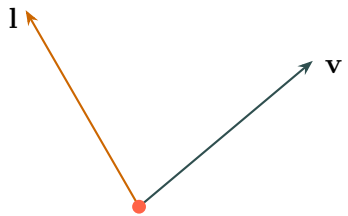
mirror configuration



the eye moved

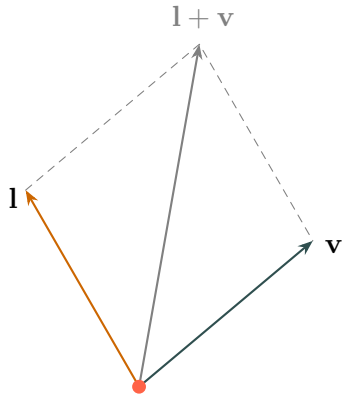
- $\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$
- Mirror configuration: $\mathbf{h}$ lies on $\mathbf{n}$
- α small: bright; α large: dim

Computing $\mathbf{h}$: add, then make length 1



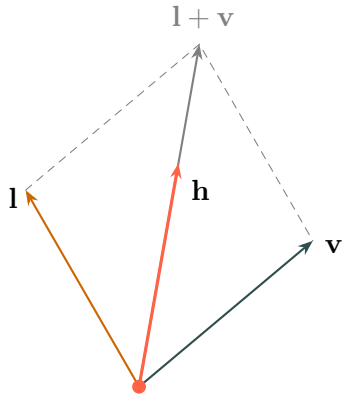
- $\mathbf{l}$ and $\mathbf{v}$: the same length

Computing $\mathbf{h}$: add, then make length 1



- $\mathbf{l}$ and $\mathbf{v}$: the same length
- So their sum halves the angle

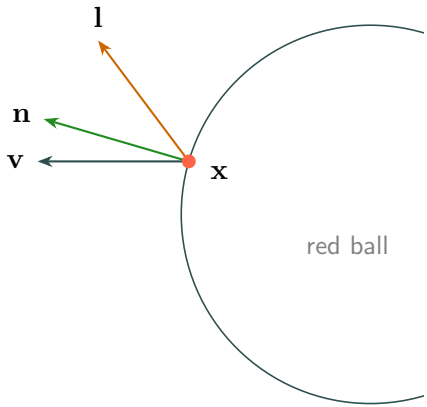
Computing $\mathbf{h}$: add, then make length 1



$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

- $\mathbf{l}$ and $\mathbf{v}$: the same length
- So their sum halves the angle
- Divide by its length: $\mathbf{h}$

Worked: $\mathbf{h}$ at our hit

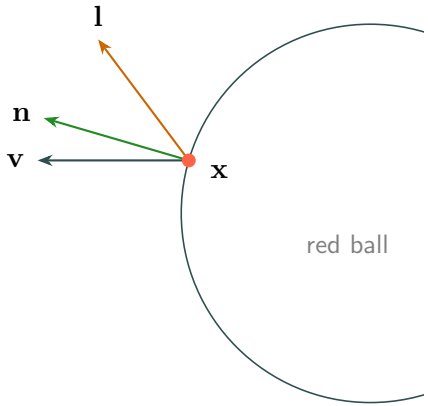


- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$

$\mathbf{n} = (0, 0.28, 0.96)$

Worked: $\mathbf{h}$ at our hit



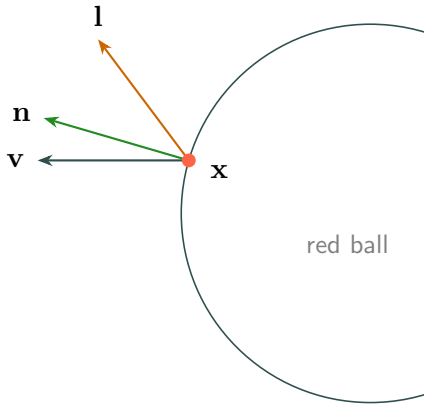
- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$

$\mathbf{n} = (0, 0.28, 0.96)$

$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$

Worked: $\mathbf{h}$ at our hit



- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

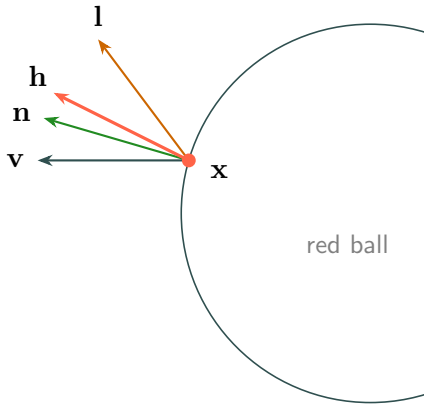
$$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$$

