Lecture 16: Point Lights and Lambertian Shading

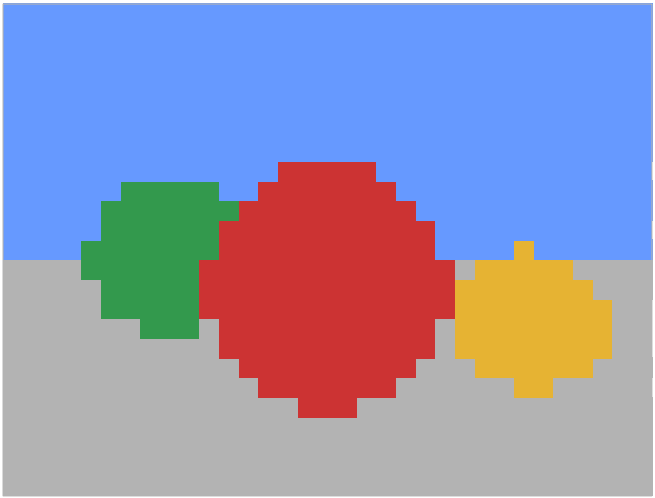
How Much Light Arrives, and How Much Comes Back

CS 453 — Computer Graphics

Fakhir Shaheen

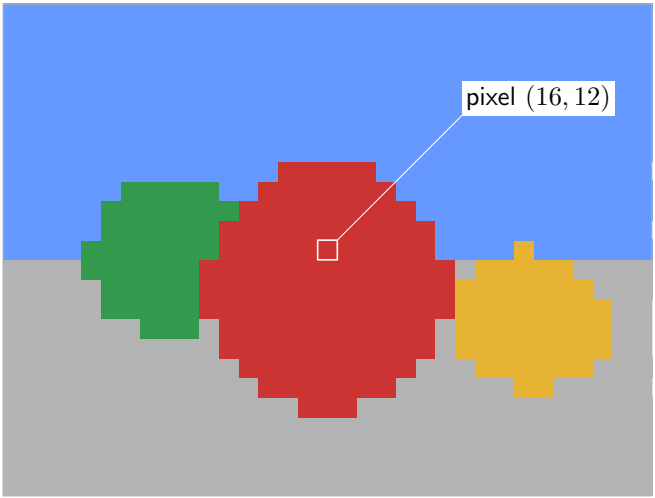
Forman Christian University

Today: how bright should each pixel be?



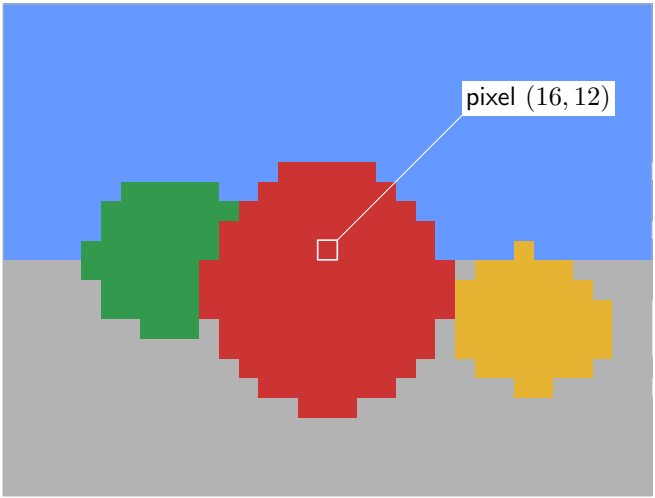
- Lecture 15: one flat colour per surface

Today: how bright should each pixel be?



- Lecture 15: one flat colour per surface
- Every red pixel: the same red

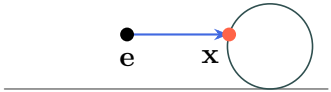
Today: how bright should each pixel be?



- Lecture 15: one flat colour per surface
- Every red pixel: the same red
- Today: a lamp sets each pixel's brightness

Our conventions in one picture

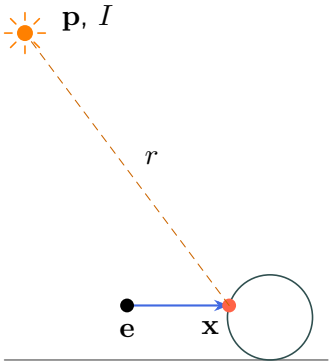
side view



- $\mathbf{x}$: the hit point (lecture 15's $\mathbf{p}$)

Our conventions in one picture

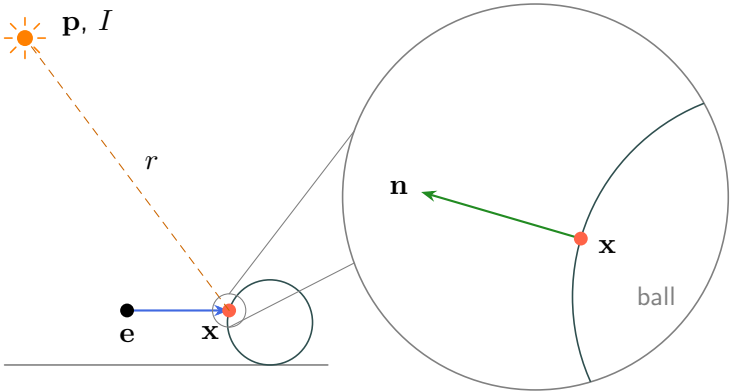
side view



- $\mathbf{x}$: the hit point (lecture 15's $\mathbf{p}$)
- $\mathbf{p}$, I: the lamp;
  r: its distance

Our conventions in one picture

side view

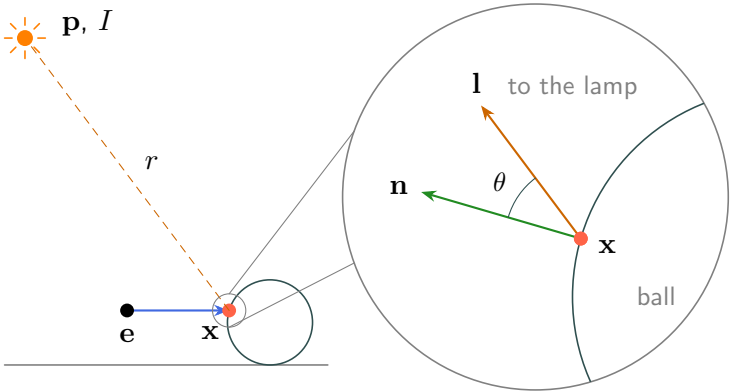


- $\mathbf{x}$: the hit point (lecture 15's $\mathbf{p}$)
- $\mathbf{p}$, I: the lamp; r: its distance
- $\mathbf{n}$, $\mathbf{l}$, $\mathbf{v}$: arrows of length 1

as in lecture 11, now in 3D

Our conventions in one picture

side view



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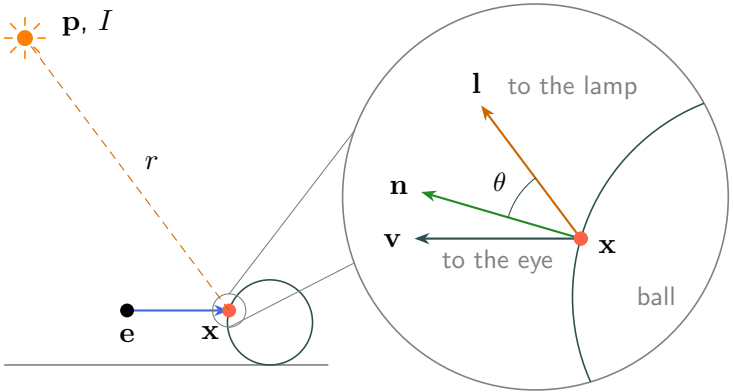


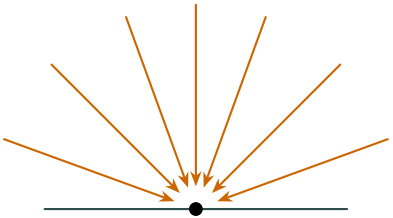
- $\mathbf{x}$: the hit point (lecture 15's $\mathbf{p}$)
- $\mathbf{p}$, I: the lamp; r: its distance
- $\mathbf{n}$, $\mathbf{l}$, $\mathbf{v}$: arrows of length 1

as in lecture 11, now in 3D

How much light arrives

Irradiance: light per unit area

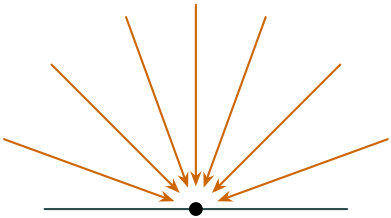
Point-like lights: small, or far away



- Real world: light from all directions

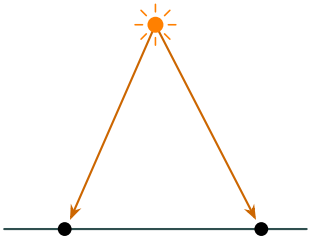
Point-like lights: small, or far away

point-like: one direction of light at each point



- Real world: light from all directions

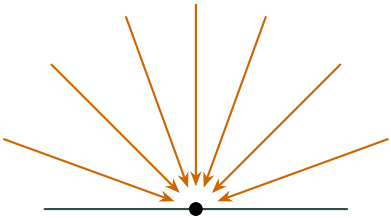
an LED flashlight



- Point source: small and close

Point-like lights: small, or far away

point-like: one direction of light at each point



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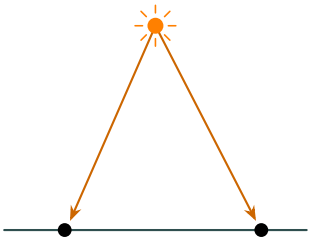
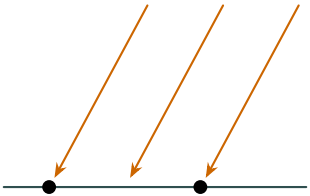


- Point source: small and close

the sun



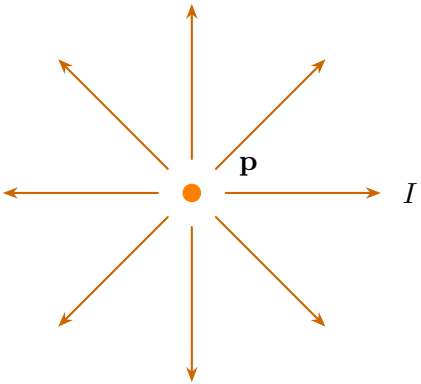
- Directional source: small and very far

A point source: a position and an intensity



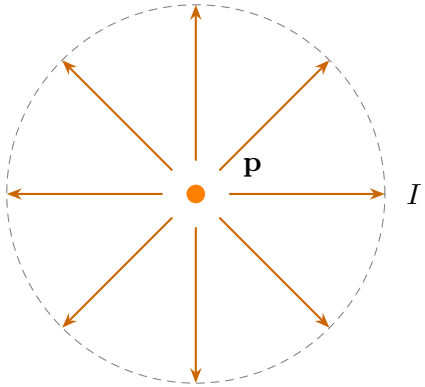
- Position $\mathbf{p}$: a point in space

A point source: a position and an intensity



- Position $\mathbf{p}$: a point in space
- Intensity I: how much light it makes

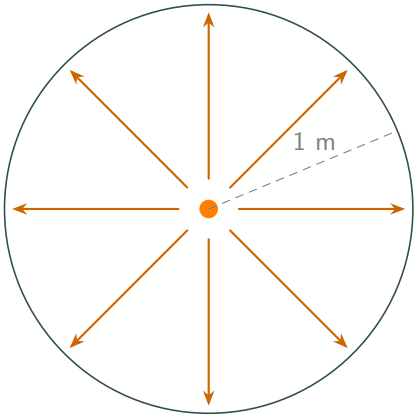
A point source: a position and an intensity



- Position $\mathbf{p}$: a point in space
- Intensity I: how much light it makes
- Isotropic: the same in all directions

One watt, spread over a one-metre sphere

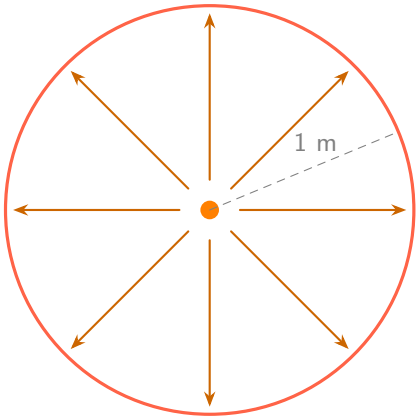
a hollow sphere, the lamp at its centre



- The lamp gives out 1 watt

One watt, spread over a one-metre sphere

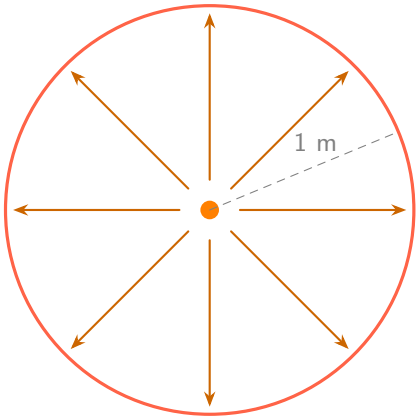
a hollow sphere, the lamp at its centre



- The lamp gives out 1 watt
- All of it lands inside: 4π m^2

One watt, spread over a one-metre sphere

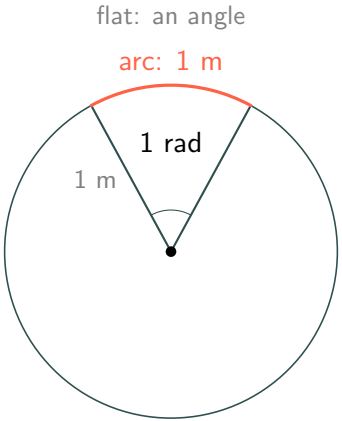
a hollow sphere, the lamp at its centre



- The lamp gives out 1 watt
- All of it lands inside: 4π m^2
- Irradiance E: power per unit area

$$E = \frac{1\ \mathrm{W}}{4\pi\ \mathrm{m}^2} \approx 0.08\ \mathrm{W/m}^2$$

Measuring a cone of light: the steradian

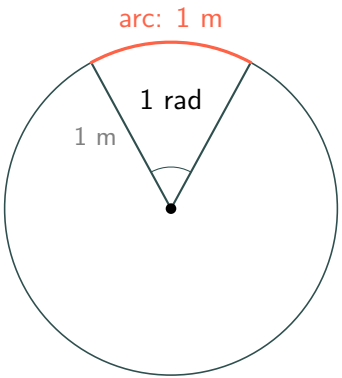


$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

- Radians: how wide an angle is

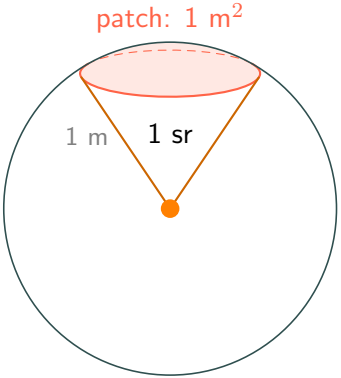
Measuring a cone of light: the steradian

flat: an angle



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

in space: a cone of the lamp's light

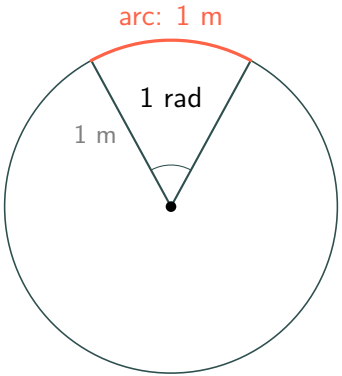


$$\text{solid angle} = \frac{\text{area}}{\text{radius}^2}$$

- Radians: how wide an angle is
- Steradians (sr): how wide a cone is

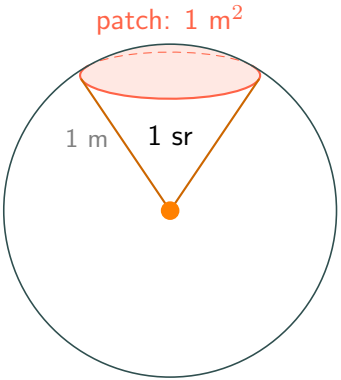
Measuring a cone of light: the steradian

flat: an angle



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

in space: a cone of the lamp's light



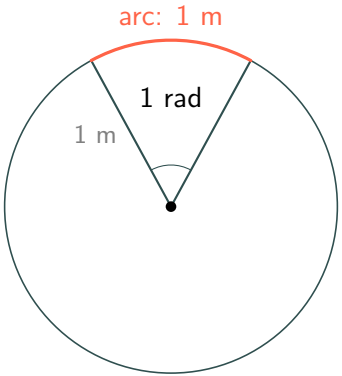
$$\text{solid angle} = \frac{\text{area}}{\text{radius}^2}$$