$$\mathbf{n} = (0, 0.28, 0.96)$$

$$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$$

$$\|\mathbf{l} + \mathbf{v}\| = \sqrt{0.64 + 2.56} = \sqrt{3.2} = 1.789$$

Worked: $\mathbf{h}$ at our hit



- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$$

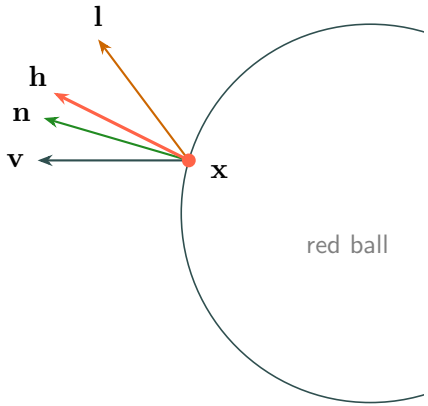
$$\mathbf{n} = (0, 0.28, 0.96)$$

$$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$$

$$\|\mathbf{l} + \mathbf{v}\| = \sqrt{0.64 + 2.56} = \sqrt{3.2} = 1.789$$

$$\mathbf{h} = (0, 0.447, 0.894)$$

Worked: $\mathbf{h}$ at our hit



- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$$

$$\mathbf{n} = (0, 0.28, 0.96)$$

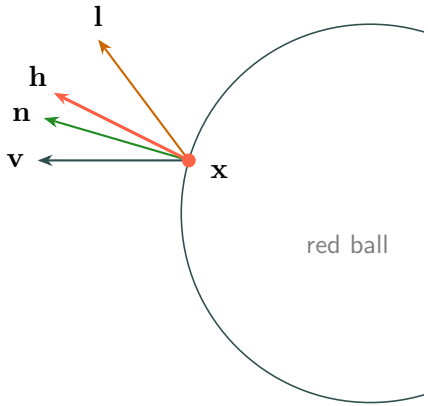
$$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$$

$$\|\mathbf{l} + \mathbf{v}\| = \sqrt{0.64 + 2.56} = \sqrt{3.2} = 1.789$$

$$\mathbf{h} = (0, 0.447, 0.894)$$

$$\mathbf{n} \cdot \mathbf{h} = 0.28 \times 0.447 + 0.96 \times 0.894$$

Worked: $\mathbf{h}$ at our hit



- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$$

$$\mathbf{n} = (0, 0.28, 0.96)$$

$$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$$

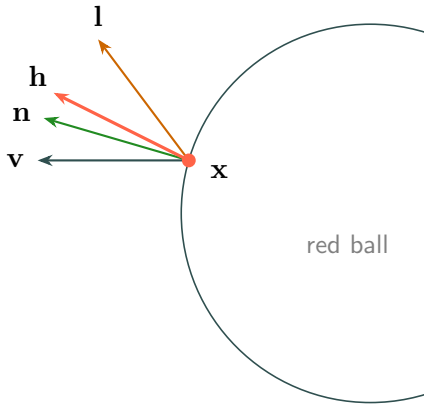
$$\|\mathbf{l} + \mathbf{v}\| = \sqrt{0.64 + 2.56} = \sqrt{3.2} = 1.789$$