- Radians: how wide an angle is
- Steradians (sr): how wide a cone is
- All directions: 4π sr

  4π m^2 of sphere, 1 m^2 each

Measuring a cone of light: the steradian

flat: an angle



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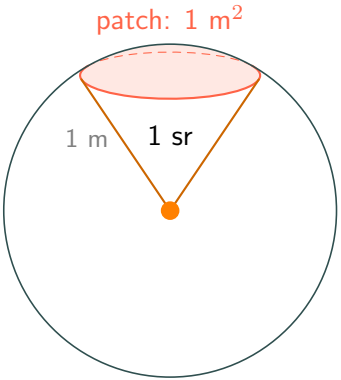

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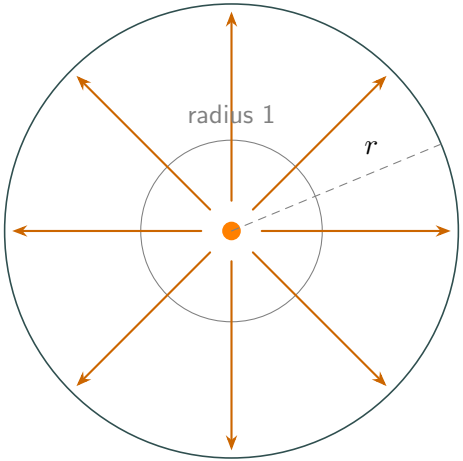
- Radians: how wide an angle is
- Steradians (sr): how wide a cone is
- All directions: 4π sr

  4π m^2 of sphere, 1 m^2 each
- Intensity I: watts per steradian

  1 W / 4π sr ≈ 0.08 W/sr

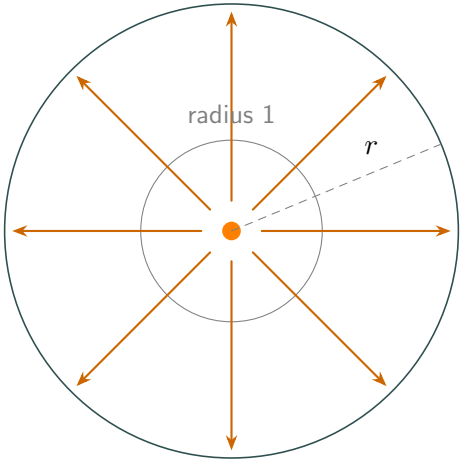
Radius r: the same power, more area



A lamp of power P, a sphere of radius r:

area $= 4\pi r^2$ a sphere's area

Radius r: the same power, more area

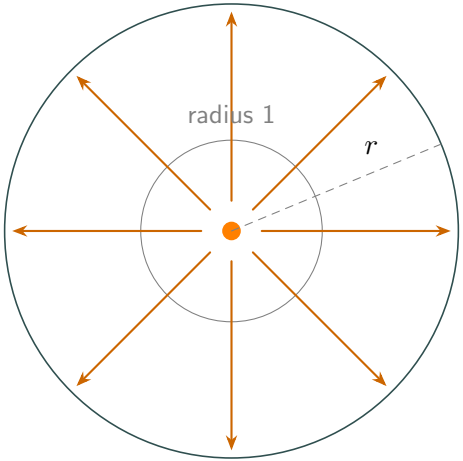


A lamp of power P, a sphere of radius r:

area $= 4\pi r^2$ — a sphere's area

$$E = \frac{P}{4\pi r^2}$$ — irradiance: power / area

Radius r: the same power, more area



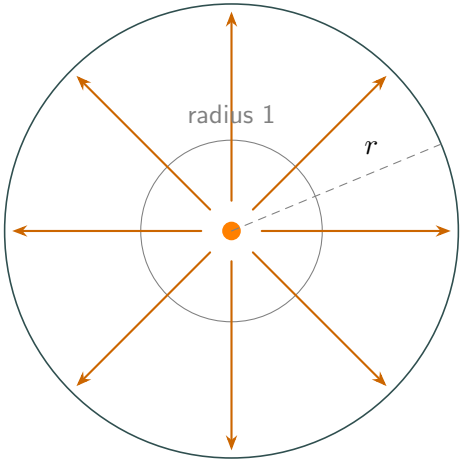
A lamp of power P, a sphere of radius r:

area $= 4\pi r^2$ — a sphere's area

$$E = \frac{P}{4\pi r^2}$$ — irradiance: power / area

$$E = \frac{P}{4\pi} \cdot \frac{1}{r^2}$$ — split the fraction

Radius r: the same power, more area



A lamp of power P, a sphere of radius r:

$\text{area} = 4\pi r^2$ — a sphere's area

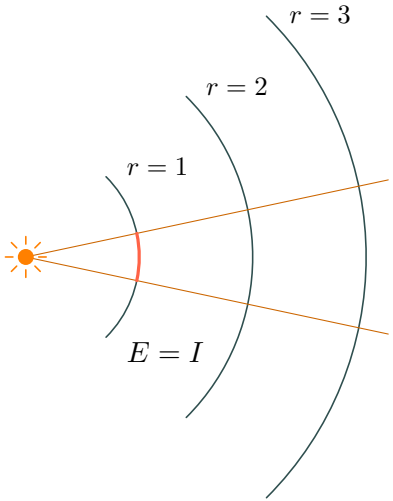
$E = \dfrac{P}{4\pi r^2}$ — irradiance: power / area

$E = \dfrac{P}{4\pi} \cdot \dfrac{1}{r^2}$ — split the fraction

$E = \dfrac{I}{r^2}, \quad I = \dfrac{P}{4\pi}$ — name the first factor

- I: the intensity of the source

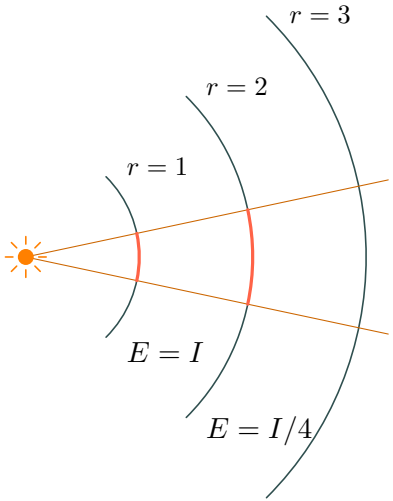
Twice as far: a quarter of the light



$$E = \frac{I}{r^2}$$

- Same light, spread over more area

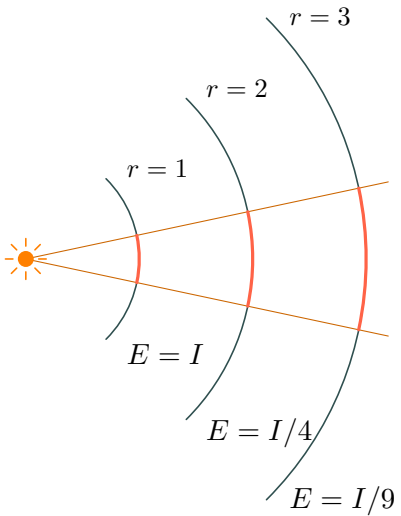
Twice as far: a quarter of the light



$$E = \frac{I}{r^2}$$

- Same light, spread over more area
- $1/r^2$: the inverse square term

Twice as far: a quarter of the light

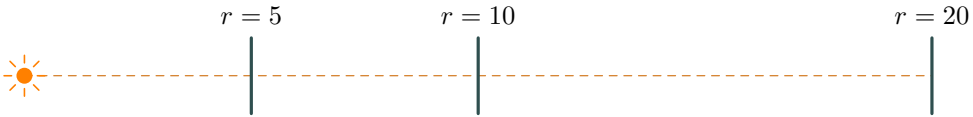


$$E = \frac{I}{r^2}$$

- Same light, spread over more area
- $1/r^2$: the inverse square term
- I belongs to the source alone

Worked: three surfaces facing our lamp

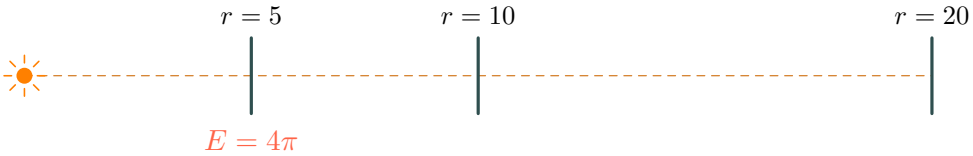
Our lamp: $I = 100\pi \approx 314$ (our choice of number)



- Try first: E at each surface

Worked: three surfaces facing our lamp

Our lamp: $I = 100\pi \approx 314$ (our choice of number)

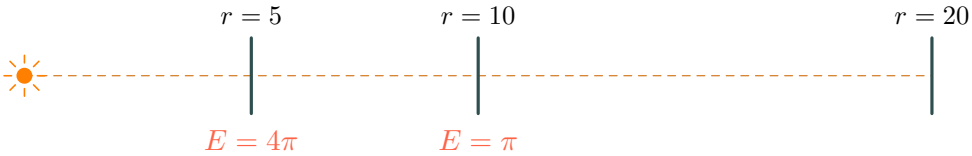


$r = 5$: $\quad E = \dfrac{100\pi}{25} = 4\pi \approx 12.6$

- Try first: E at each surface

Worked: three surfaces facing our lamp

Our lamp: $I = 100\pi \approx 314$ (our choice of number)



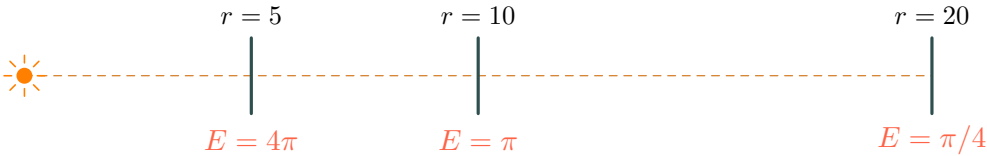
$r = 5$: $E = \dfrac{100\pi}{25} = 4\pi \approx 12.6$

$r = 10$: $E = \dfrac{100\pi}{100} = \pi \approx 3.14$

- Try first: E at each surface

Worked: three surfaces facing our lamp

Our lamp: $I = 100\pi \approx 314$ (our choice of number)



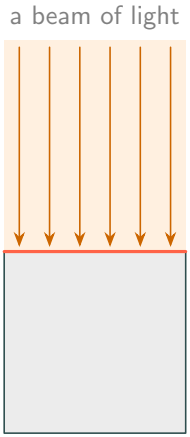
$r = 5$: $E = \dfrac{100\pi}{25} = 4\pi \approx 12.6$

$r = 10$: $E = \dfrac{100\pi}{100} = \pi \approx 3.14$

$r = 20$: $E = \dfrac{100\pi}{400} = \dfrac{\pi}{4} \approx 0.79$

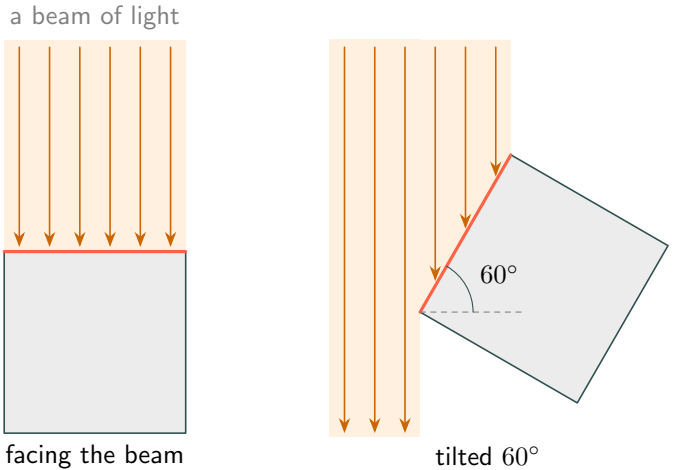
- Try first: E at each surface
- Twice as far: a quarter

Tilt a surface: it catches less light



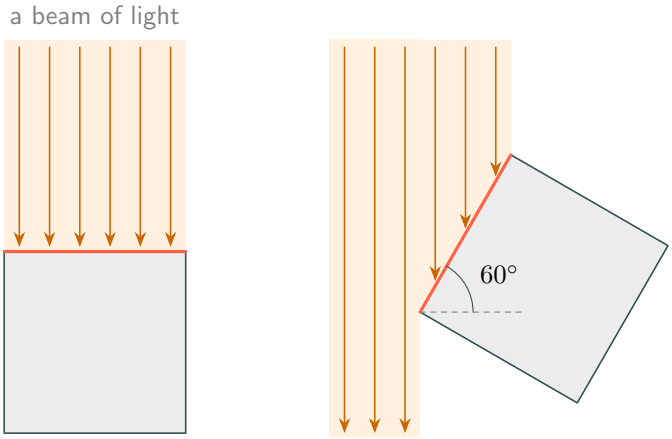
- Facing the beam: all of it lands

Tilt a surface: it catches less light



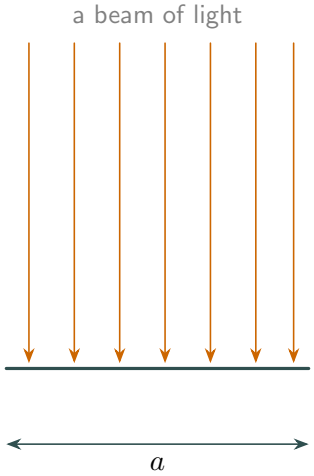
- Facing the beam: all of it lands
- Tilted $60°$: only half lands