$$\mathbf{h} = (0, 0.447, 0.894)$$

$$\mathbf{n} \cdot \mathbf{h} = 0.28 \times 0.447 + 0.96 \times 0.894$$

$$\mathbf{n} \cdot \mathbf{h} = 0.125 + 0.859 = 0.984$$

Worked: $\mathbf{h}$ at our hit



- Try first: $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$

$$\mathbf{l} = (0, 0.8, 0.6), \quad \mathbf{v} = (0, 0, 1)$$

$$\mathbf{n} = (0, 0.28, 0.96)$$

$$\mathbf{l} + \mathbf{v} = (0, 0.8, 1.6)$$

$$\|\mathbf{l} + \mathbf{v}\| = \sqrt{0.64 + 2.56} = \sqrt{3.2} = 1.789$$

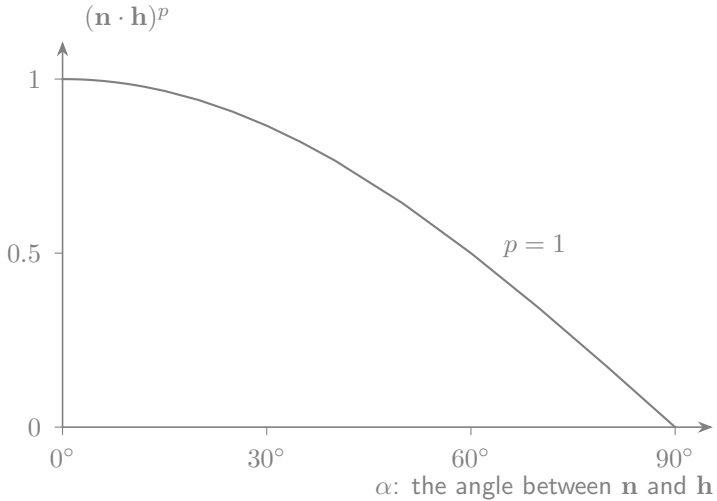
$$\mathbf{h} = (0, 0.447, 0.894)$$

$$\mathbf{n} \cdot \mathbf{h} = 0.28 \times 0.447 + 0.96 \times 0.894$$

$$\mathbf{n} \cdot \mathbf{h} = 0.125 + 0.859 = 0.984$$

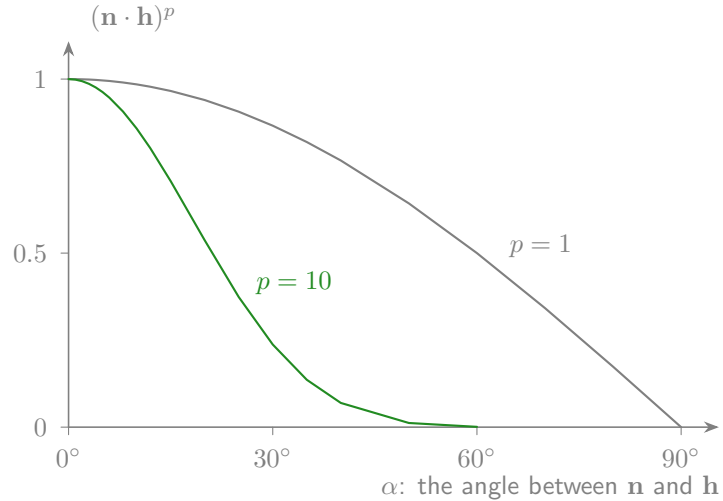
- $\alpha = 10.3°$: close to the peak

A power makes the spot small



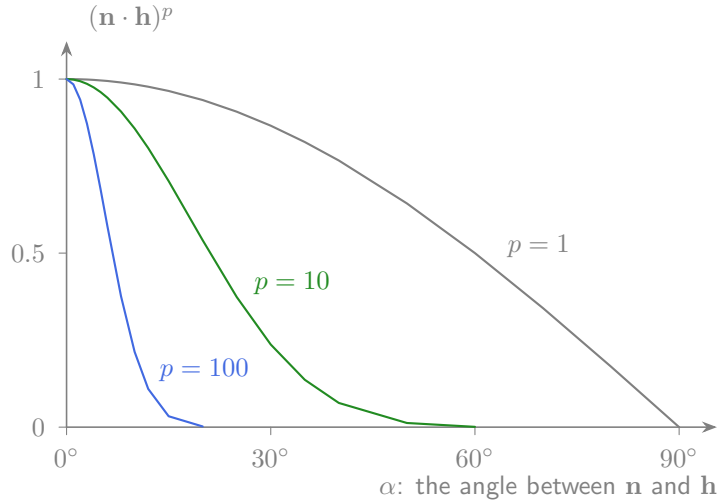
- $\mathbf{n} \cdot \mathbf{h}$: 1 when $\mathbf{h}$ lies on $\mathbf{n}$

A power makes the spot small



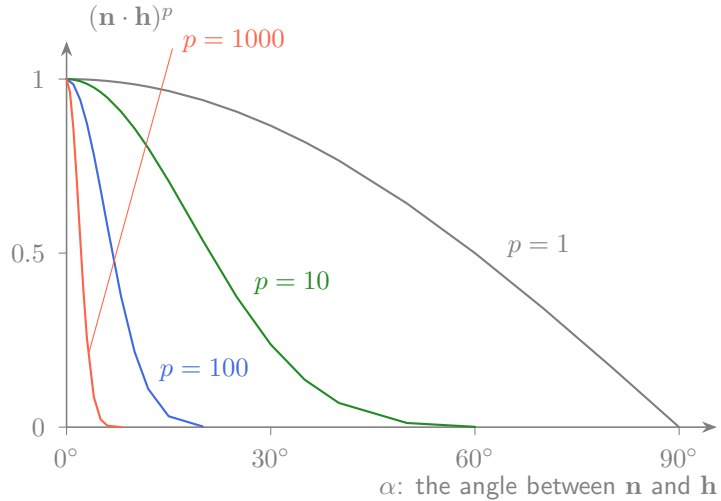
- $\mathbf{n} \cdot \mathbf{h}$: 1 when $\mathbf{h}$ lies on $\mathbf{n}$
- To the power p: it falls faster

A power makes the spot small



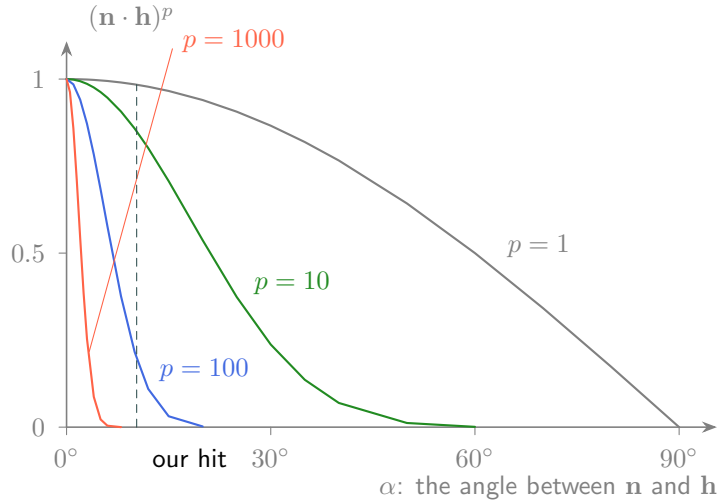
- $\mathbf{n} \cdot \mathbf{h}$: 1 when $\mathbf{h}$ lies on $\mathbf{n}$
- To the power p: it falls faster

A power makes the spot small



- $\mathbf{n} \cdot \mathbf{h}$: 1 when $\mathbf{h}$ lies on $\mathbf{n}$
- To the power p: it falls faster
- Bigger p: a smaller, sharper spot
- p: the Phong exponent

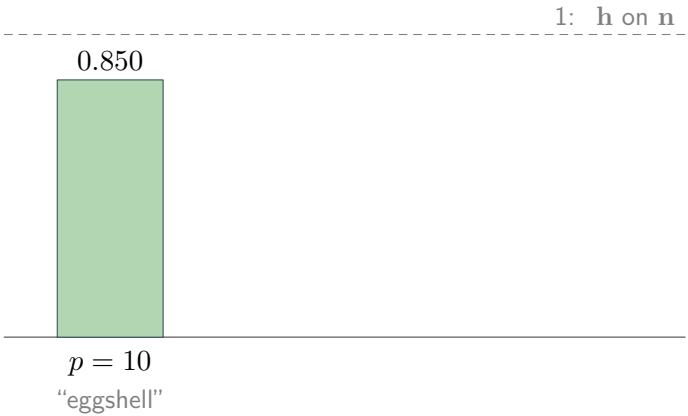
A power makes the spot small



- $\mathbf{n} \cdot \mathbf{h}$: 1 when $\mathbf{h}$ lies on $\mathbf{n}$
- To the power p: it falls faster
- Bigger p: a smaller, sharper spot
- p: the Phong exponent

Worked: three exponents at our hit

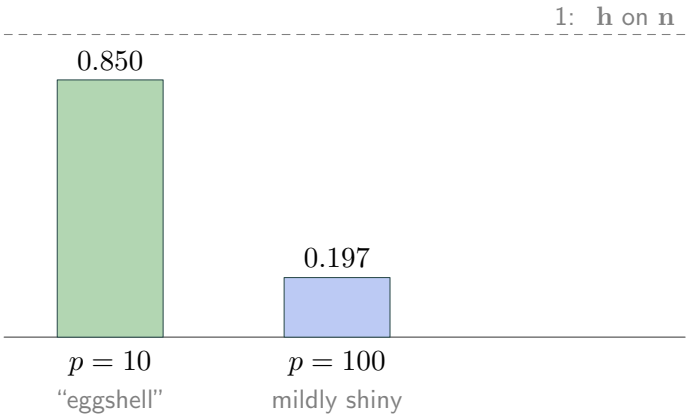
our hit: $\mathbf{n} \cdot \mathbf{h} = 0.984$



- Our hit: 10 degrees off the peak

Worked: three exponents at our hit

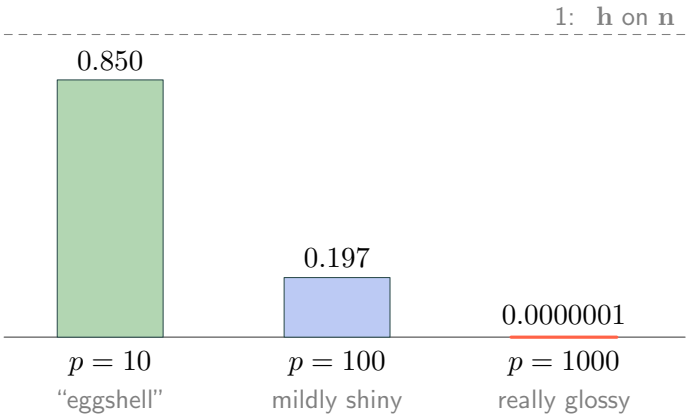
our hit: $\mathbf{n} \cdot \mathbf{h} = 0.984$



- Our hit: 10 degrees off the peak
- Bigger p: our hit leaves the spot

Worked: three exponents at our hit

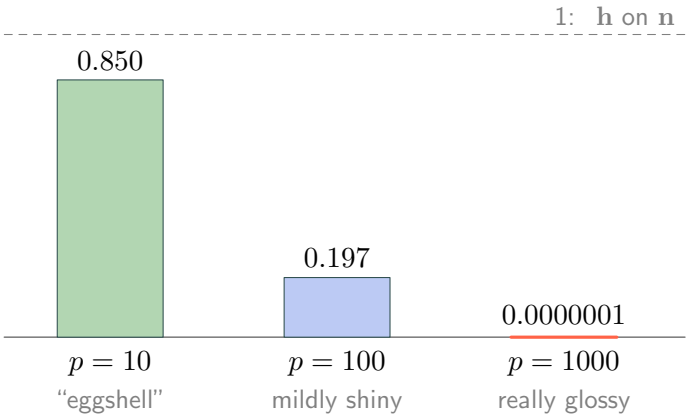
our hit: $\mathbf{n} \cdot \mathbf{h} = 0.984$



- Our hit: 10 degrees off the peak
- Bigger p: our hit leaves the spot

Worked: three exponents at our hit

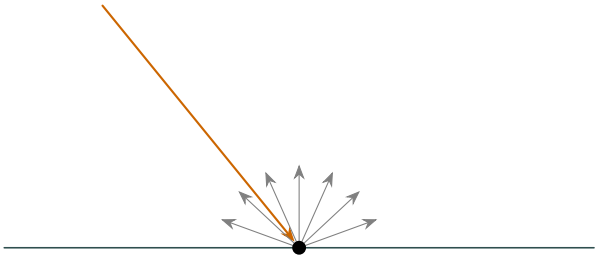
our hit: $\mathbf{n} \cdot \mathbf{h} = 0.984$



- Our hit: 10 degrees off the peak
- Bigger p: our hit leaves the spot
- $p = 10{,}000$: nearly a mirror