Tilt a surface: it catches less light



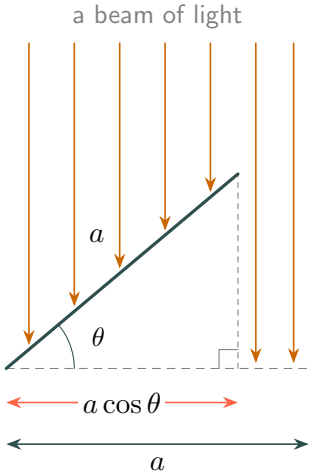
- Facing the beam: all of it lands
- Tilted $60°$: only half lands
- Same area, half the power: E halves

Why cosine: the beam width the surface blocks



facing the beam: width caught $= a$

Why cosine: the beam width the surface blocks

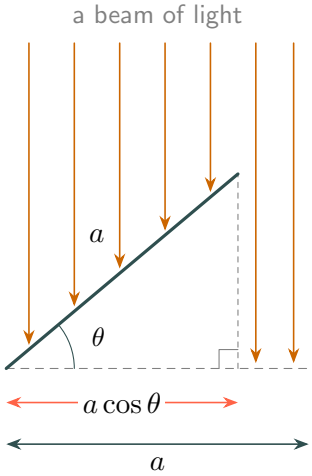


facing the beam: width caught $= a$

tilted by θ: width caught $= a\cos\theta$

a right triangle

Why cosine: the beam width the surface blocks



facing the beam: width caught $= a$

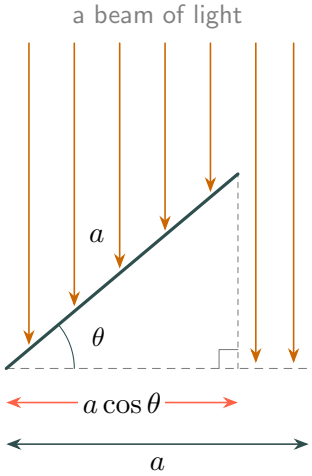
tilted by θ: width caught $= a \cos \theta$

a right triangle

power caught $\propto \cos \theta$

an even beam

Why cosine: the beam width the surface blocks



facing the beam: width caught $= a$

tilted by θ: width caught $= a\cos\theta$

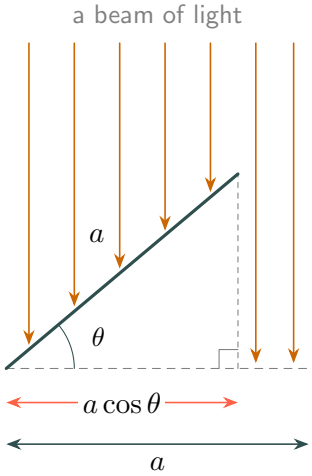
a right triangle

power caught $\propto \cos\theta$

an even beam

$$E = \frac{\text{power}}{\text{area}} \propto \cos\theta$$ same area

Why cosine: the beam width the surface blocks



facing the beam: width caught $= a$

tilted by θ: width caught $= a\cos\theta$

a right triangle

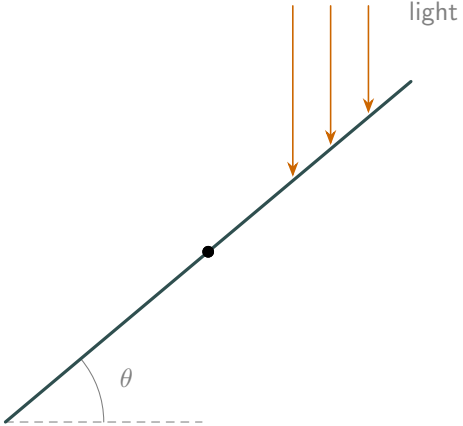
power caught $\propto \cos\theta$

an even beam

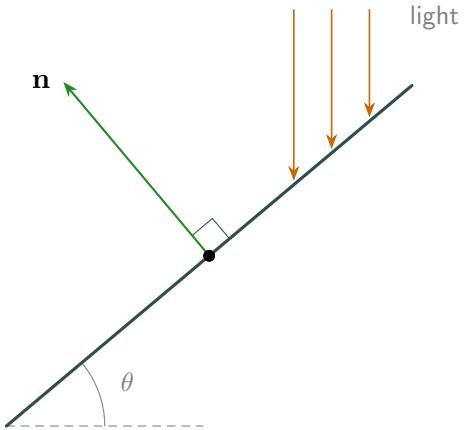
$E = \dfrac{\text{power}}{\text{area}} \propto \cos\theta$ same area

check: $\cos 60^\circ = 0.5$ the cube: half

θ: the angle between $\mathbf{n}$ and $\mathbf{l}$

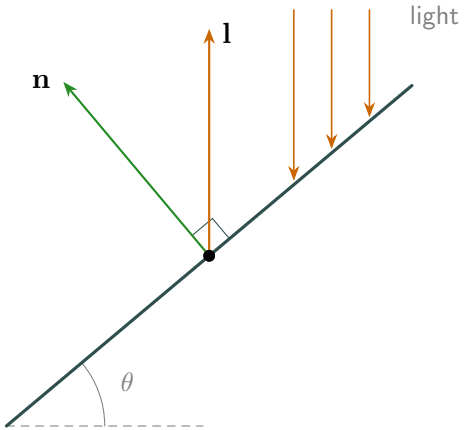


θ: the angle between $\mathbf{n}$ and $\mathbf{l}$



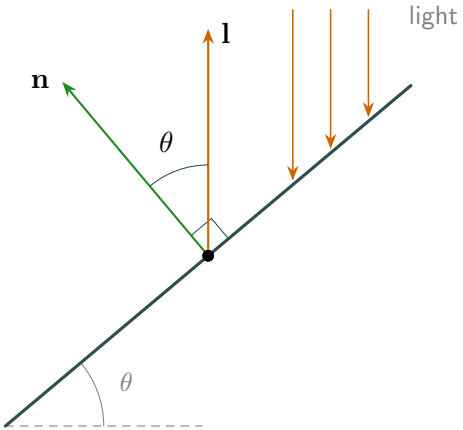
- $\mathbf{n}$: perpendicular to the surface

θ: the angle between $\mathbf{n}$ and $\mathbf{l}$



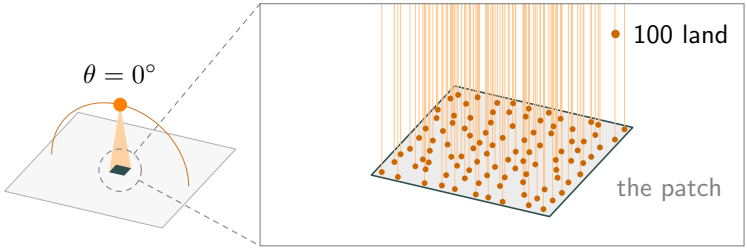
- $\mathbf{n}$: perpendicular to the surface
- $\mathbf{l}$: points toward the light

θ: the angle between $\mathbf{n}$ and $\mathbf{l}$



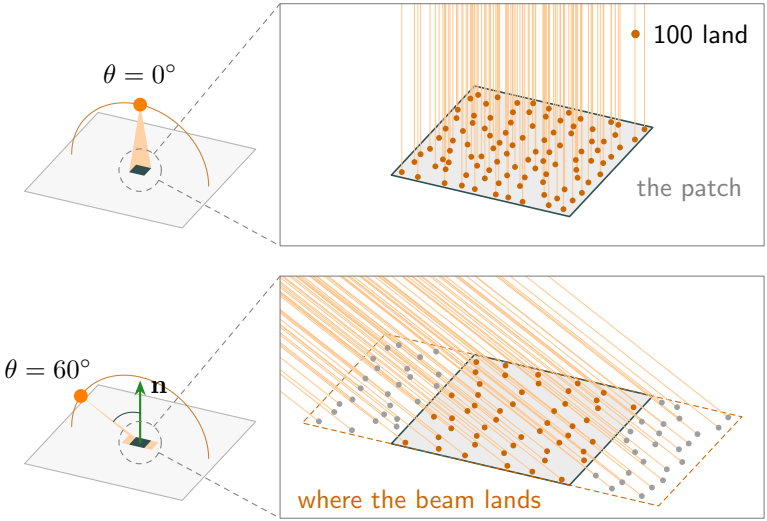
- $\mathbf{n}$: perpendicular to the surface
- $\mathbf{l}$: points toward the light
- Tilting by θ turns $\mathbf{n}$ by θ
- θ: the angle of incidence

Counting rays: a low lamp spreads them out



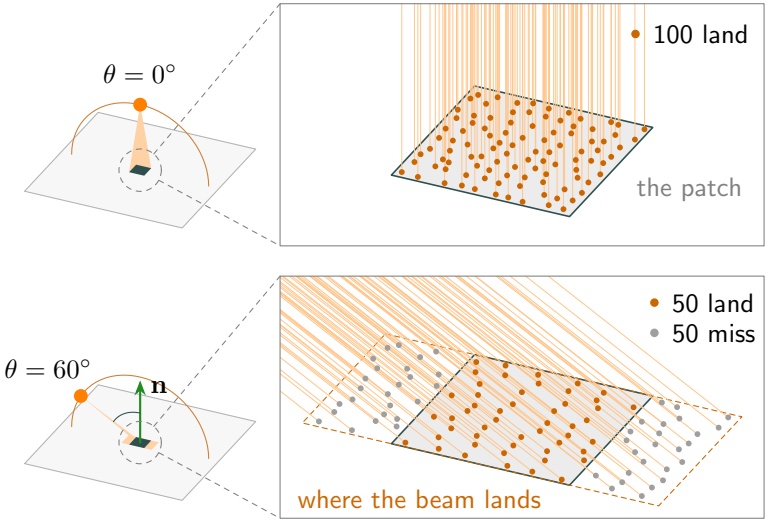
- Lamp overhead: all 100 rays land

Counting rays: a low lamp spreads them out



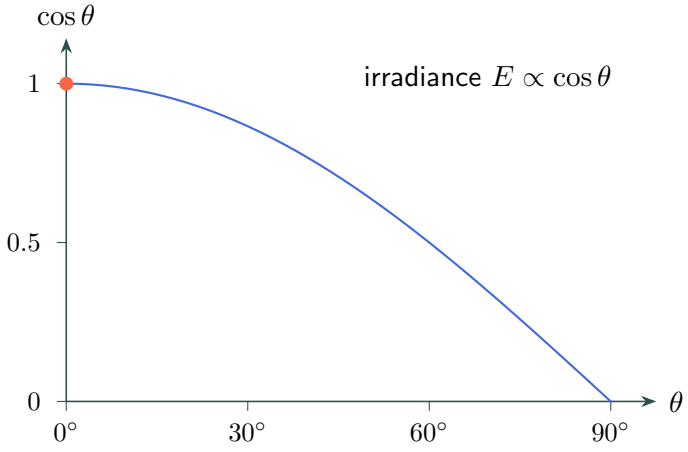
- Lamp overhead: all 100 rays land
- At $60°$: same rays, twice the floor

Counting rays: a low lamp spreads them out



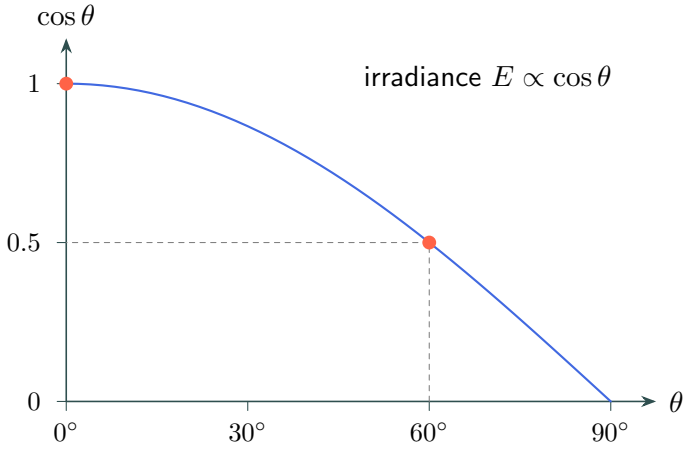
- Lamp overhead: all 100 rays land
- At $60°$: same rays, twice the floor
- Only 50 land: $\cos 60° = 0.5$
- Same patch, half the rays: E halves

Lambert's cosine law: E goes as $\cos\theta$



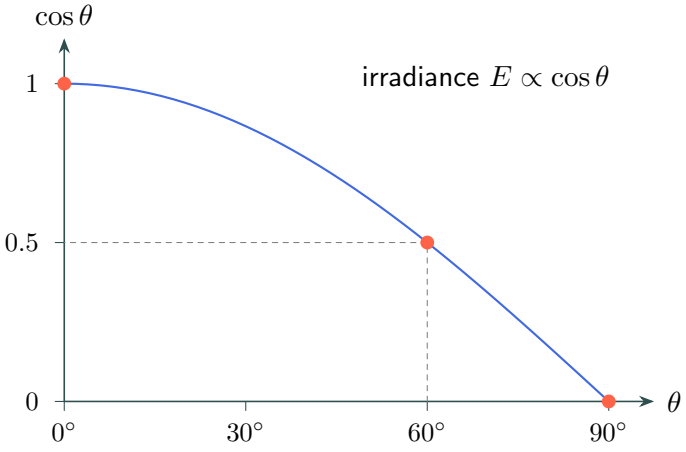
- Facing the light: all of it

Lambert's cosine law: E goes as $\cos\theta$



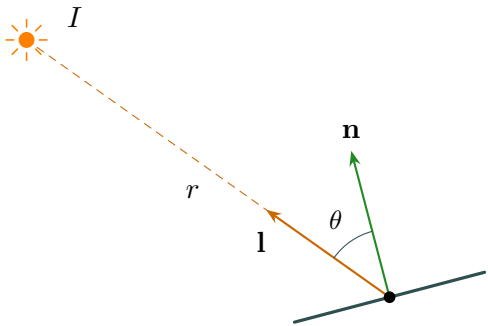
- Facing the light: all of it
- Tilted $60°$: half

Lambert's cosine law: E goes as $\cos\theta$



- Facing the light: all of it
- Tilted $60°$: half
- Edge on, $90°$: nothing
- Lambert's cosine law, 1760

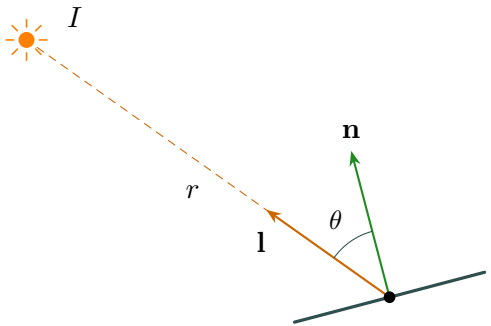
Distance and tilt together: one formula



$$E = I \frac{\cos\theta}{r^2}$$

E: irradiance, power per unit area

Distance and tilt together: one formula

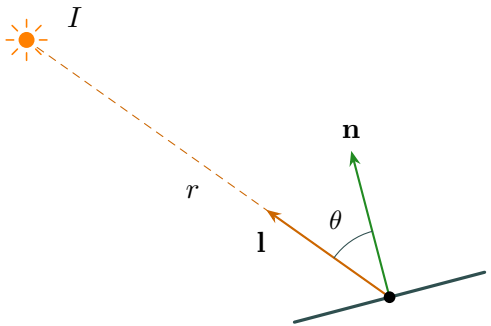


$$E = I \frac{\cos\theta}{r^2}$$

E: irradiance, power per unit area

- I: what the source gives

Distance and tilt together: one formula

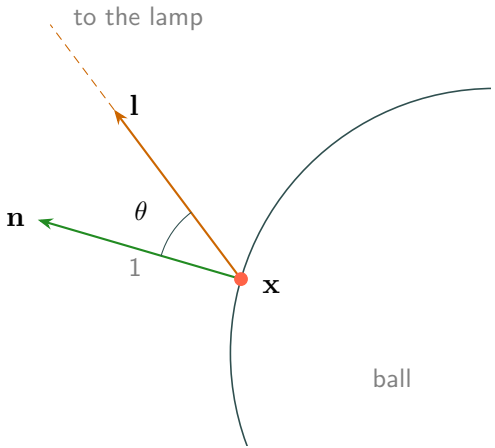


$$E = I \frac{\cos\theta}{r^2}$$

E: irradiance, power per unit area

- I: what the source gives
- $\cos\theta / r^2$: the geometry factor
- Geometry: where things are, nothing more

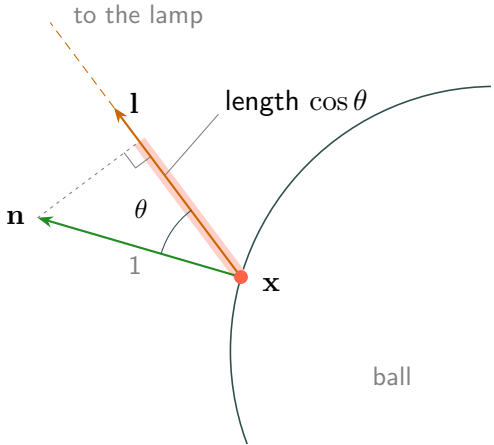
No angle needed: $\cos\theta = \mathbf{n} \cdot \mathbf{l}$



$$\cos\theta = \mathbf{n} \cdot \mathbf{l}$$

- Only for arrows of length 1

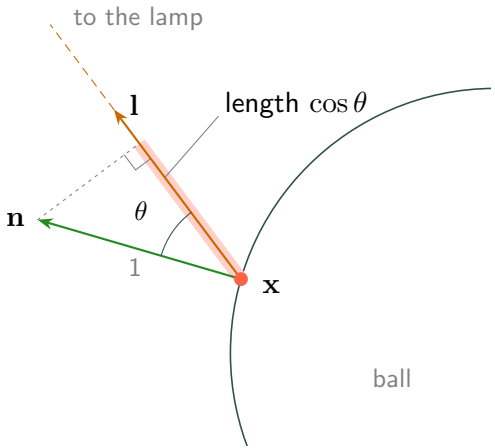
No angle needed: $\cos\theta = \mathbf{n} \cdot \mathbf{l}$



$$\cos\theta = \mathbf{n} \cdot \mathbf{l}$$

- Only for arrows of length 1
- $\mathbf{n} \cdot \mathbf{l}$: the shadow of $\mathbf{n}$ on $\mathbf{l}$

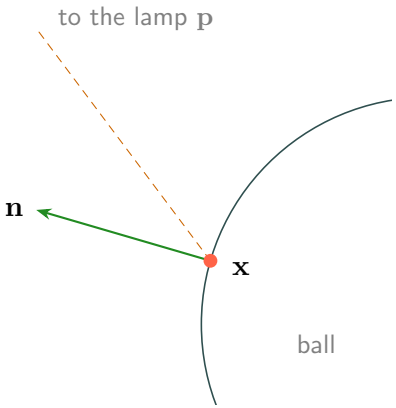
No angle needed: $\cos\theta = \mathbf{n} \cdot \mathbf{l}$



$$\cos\theta = \mathbf{n} \cdot \mathbf{l}$$

- Only for arrows of length 1
- $\mathbf{n} \cdot \mathbf{l}$: the shadow of $\mathbf{n}$ on $\mathbf{l}$
- One dot product, no trigonometry

Worked: the irradiance E at our hit

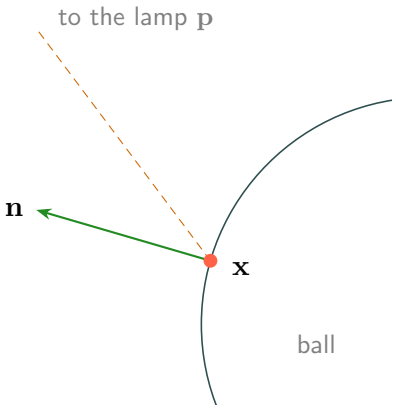


$\mathbf{p} = (0,\ 8,\ 3)$ $\mathbf{x} = (0,\ 0,\ -3)$

$\mathbf{n} = (0,\ 0.28,\ 0.96)$ $I = 100\pi$

- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

Worked: the irradiance E at our hit



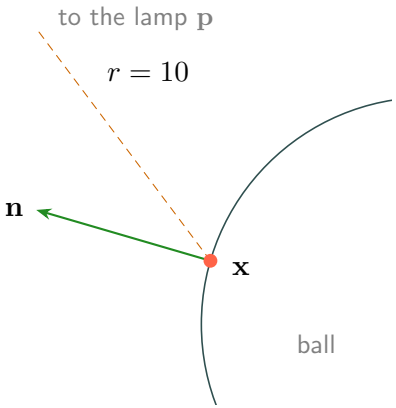
$\mathbf{p} = (0,\ 8,\ 3)$ $\mathbf{x} = (0,\ 0,\ -3)$

$\mathbf{n} = (0,\ 0.28,\ 0.96)$ $I = 100\pi$

- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

$\mathbf{p} - \mathbf{x} = (0,\ 8,\ 6)$

Worked: the irradiance E at our hit



$\mathbf{p} = (0,\ 8,\ 3)$ $\quad$ $\mathbf{x} = (0,\ 0,\ -3)$

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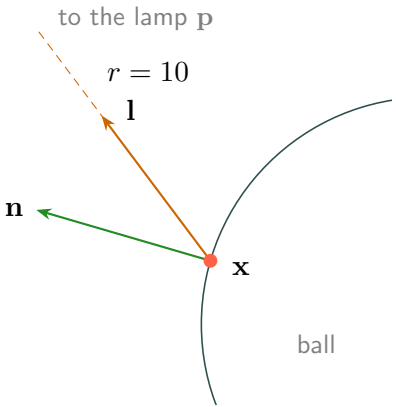
- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

$\mathbf{p} - \mathbf{x} = (0,\ 8,\ 6)$

$r = \sqrt{0 + 64 + 36} = 10$

Worked: the irradiance E at our hit



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$\mathbf{n} = (0,\ 0.28,\ 0.96)$ $I = 100\pi$

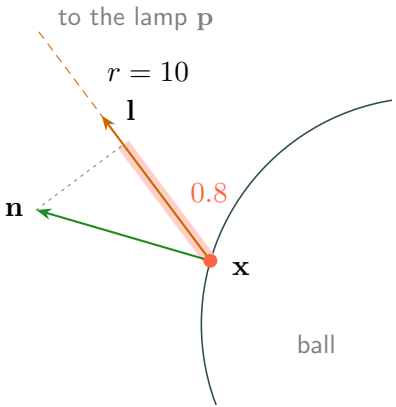
- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

$\mathbf{p} - \mathbf{x} = (0,\ 8,\ 6)$

$r = \sqrt{0 + 64 + 36} = 10$

$\mathbf{l} = (0,\ 8,\ 6)/10 = (0,\ 0.8,\ 0.6)$

Worked: the irradiance E at our hit



$\mathbf{p} = (0,\ 8,\ 3)$ $\quad$ $\mathbf{x} = (0,\ 0,\ -3)$

$\mathbf{n} = (0,\ 0.28,\ 0.96)$ $\quad$ $I = 100\pi$

- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

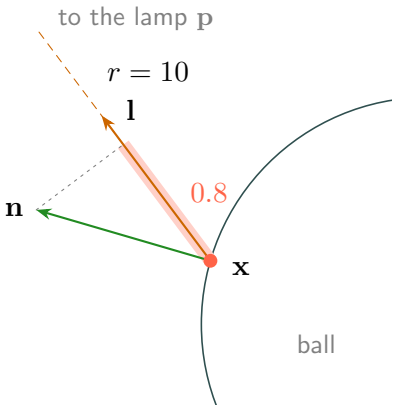
$\mathbf{p} - \mathbf{x} = (0,\ 8,\ 6)$

$r = \sqrt{0 + 64 + 36} = 10$

$\mathbf{l} = (0,\ 8,\ 6)/10 = (0,\ 0.8,\ 0.6)$

$\mathbf{n} \cdot \mathbf{l} = 0 + 0.224 + 0.576 = 0.8$

Worked: the irradiance E at our hit



$\mathbf{p} = (0,\ 8,\ 3)$ $\quad$ $\mathbf{x} = (0,\ 0,\ -3)$

$\mathbf{n} = (0,\ 0.28,\ 0.96)$ $\quad$ $I = 100\pi$

- Try first: $\mathbf{l}$, r, then E at $\mathbf{x}$

$$\mathbf{p} - \mathbf{x} = (0,\ 8,\ 6)$$

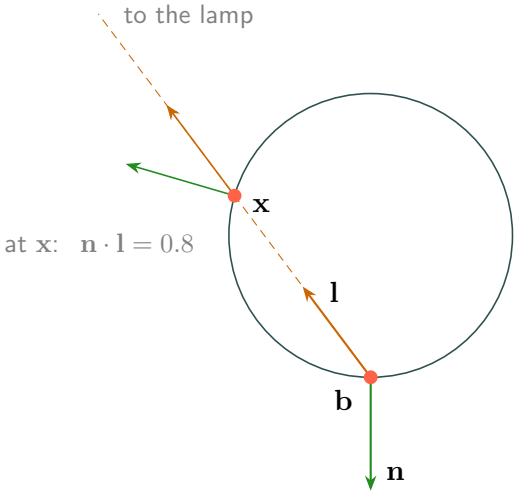
$$r = \sqrt{0 + 64 + 36} = 10$$

$$\mathbf{l} = (0,\ 8,\ 6)/10 = (0,\ 0.8,\ 0.6)$$

$$\mathbf{n} \cdot \mathbf{l} = 0 + 0.224 + 0.576 = 0.8$$

$$E = \frac{100\pi \times 0.8}{10^2} = 0.8\pi \approx 2.51$$

Lamp behind the surface: no light arrives

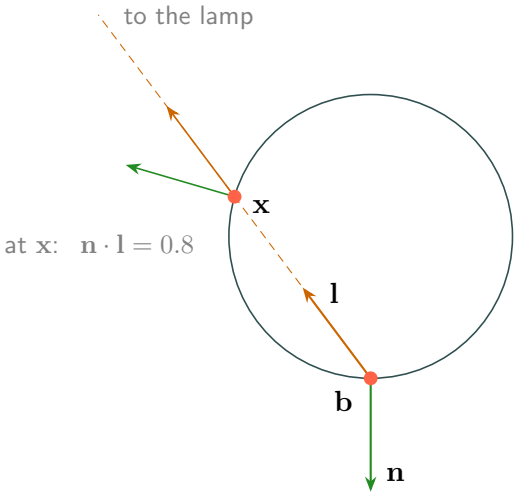


- **b**: the lowest point of the ball

  $\mathbf{n} = (0,\ -1,\ 0)$

  $\mathbf{l} = (0,\ 0.8,\ 0.6), \quad r = 12$

Lamp behind the surface: no light arrives



- **b**: the lowest point of the ball

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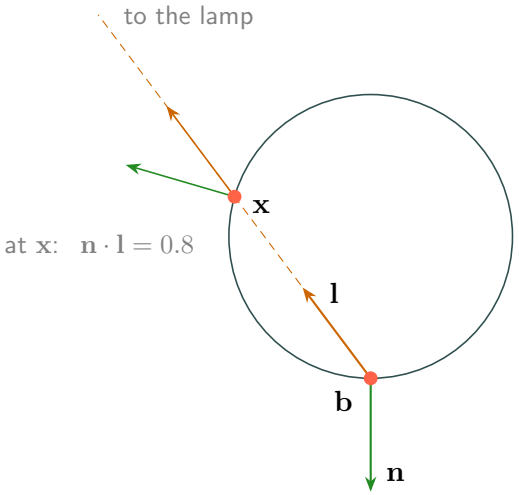
  $\mathbf{l} = (0,\ 0.8,\ 0.6), \quad r = 12$

  $\mathbf{n} \cdot \mathbf{l} = 0 - 0.8 + 0 = -0.8$

- Negative: the lamp is behind

Lamp behind the surface: no light arrives



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  $\mathbf{n} \cdot \mathbf{l} = 0 - 0.8 + 0 = -0.8$

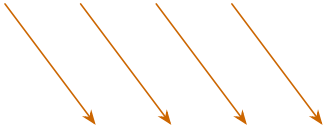
- Negative: the lamp is behind

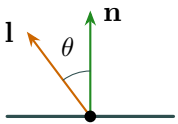
  $$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\ \frac{I}{r^2} = 0$$