Blinn–Phong: add a shiny term to Lambert

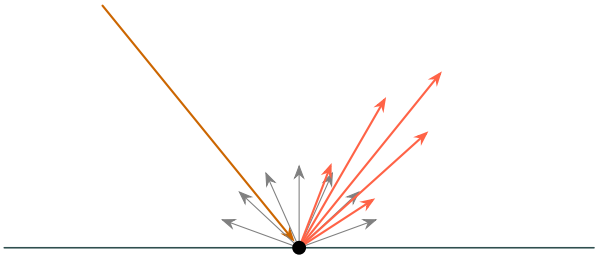
$$L_r = \frac{R}{\pi} E$$



- Diffuse part: lecture 16's fan

Blinn–Phong: add a shiny term to Lambert

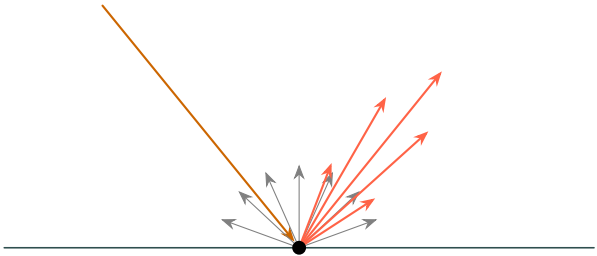
$$L_r = \left(\frac{R}{\pi} + R\,\max(0,\ \mathbf{n}\cdot\mathbf{h})^p\right) E$$



- Diffuse part: lecture 16's fan
- Specular part: a narrow spread

Blinn–Phong: add a shiny term to Lambert

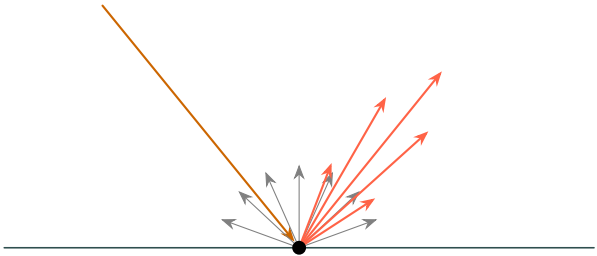
$$L_r = \left(\frac{R}{\pi} + R\,\max(0,\ \mathbf{n}\cdot\mathbf{h})^p\right) E$$



- Diffuse part: lecture 16's fan
- Specular part: a narrow spread
- R again: it scales the spot too

Blinn–Phong: add a shiny term to Lambert

$$L_r = \left(\frac{R}{\pi} + R\,\max(0,\ \mathbf{n}\cdot\mathbf{h})^p\right) E$$



- Diffuse part: lecture 16's fan
- Specular part: a narrow spread
- R again: it scales the spot too
- So the spot has the ball's colour

Worked: our hit, with a highlight

- Try first: $E = 0.8\pi$, $R = (0.8,\ 0.2,\ 0.2)$, $(\mathbf{n} \cdot \mathbf{h})^{100} = 0.197$



$(0.64,\ 0.16,\ 0.16)$

diffuse: $(R/\pi)\, E$

Worked: our hit, with a highlight

- Try first: $E = 0.8\pi$, $R = (0.8,\ 0.2,\ 0.2)$, $(\mathbf{n} \cdot \mathbf{h})^{100} = 0.197$

$$R\,(\mathbf{n} \cdot \mathbf{h})^{p}\,E = R \times 0.197 \times 0.8\pi = 0.495\,R$$



$(0.64,\ 0.16,\ 0.16)$

diffuse: $(R/\pi)\,E$

Worked: our hit, with a highlight

- Try first: $E = 0.8\pi$, $R = (0.8,\ 0.2,\ 0.2)$, $(\mathbf{n} \cdot \mathbf{h})^{100} = 0.197$

  $R\,(\mathbf{n} \cdot \mathbf{h})^{p}\,E = R \times 0.197 \times 0.8\pi = 0.495\,R$



$(0.64,\ 0.16,\ 0.16)$ $+$



$(0.40,\ 0.10,\ 0.10)$

diffuse: $(R/\pi)\,E$ specular

- The specular share is red too

Worked: our hit, with a highlight

- Try first: $E = 0.8\pi$, $R = (0.8,\ 0.2,\ 0.2)$, $(\mathbf{n} \cdot \mathbf{h})^{100} = 0.197$

  $R\,(\mathbf{n} \cdot \mathbf{h})^p\,E = R \times 0.197 \times 0.8\pi = 0.495\,R$



$(0.64,\ 0.16,\ 0.16)$ +



$(0.40,\ 0.10,\ 0.10)$ =



$(1.04,\ 0.26,\ 0.26)$

diffuse: $(R/\pi)\,E$ | specular | the lamp's share

- The specular share is red too
- The red gets brighter, and stays red

Worked: our hit, with a highlight

- Try first: $E = 0.8\pi$, $R = (0.8,\ 0.2,\ 0.2)$, $(\mathbf{n} \cdot \mathbf{h})^{100} = 0.197$

  $R\,(\mathbf{n} \cdot \mathbf{h})^p\,E = R \times 0.197 \times 0.8\pi = 0.495\,R$



$(0.64,\ 0.16,\ 0.16)$ +



$(0.40,\ 0.10,\ 0.10)$ =



$(1.04,\ 0.26,\ 0.26)$

diffuse: $(R/\pi)\,E$ specular the lamp's share

the pixel, with the ambient share $(0.08, 0.02, 0.02)$:



$(1.12,\ 0.28,\ 0.28)$

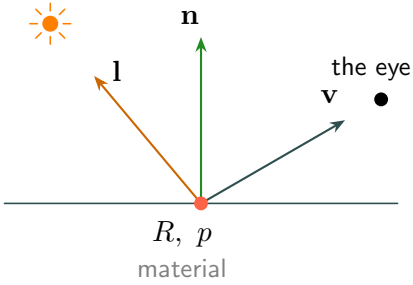
stored: red clamped to 1

- The specular share is red too
- The red gets brighter, and stays red

In code: the material now uses $\mathbf{v}$

```
class Lambertian(Material):
    def evaluate(self, l, v, n):
        return self.albedo / math.pi

class BlinnPhong(Material):
    def evaluate(self, l, v, n):
```

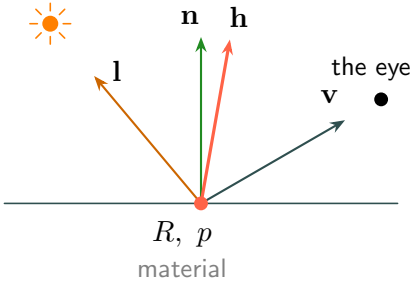


- Grey lines: lecture 16, $\mathbf{v}$ not used

In code: the material now uses $\mathbf{v}$

```
class Lambertian(Material):
    def evaluate(self, l, v, n):
        return self.albedo / math.pi

class BlinnPhong(Material):
    def evaluate(self, l, v, n):
        h = (l + v) / (l + v).length()
```



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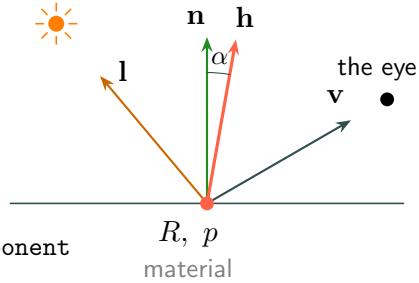

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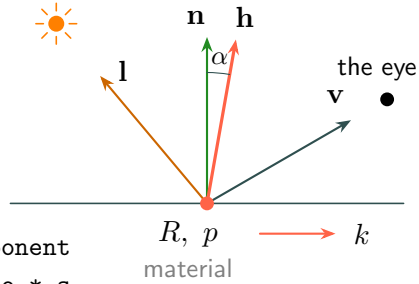
        s = max(0, n.dot(h)) ** self.specular_exponent
        return self.albedo / math.pi + self.albedo * s
```



- Grey lines: lecture 16, $\mathbf{v}$ not used
- k now depends on $\mathbf{l}$ and $\mathbf{v}$

In code: the material now uses $\mathbf{v}$

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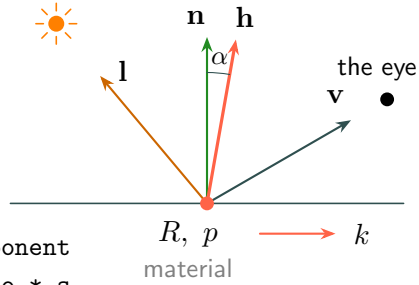

- Grey lines: lecture 16, $\mathbf{v}$ not used
- Its name: the BRDF
- k now depends on $\mathbf{l}$ and $\mathbf{v}$
- The light's code does not change

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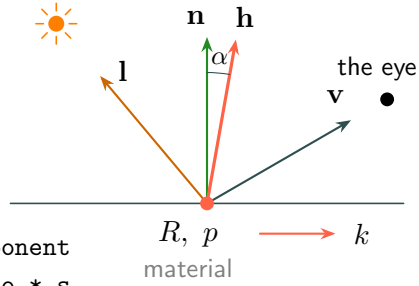
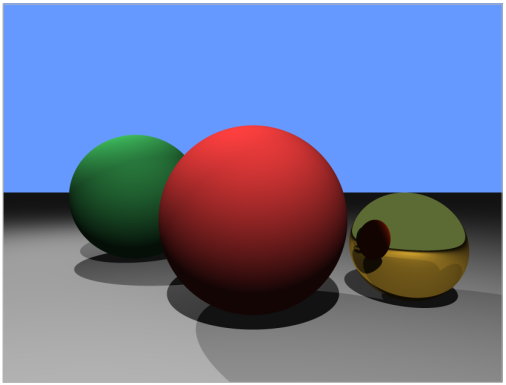


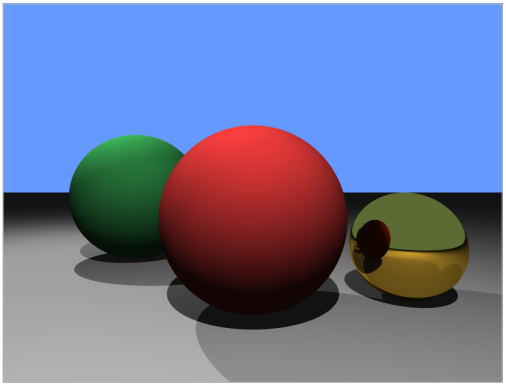
- Grey lines: lecture 16, $\mathbf{v}$ not used
- Its name: the BRDF
- k now depends on $\mathbf{l}$ and $\mathbf{v}$
- The light's code does not change
- `trace` must add up the lamps for this kind too. Its first test becomes:
  `if isinstance(material, (Lambertian, BlinnPhong)):`

Highlights: small on red, wide on green



part two's picture

Highlights: small on red, wide on green



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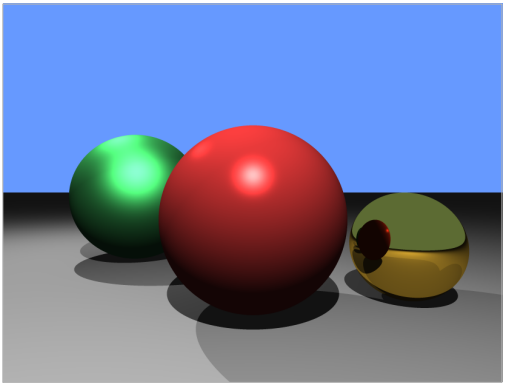
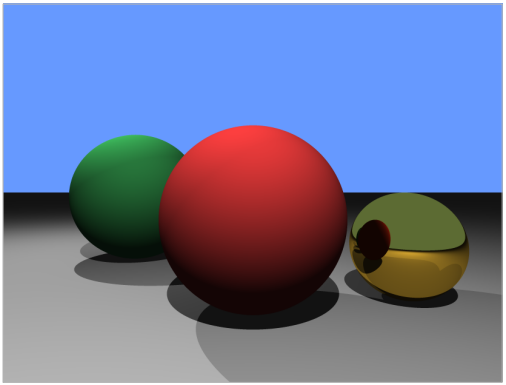


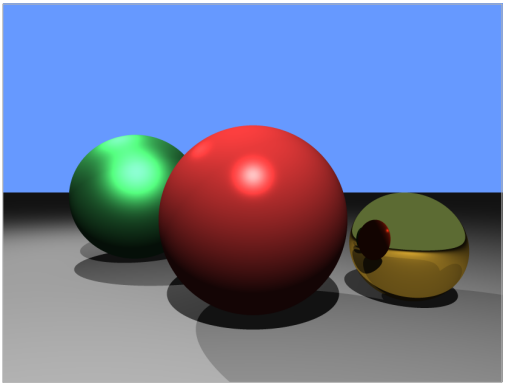
with the specular term

- Red: $p = 100$, a small spot

Highlights: small on red, wide on green



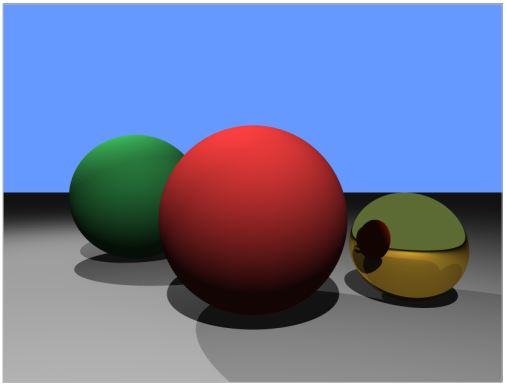
part two's picture



with the specular term

- Red: $p = 100$, a small spot
- Green: $p = 10$, a wide glow

Highlights: small on red, wide on green



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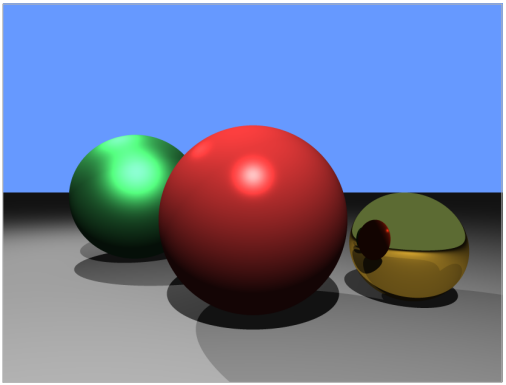
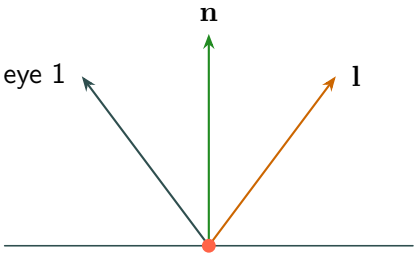


with the specular term

- Red: $p = 100$, a small spot
- Green: $p = 10$, a wide glow
- The gold ball gets no spot of its own: a mirror hit asks no lamp.

Quick check 2: find $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$



$\mathbf{n} = (0, 1, 0), \quad \mathbf{l} = (0.6, 0.8, 0)$

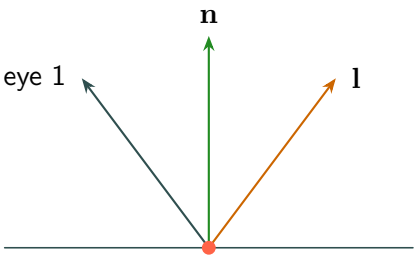
1. Eye 1: $\mathbf{v} = (-0.6, 0.8, 0)$

---

2. Eye 2 sits at the lamp: $\mathbf{v} = \mathbf{l}$