- No light, and never negative light

A directional source: a direction, no distance

light from very far away

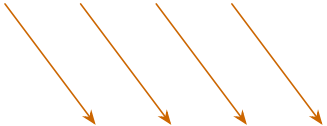


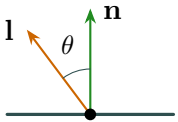


- Very far away: I/r^2 is the same everywhere
- No position, so no r to compute

A directional source: a direction, no distance

light from very far away





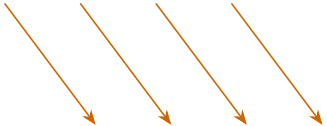
$E = H \cos\theta$

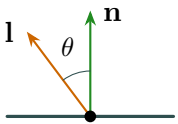
- Very far away: I/r^2 is the same everywhere
- No position, so no r to compute
- Store the ratio: $H = I/r^2$
- H: the normal irradiance, E at $\theta = 0$

the sun: $I \approx 3 \times 10^{25}$ W/sr, $r \approx 1.5 \times 10^{11}$ m, $H \approx 1360$ W/m^2

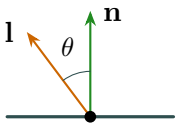
A directional source: a direction, no distance

light from very far away





$E = H \cos\theta$

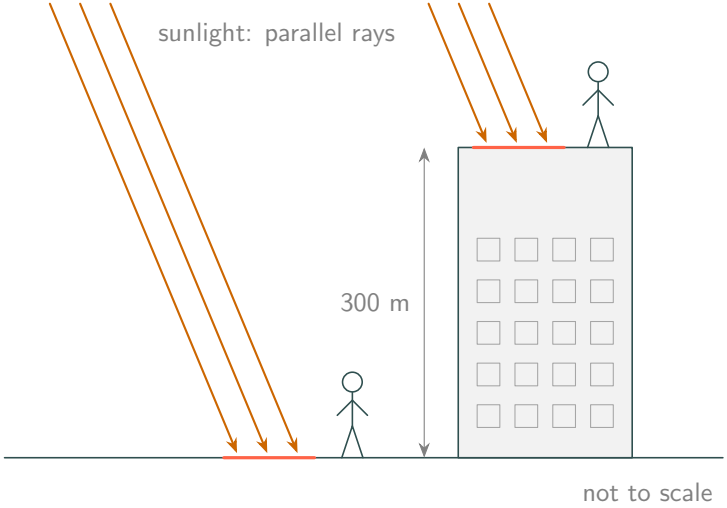


farther away: same $\mathbf{l}$, same E

- Very far away: I/r^2 is the same everywhere
- No position, so no r to compute
- Store the ratio: $H = I/r^2$
- H: the normal irradiance, E at $\theta = 0$

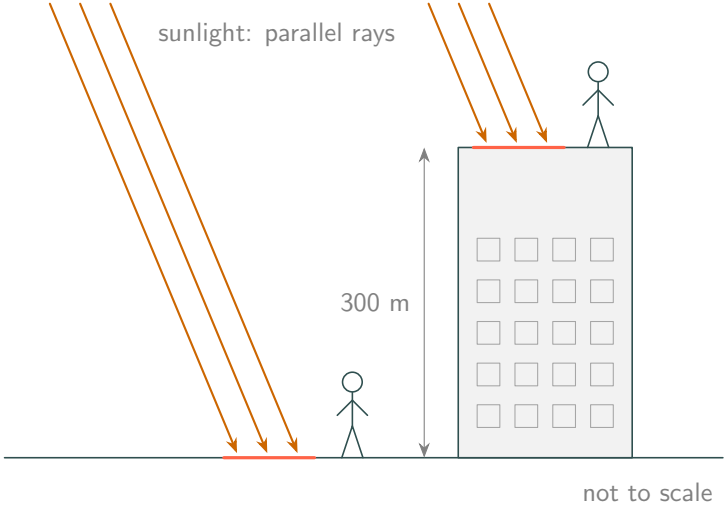
the sun: $I \approx 3 \times 10^{25}$ W/sr, $r \approx 1.5 \times 10^{11}$ m, $H \approx 1360$ W/m^2

Roof or ground: who gets more sunlight?



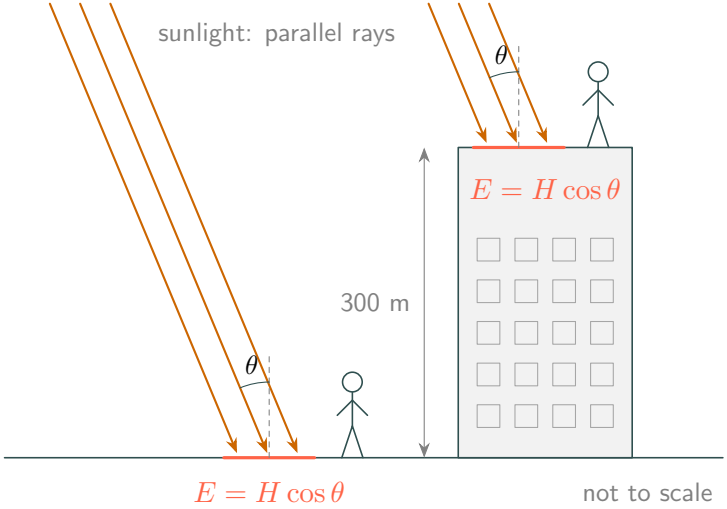
- The sun:
  $150\,000\,000$ km away
- The roof:
  only 300 m closer

Roof or ground: who gets more sunlight?



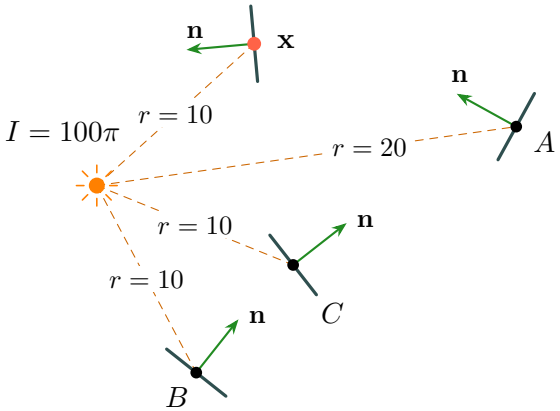
- The sun:
  $150\,000\,000$ km away
- The roof:
  only 300 m closer
- I/r^2 grows by
  $0.000\,000\,4\,\%$

Roof or ground: who gets more sunlight?



- The sun:
  $150\,000\,000$ km away
- The roof:
  only 300 m closer
- I/r^2 grows by
  $0.000\,000\,4\,\%$
- Same H, same $\mathbf{l}$:
  the same E

Quick check 1: farther, or more tilted

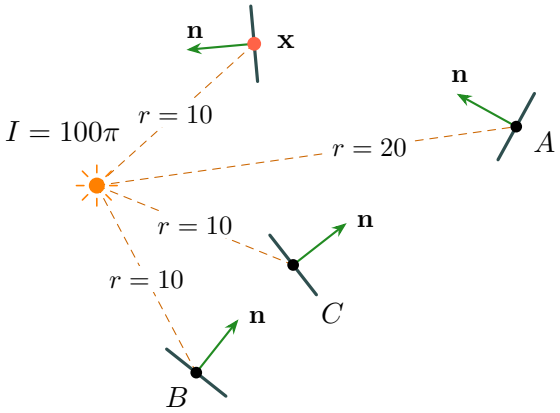


- Try first: E at A, B and C

point	r	$\mathbf{n} \cdot \mathbf{l}$	E	
$\mathbf{x}$	10	0.8	0.8π	known
A	20	0.8	?	
B	10	0.4	?	
C	10	-0.5	?	

$$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\ \frac{I}{r^2}$$

Quick check 1: farther, or more tilted



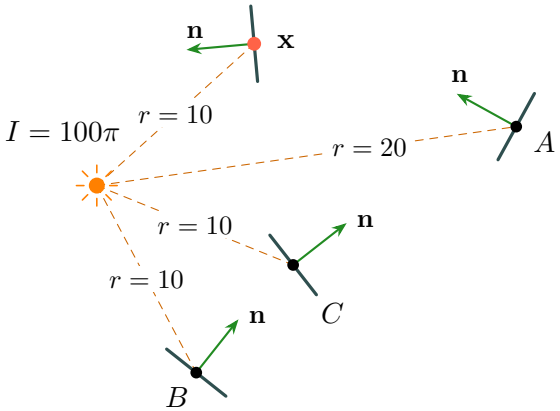
- A: twice as far, a quarter

- Try first: E at A, B and C

point	r	$\mathbf{n} \cdot \mathbf{l}$	E	
$\mathbf{x}$	10	0.8	0.8π	known
A	20	0.8	0.2π	
B	10	0.4	?	
C	10	-0.5	?	

$$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\ \frac{I}{r^2}$$

Quick check 1: farther, or more tilted



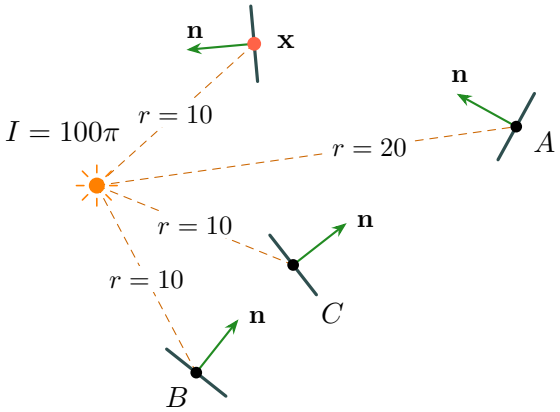
- Try first: E at A, B and C

point	r	$\mathbf{n} \cdot \mathbf{l}$	E	
$\mathbf{x}$	10	0.8	0.8π	known
A	20	0.8	0.2π	
B	10	0.4	0.4π	
C	10	-0.5	?	

$$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\ \frac{I}{r^2}$$

- A: twice as far, a quarter
- B: half the cosine, half

Quick check 1: farther, or more tilted



- Try first: E at A, B and C

point	r	$\mathbf{n} \cdot \mathbf{l}$	E	
$\mathbf{x}$	10	0.8	0.8π	known
A	20	0.8	0.2π	
B	10	0.4	0.4π	
C	10	-0.5	0	

$$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\ \frac{I}{r^2}$$

- A: twice as far, a quarter
- B: half the cosine, half
- C: lamp behind, nothing

Recap: the light arriving from a point source

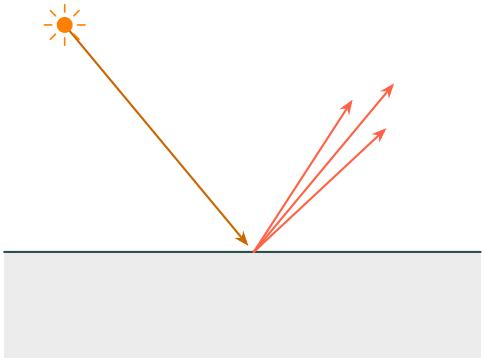
$$E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\, \frac{I}{r^2}$$

$$\mathbf{l} = \frac{\mathbf{p} - \mathbf{x}}{r}, \qquad r = \|\mathbf{p} - \mathbf{x}\|$$

What the surface sends back

Lambertian reflection: the same in every direction

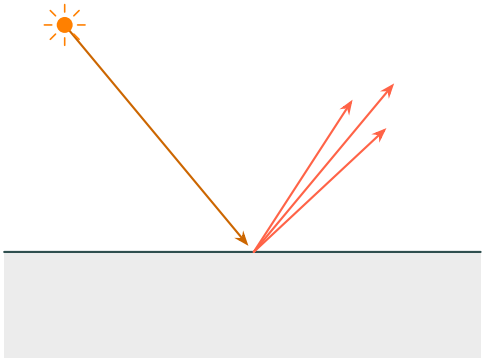
A material: shiny surface over coloured base



- Specular: off the top surface

  near the mirror direction

A material: shiny surface over coloured base



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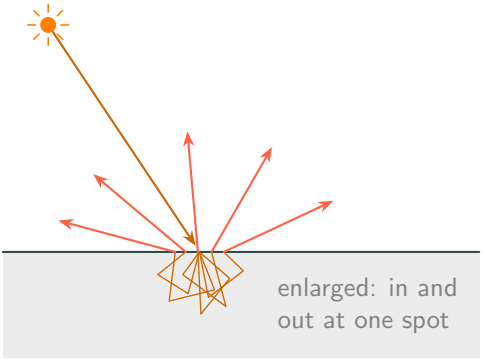
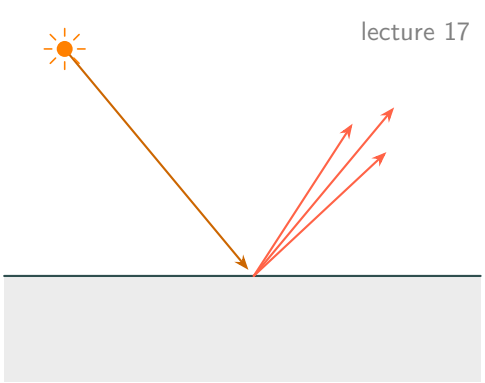


- Diffuse: from inside, in all directions

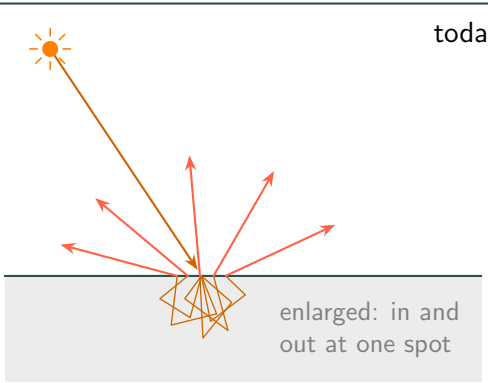
  gives the object its colour

A material: shiny surface over coloured base



- Specular: off the top surface

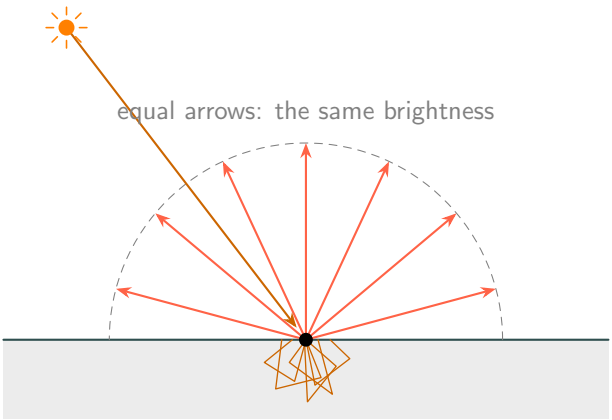
  near the mirror direction



- Diffuse: from inside, in all directions

  gives the object its colour

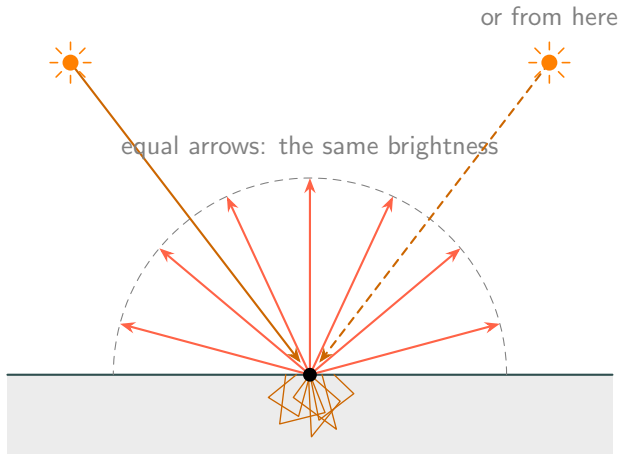
Lambertian: equal light out in every direction



an ideal diffuse surface: close to paper, flat paint, stone

- Inside: the light forgets its direction
- So it leaves equally, in all directions

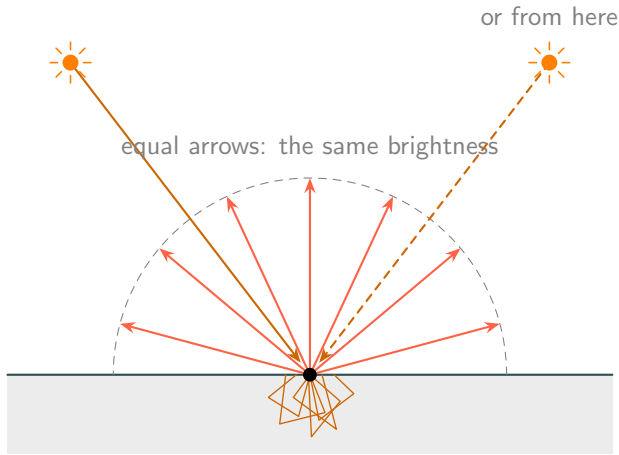
Lambertian: equal light out in every direction



an ideal diffuse surface: close to paper, flat paint, stone

- Inside: the light forgets its direction
- So it leaves equally, in all directions
- Wherever the light came from

Lambertian: equal light out in every direction

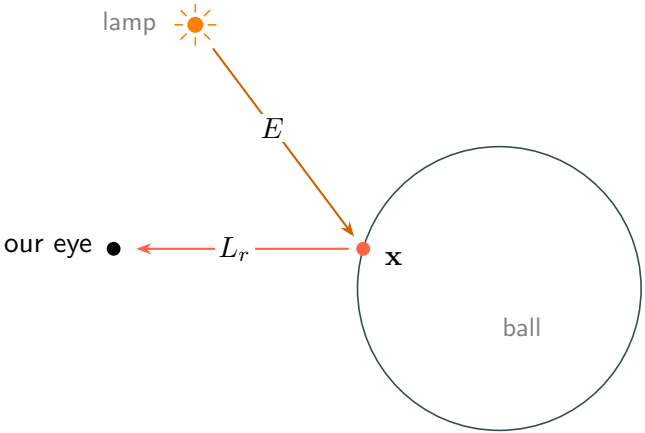


an ideal diffuse surface: close to paper, flat paint, stone

- Inside: the light forgets its direction
- So it leaves equally, in all directions
- Wherever the light came from
- Light in = all light out + absorbed

conservation of energy

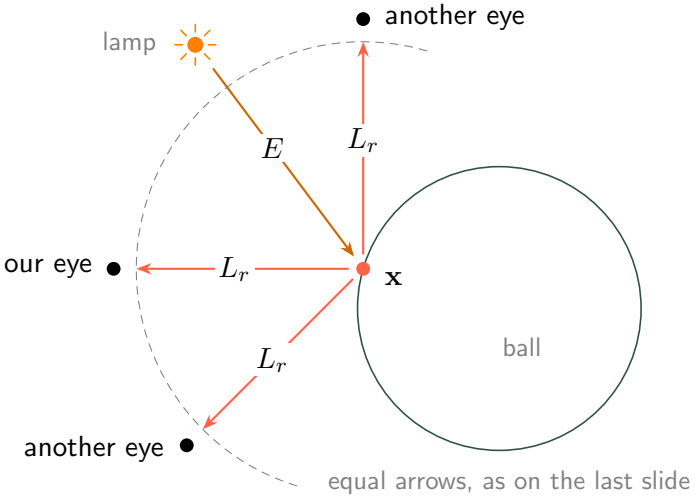
Every eye sees the same brightness, L_r



- L_r: the reflected light an eye sees at $\mathbf{x}$

  for our eye, L_r is the pixel's colour

Every eye sees the same brightness, L_r



- L_r: the reflected light an eye sees at $\mathbf{x}$

  for our eye, L_r is the pixel's colour

- Every eye: the same L_r

- The textbook's word: view independent

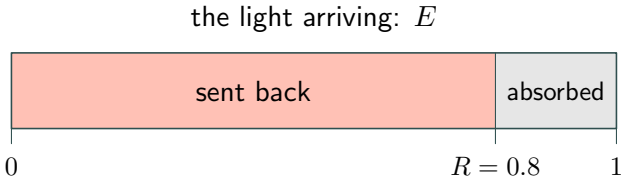
Reflectance R: the fraction sent back

the light arriving: E



- A surface keeps some of the light

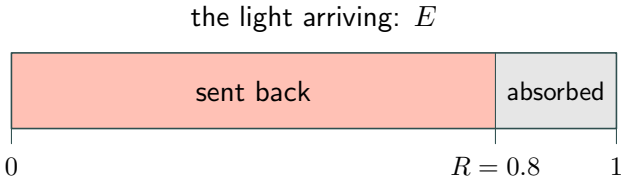
Reflectance R: the fraction sent back



- A surface keeps some of the light
- R: the fraction it sends back

today R is the reflectance, not a radius

Reflectance R: the fraction sent back

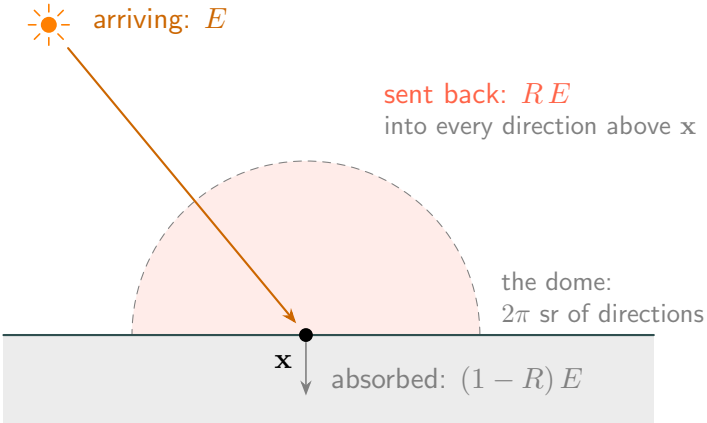


sent back: $R\,E$ absorbed: $(1 - R)\,E$

- A surface keeps some of the light
- R: the fraction it sends back
- $R\,E$ leaves, in all directions together

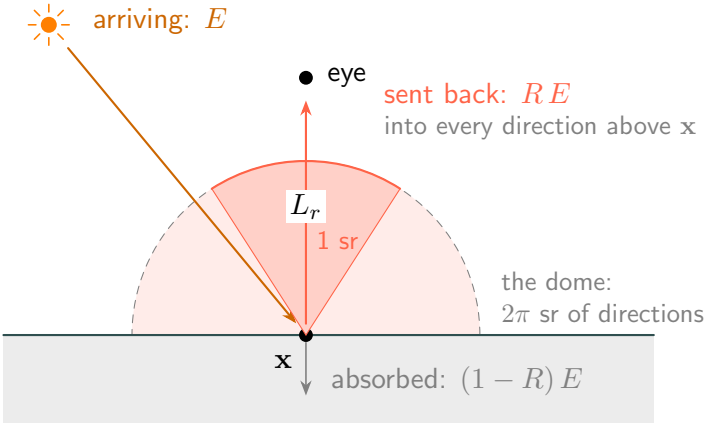
today R is the reflectance, not a radius

Why divide by π? Share $R\,E$ out over the dome



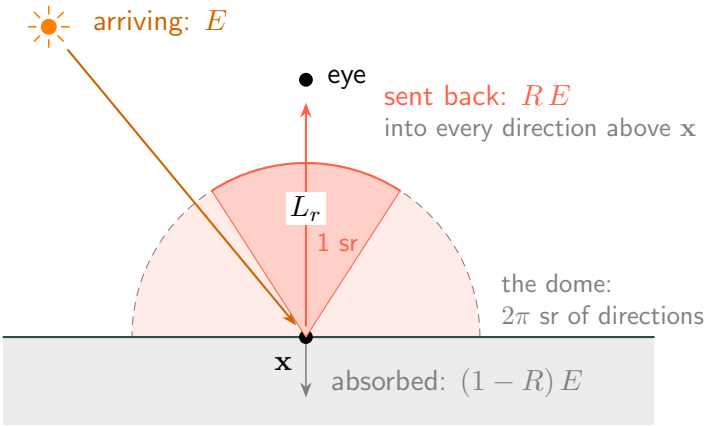
- The surface sends $R\,E$ back into the dome.

Why divide by π? Share $R\,E$ out over the dome



- The surface sends $R\,E$ back into the dome.
- L_r is the light into one steradian, straight up.

Why divide by π? Share RE out over the dome



- The surface sends RE back into the dome.
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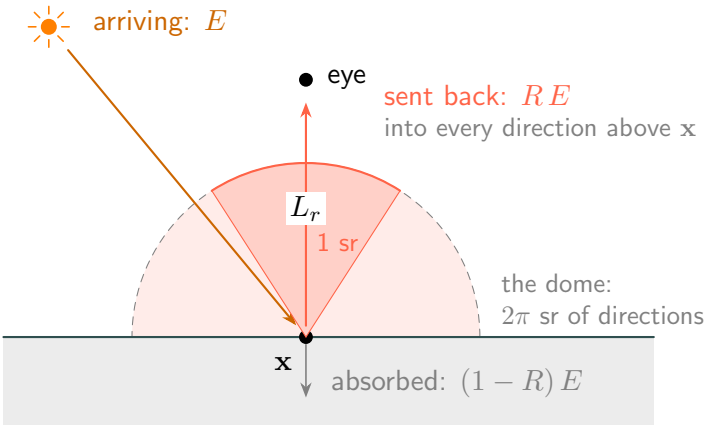
- Equal shares would give $L_r = RE/2\pi$.

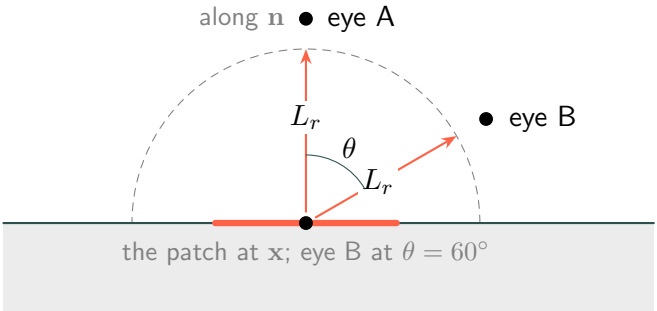
Why divide by π? Share RE out over the dome



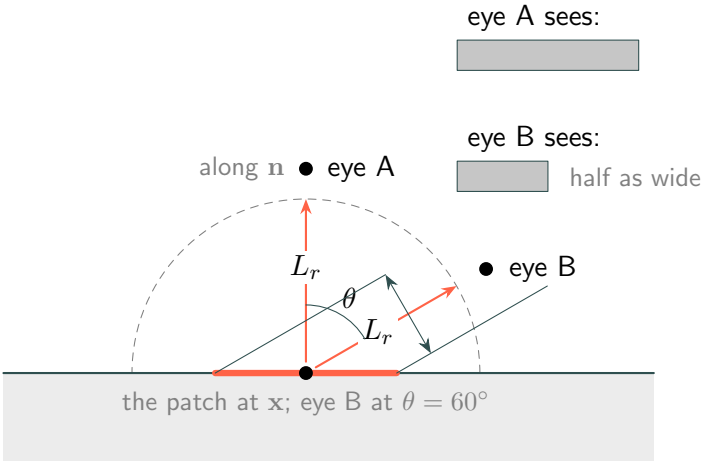
- The surface sends RE back into the dome.
- L_r is the light into one steradian, straight up.
- Equal shares would give $L_r = RE/2\pi$.
- The answer will be $L_r = RE/\pi$. The next three slides say why.

The shares are not equal: a slanted direction gets $\cos\theta$

- Both eyes see the same brightness, L_r.

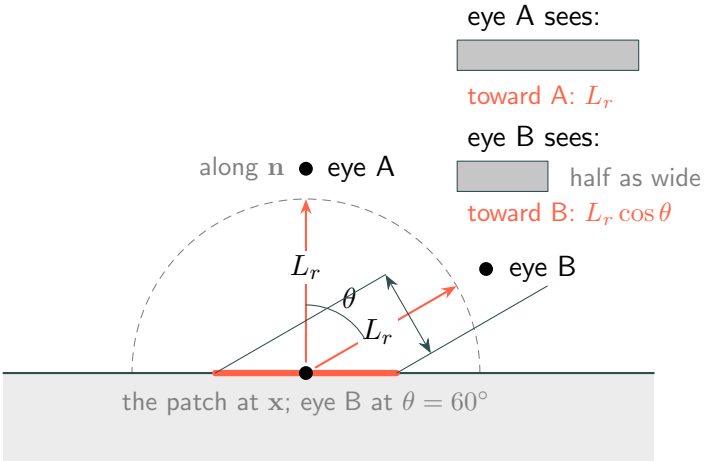


The shares are not equal: a slanted direction gets $\cos\theta$



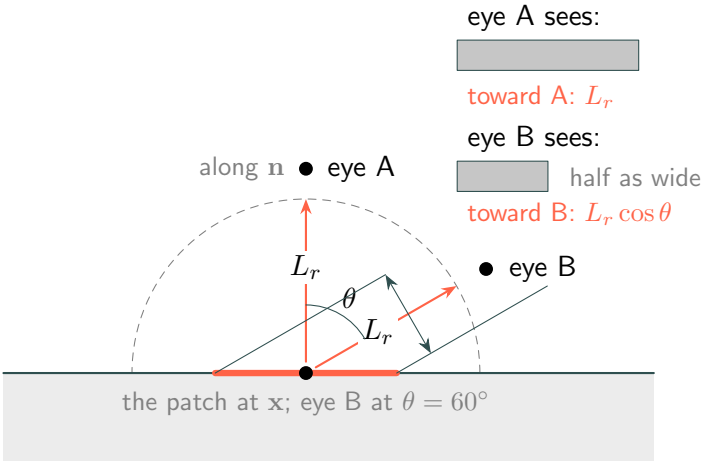
- Both eyes see the same brightness, L_r.
- Eye B sees the patch $\cos\theta$ as wide.

The shares are not equal: a slanted direction gets $\cos\theta$



- Both eyes see the same brightness, L_r.
- Eye B sees the patch $\cos\theta$ as wide.
- So the light toward B is $L_r \cos\theta$, half of A's.