- For each eye: $\mathbf{h}$ and $\mathbf{n} \cdot \mathbf{h}$

Quick check 2: find $\mathbf{h}$, then $\mathbf{n} \cdot \mathbf{h}$



$\mathbf{n} = (0, 1, 0), \quad \mathbf{l} = (0.6, 0.8, 0)$

1. Eye 1: $\mathbf{v} = (-0.6, 0.8, 0)$

   $\mathbf{h} = (0, 1, 0), \ \mathbf{n} \cdot \mathbf{h} = 1$: the peak

2. Eye 2 sits at the lamp: $\mathbf{v} = \mathbf{l}$

   $\mathbf{h} = \mathbf{l}, \ \mathbf{n} \cdot \mathbf{h} = 0.8; \ 0.8^{10} = 0.11$

- For each eye: $\mathbf{h}$ and $\mathbf{n} \cdot \mathbf{h}$
- Nearly 1 is not enough

Recap: the Blinn–Phong colour

$$L_r = \left(\frac{R}{\pi} + R\,\max(0,\ \mathbf{n}\cdot\mathbf{h})^p\right) E \qquad \mathbf{h} = \frac{\mathbf{l}+\mathbf{v}}{\|\mathbf{l}+\mathbf{v}\|}$$

E: lecture 16's irradiance, or 0 in shadow

Summary

1 Shadow: is the lamp blocked?

$\mathbf{x} + t\,\mathbf{l}$, a hit with t in $[\epsilon,\ r]$

one ray for each lamp; never start at 0

2 The mirror direction

$$\mathbf{r} = \mathbf{d} - 2\,(\mathbf{d} \cdot \mathbf{n})\,\mathbf{n}$$

$\mathbf{d}$ points in; $\mathbf{n}$ has length 1

3 A mirror's colour

$c = k_m$ trace(Ray$(\mathbf{x}, \mathbf{r})$)

a mirror hit adds up no lamps

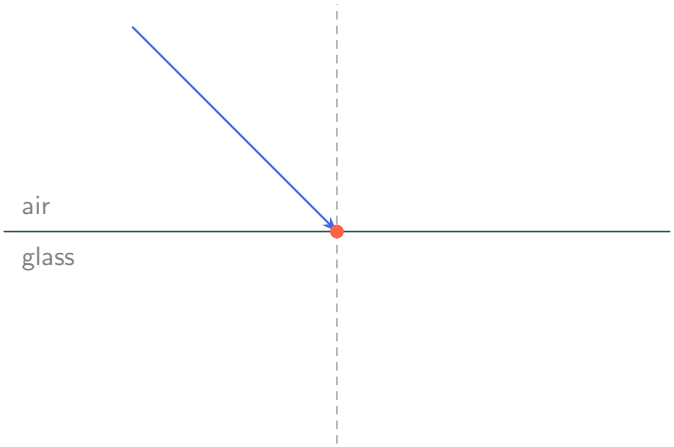
a maximum depth ends the bouncing

4 The highlight

$$L_r = \left(\tfrac{R}{\pi} + R \max(0,\ \mathbf{n} \cdot \mathbf{h})^p\right) E$$

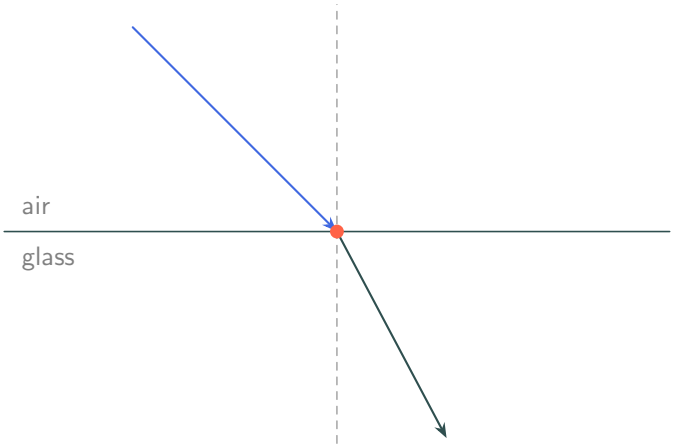
$\mathbf{h}$: halfway between $\mathbf{l}$ and $\mathbf{v}$

Next lecture: glass



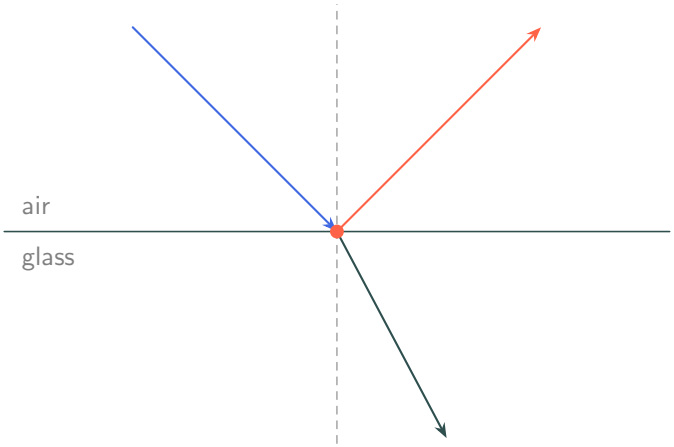
- A ray meets glass

Next lecture: glass



- A ray meets glass
- Part goes through, and bends

Next lecture: glass



- A ray meets glass
- Part goes through, and bends
- Part is mirrored, as today

Exit Ticket

Today's question *(2 minutes)*

A ray with direction $\mathbf{d} = (0.6, -0.8, 0)$ hits a mirror floor. The floor's normal is $\mathbf{n} = (0, 1, 0)$.

(a) Find $\mathbf{d} \cdot \mathbf{n}$ and the mirror direction $\mathbf{r}$.

(b) A lamp is 5 away from the hit. The shadow ray's only hit is at $t = 7$. Is the point lit, or in shadow?

CS 453 — Exit Ticket #17

Name: _____

Roll #: _____ Date: _____

Answer:

Hand it to me on your way out.
No name, no attendance.