The shares are not equal: a slanted direction gets $\cos\theta$

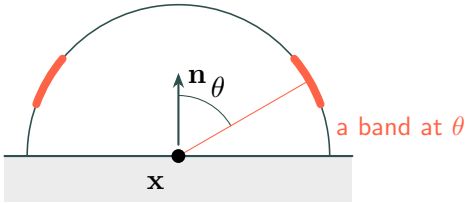


- Both eyes see the same brightness, L_r.
- Eye B sees the patch $\cos\theta$ as wide.
- So the light toward B is $L_r \cos\theta$, half of A's.
- The 2π steradians do not share RE equally.

  this is the tilt slide's $\cos\theta$, now on the way out

The dome, seen from straight above, is a disc of area π

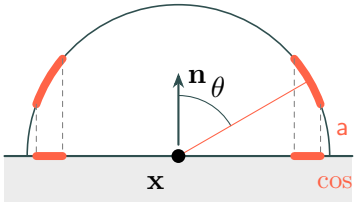
- A band at θ gets $L_r \cos\theta$ times its area.



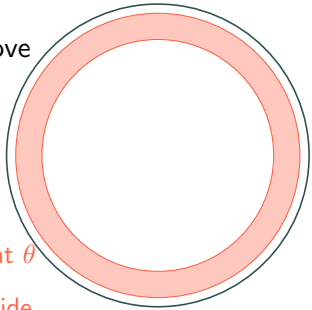
the dome, radius 1: area = steradians

The dome, seen from straight above, is a disc of area π

- an eye straight above



the eye above sees:

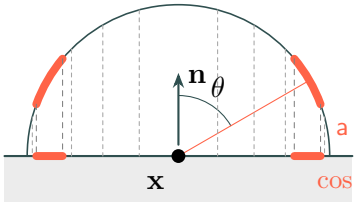


- A band at θ gets $L_r \cos\theta$ times its area.
- From above the band is a ring, $\cos\theta$ as big.

the dome, radius 1: area = steradians

The dome, seen from straight above, is a disc of area π

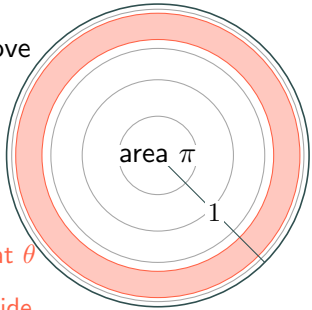
- an eye straight above



a band at θ

$\cos\theta$ as wide

the eye above sees:



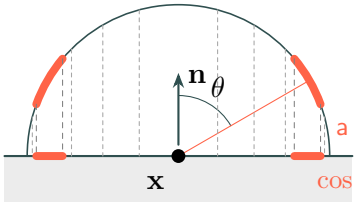
a ring, $\cos\theta$ as wide

the dome, radius 1: area = steradians

- A band at θ gets $L_r \cos\theta$ times its area.
- From above the band is a ring, $\cos\theta$ as big.
- The rings fill a disc of radius 1. Its area is π.

The dome, seen from straight above, is a disc of area π

- an eye straight above

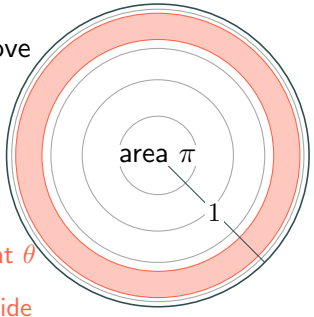


a band at θ

$\cos\theta$ as wide

the dome, radius 1: area = steradians

the eye above sees:

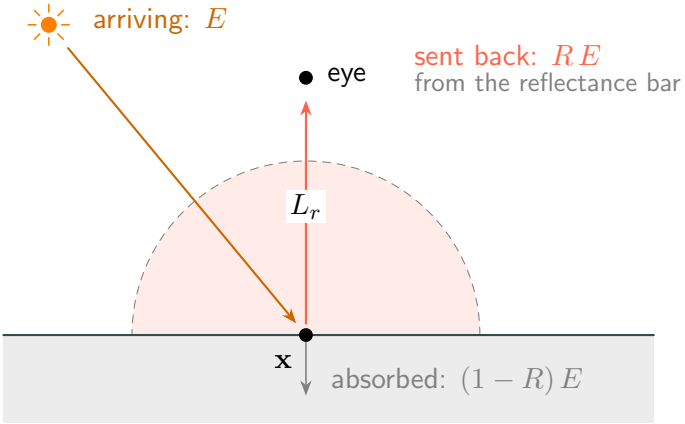


a ring, $\cos\theta$ as wide

- A band at θ gets $L_r \cos\theta$ times its area.
- From above the band is a ring, $\cos\theta$ as big.
- The rings fill a disc of radius 1. Its area is π.
- So all the light sent back is L_r times π.

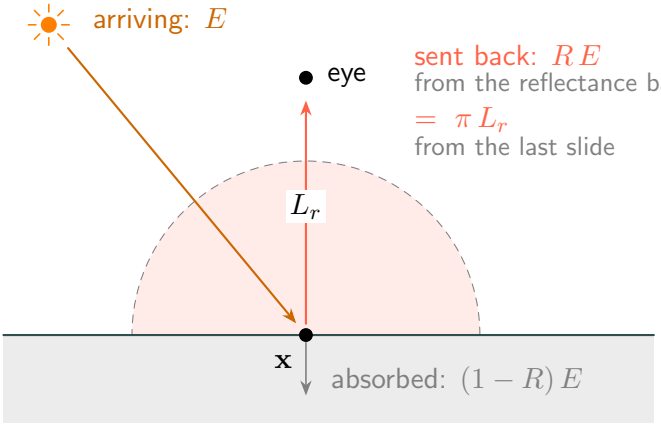
without the slant it would be L_r times 2π

So $\pi L_r = R E$: divide by π, and $k = R/\pi$



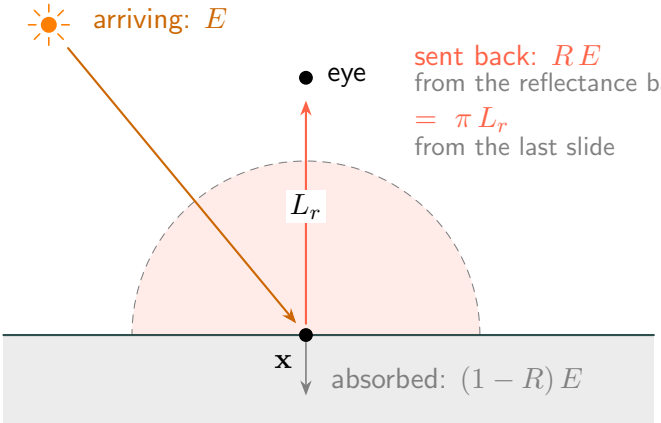
- The bar says the surface sends back $R E$.

So $\pi L_r = R E$: divide by π, and $k = R/\pi$



- The bar says the surface sends back $R E$.
- The dome says the same light is πL_r.

So $\pi L_r = R E$: divide by π, and $k = R/\pi$



- The bar says the surface sends back $R E$.
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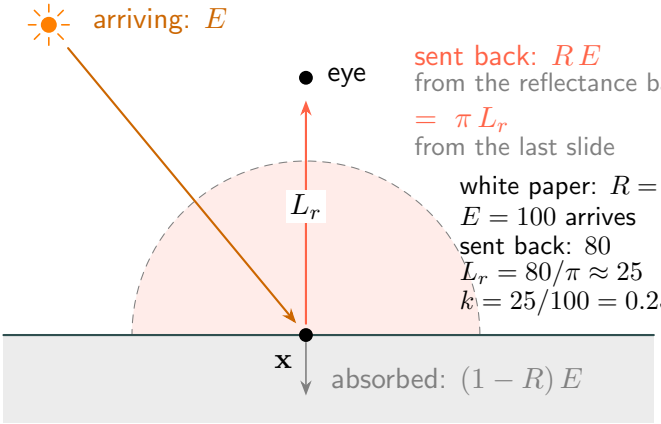
- So $\pi L_r = R E$. Divide by π:

$$L_r = \underbrace{\frac{R}{\pi}}_{k} E$$

So $\pi L_r = R E$: divide by π, and $k = R/\pi$

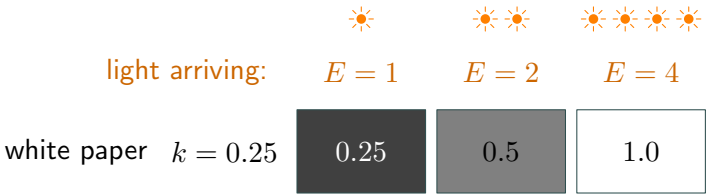


- The bar says the surface sends back $R E$.
- The dome says the same light is πL_r.
- So $\pi L_r = R E$. Divide by π:

$$L_r = \underbrace{\frac{R}{\pi}}_{k} E$$

- We call $k = R/\pi$ the material's factor.

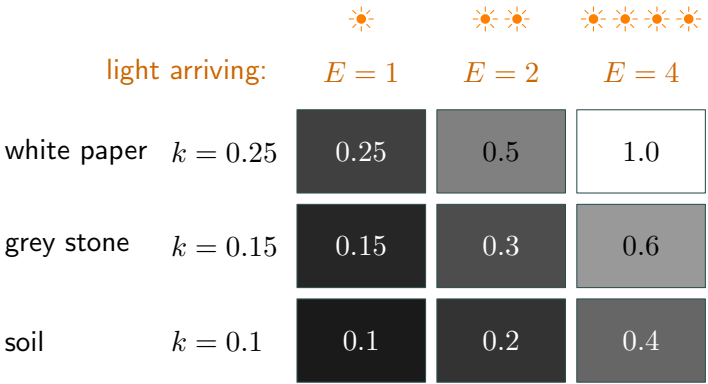
The material's factor k: one number per material



- k is the number that multiplies E

each box: the light to the eye, $L_r = k \times E$

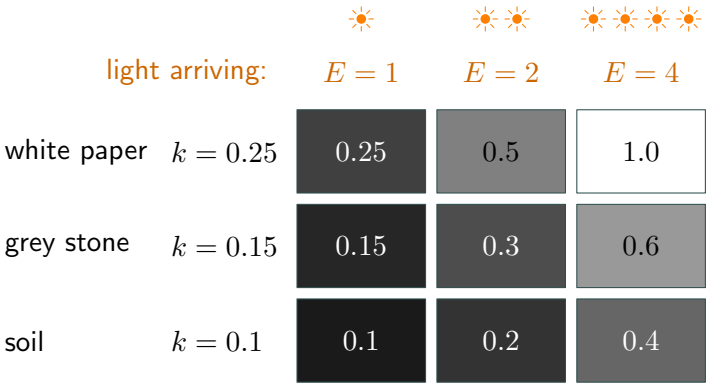
The material's factor k: one number per material



each box: the light to the eye, $L_r = k \times E$

- k is the number that multiplies E
- Every material has its own value of k
- A bigger k looks brighter under the same light

The material's factor k: one number per material



each box: the light to the eye, $L_r = k \times E$

Lambertian: $k = R/\pi$. White paper: $R = 0.8$, $k = 0.8/\pi \approx 0.25$

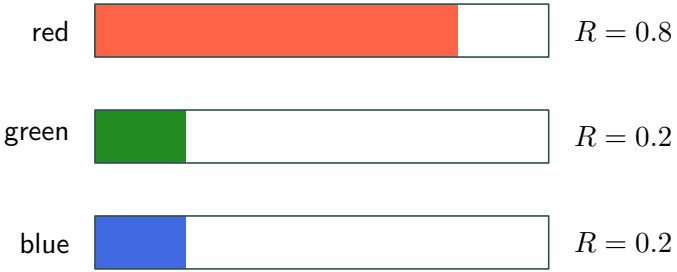
- k is the number that multiplies E
- Every material has its own value of k
- A bigger k looks brighter under the same light
- The lamp sets E. The material sets k.

Colour: one reflectance for each channel

red  $R = 0.8$

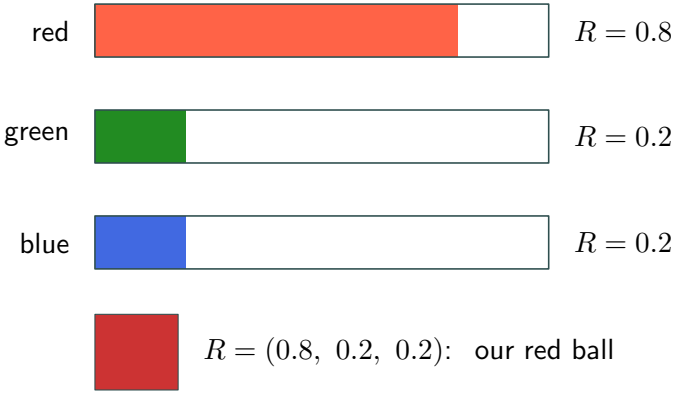
- Three reflectances: red, green, blue

Colour: one reflectance for each channel



- Three reflectances: red, green, blue
- The same equation, once per channel

Colour: one reflectance for each channel



- Three reflectances: red, green, blue
- The same equation, once per channel
- Neutral lamp: the same I in each

Worked: the light leaving our hit

its reflectance R



$(0.8,\ 0.2,\ 0.2)$

$E = 0.8\pi \qquad R = (0.8,\ 0.2,\ 0.2)$

- Try first: the three numbers of L_r

Worked: the light leaving our hit

its reflectance R



$(0.8,\ 0.2,\ 0.2)$

$E = 0.8\pi \qquad R = (0.8,\ 0.2,\ 0.2)$

- Try first: the three numbers of L_r

$k = R/\pi = (0.8/\pi,\ 0.2/\pi,\ 0.2/\pi)$

Worked: the light leaving our hit

its reflectance R



$(0.8,\ 0.2,\ 0.2)$

$E = 0.8\pi \qquad R = (0.8,\ 0.2,\ 0.2)$

- Try first: the three numbers of L_r

$k = R/\pi = (0.8/\pi,\ 0.2/\pi,\ 0.2/\pi)$

red: $\dfrac{0.8}{\pi} \times 0.8\pi = 0.64$

Worked: the light leaving our hit

its reflectance R



$(0.8,\ 0.2,\ 0.2)$

$E = 0.8\pi \qquad R = (0.8,\ 0.2,\ 0.2)$

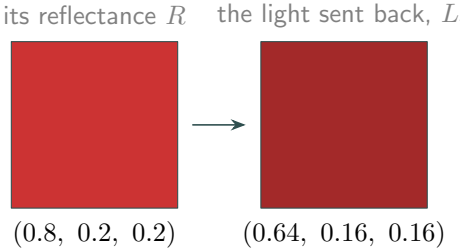
- Try first: the three numbers of L_r

$k = R/\pi = (0.8/\pi,\ 0.2/\pi,\ 0.2/\pi)$

red: $\dfrac{0.8}{\pi} \times 0.8\pi = 0.64$

green, blue: $\dfrac{0.2}{\pi} \times 0.8\pi = 0.16$

Worked: the light leaving our hit



$E = 0.8\pi \qquad R = (0.8,\ 0.2,\ 0.2)$

- Try first: the three numbers of L_r

$k = R/\pi = (0.8/\pi,\ 0.2/\pi,\ 0.2/\pi)$

red: $\dfrac{0.8}{\pi} \times 0.8\pi = 0.64$

green, blue: $\dfrac{0.2}{\pi} \times 0.8\pi = 0.16$

$L_r = (0.64,\ 0.16,\ 0.16)$

- $I = 100\pi$: chosen so that π cancels

Recap: from the lamp to the eye

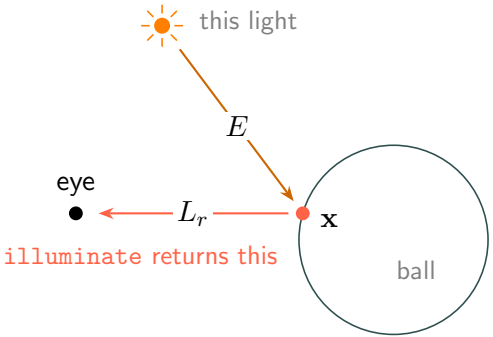
$$L_r = \frac{R}{\pi} \times \max(0,\ \mathbf{n} \cdot \mathbf{l}) \times \frac{I}{r^2}$$

the surface — the tilt — the lamp, the distance

Shading in the ray tracer

Lights, materials, and one loop

Lights and materials: two kinds of objects



Light `illuminate(ray, rec)`

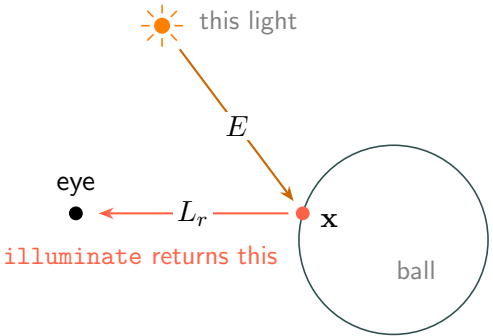
gets the viewing ray and the hit record

returns the colour of the light that $\mathbf{x}$ sends to the eye because of this light

at our hit: $(0.64,\ 0.16,\ 0.16)$

- We call this colour the light's share of the pixel's colour

Lights and materials: two kinds of objects



Light `illuminate(ray, rec)`

gets the viewing ray and the hit record

returns the colour of the light that $\mathbf{x}$ sends to the eye because of this light

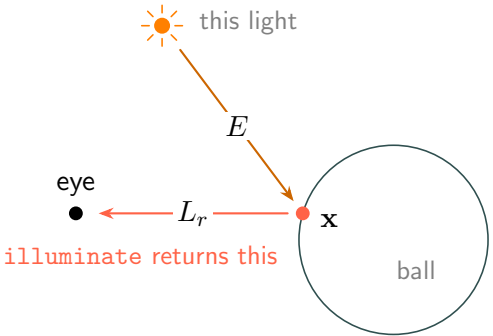
Material `evaluate(l, v, n)`

gets the unit arrows $\mathbf{l}$, $\mathbf{v}$, $\mathbf{n}$ at $\mathbf{x}$

returns the material's factor k, as in $L_r = k\,E$

- We call this colour the light's share of the pixel's colour

Lights and materials: two kinds of objects



Light `illuminate(ray, rec)`

gets the viewing ray and the hit record

returns the colour of the light that $\mathbf{x}$ sends to the eye because of this light

calls `rec.surface.material.evaluate`

Material `evaluate(l, v, n)`

gets the unit arrows $\mathbf{l}$, $\mathbf{v}$, $\mathbf{n}$ at $\mathbf{x}$

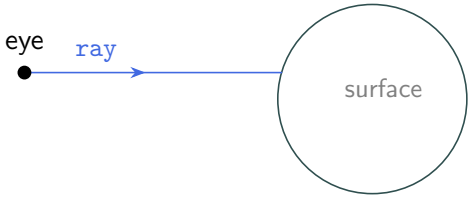
returns the material's factor k, as in $L_r = k\,E$

- We call this colour the light's share of the pixel's colour
- A point light's `illuminate` finds E, calls `evaluate` for k, and returns $k\,E$

A point light's share, in Python

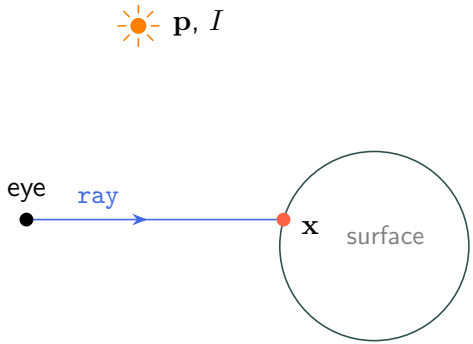
```
class PointLight(Light):
    def illuminate(self, ray, rec):
```





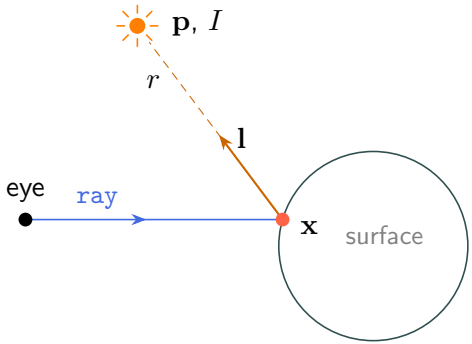
A point light's share, in Python

```
class PointLight(Light):
    def illuminate(self, ray, rec):
        x = ray.evaluate(rec.t)
```



A point light's share, in Python

```
class PointLight(Light):
    def illuminate(self, ray, rec):
        x = ray.evaluate(rec.t)
        r = (self.position - x).length()
        l = (self.position - x) / r
```



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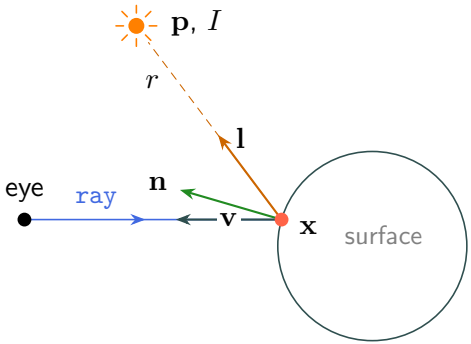
        v = -ray.d / ray.d.length()
```



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        v = -ray.d / ray.d.length()
        E = max(0, n.dot(l)) * self.intensity / r**2
```

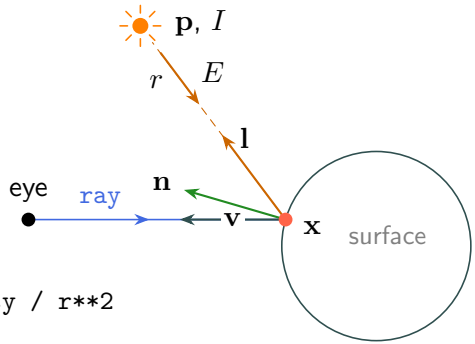

- The max: no light from behind

A point light's share, in Python

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class PointLight(Light):
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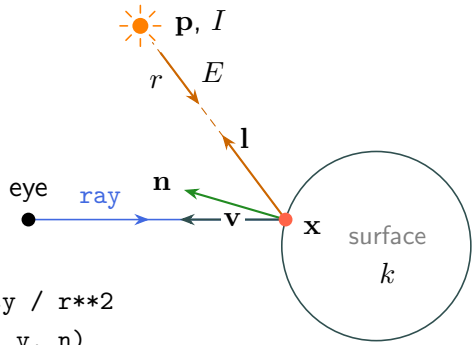
        E = max(0, n.dot(l)) * self.intensity / r**2
        k = rec.surface.material.evaluate(l, v, n)
```



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        E = max(0, n.dot(l)) * self.intensity / r**2
        k = rec.surface.material.evaluate(l, v, n)
        return k * E
```
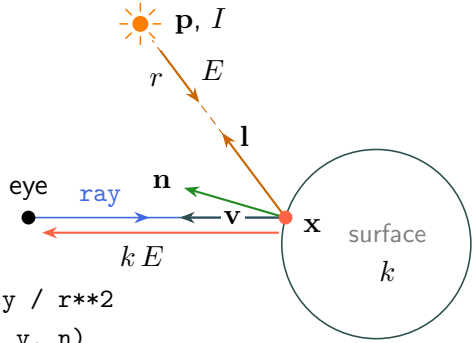
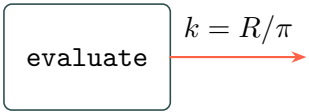



- The max: no light from behind
- k times E, channel by channel

A Lambertian material answers R/π

```
class Lambertian(Material):          # albedo:  the reflectance R
    def evaluate(self, l, v, n):     # the factor k
        return self.albedo / math.pi # the same for any l, v, n
```

- $k = R/\pi$, whatever the directions



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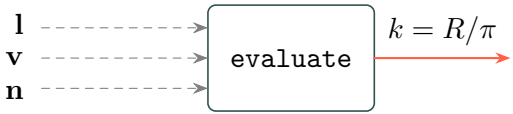

- $k = R/\pi$, whatever the directions
- $\mathbf{l}$, $\mathbf{v}$, $\mathbf{n}$ arrive, but are not used

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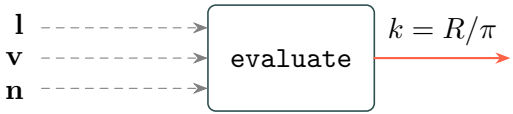
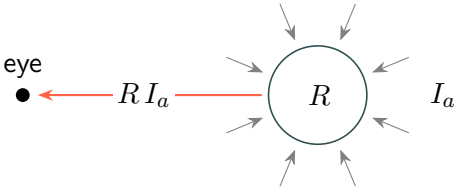


our red ball: $k = (0.8,\ 0.2,\ 0.2)/\pi$

- $k = R/\pi$, whatever the directions
- $\mathbf{l}$, $\mathbf{v}$, $\mathbf{n}$ arrive, but are not used
- Shiny materials will use them: lecture 17

Ambient light: a constant share, $R\,I_a$

```
class AmbientLight(Light):
    def illuminate(self, ray, rec):
        R = rec.surface.material.albedo
        return R * self.intensity
```



- I_a is the amount of ambient light (lecture 11). It is the same at every point
- The material sends the fraction R of it to the eye: its reflectance, as for a lamp
- In the code, I_a is the light's `intensity` and R is the material's `albedo`

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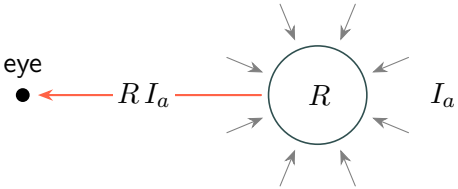

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our numbers: $R = (0.8,\ 0.2,\ 0.2), \quad I_a = 0.1$

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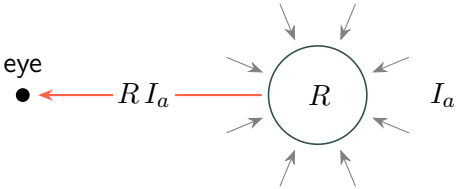

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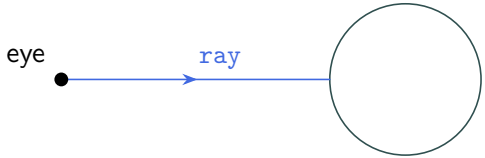
our numbers: $R = (0.8,\ 0.2,\ 0.2)$, $I_a = 0.1$

$R\,I_a = (0.08,\ 0.02,\ 0.02)$ 

- Every hit on the red ball gets this share

Shading a ray: add every light's share

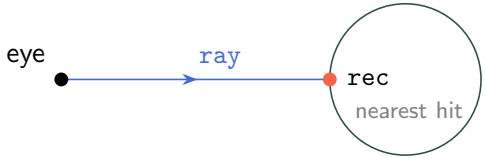
```
def trace(ray):
```



- Lecture 11's steps, written as code

Shading a ray: add every light's share

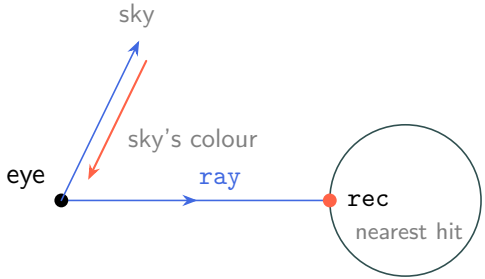
```
def trace(ray):
    rec = scene.hit(ray)
```



- Lecture 11's steps, written as code

Shading a ray: add every light's share

```
def trace(ray):
    rec = scene.hit(ray)
    if rec.t == math.inf:
        a = 0.5 * (ray.d.y + 1.0)
        return Vec3(1.0, 1.0, 1.0).lerp(
            Vec3(0.5, 0.7, 1.0), a)
```



- Lecture 11's steps, written as code
- A miss is tested first, and it returns the sky's colour

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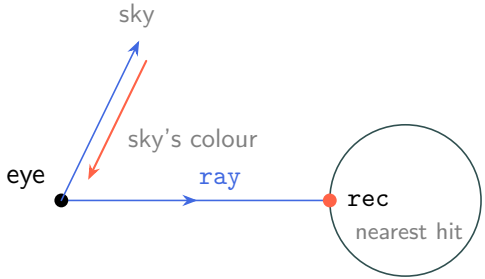
    c = Vec3(0,0,0)
```



$c = \blacksquare$

- Lecture 11's steps, written as code
- A miss is tested first, and it returns the sky's colour

Shading a ray: add every light's share

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        return Vec3(1.0, 1.0, 1.0).lerp(
            Vec3(0.5, 0.7, 1.0), a)
    c = Vec3(0,0,0)
    for light in lights:
        c = c + light.illuminate(ray, rec)
```

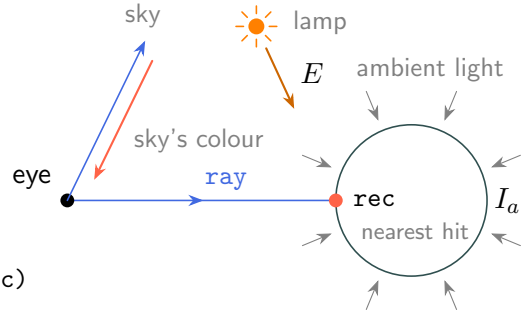

- Lecture 11's steps, written as code
- A miss is tested first, and it returns the sky's colour
- Ambient: one more light in the list

Shading a ray: add every light's share

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    return c
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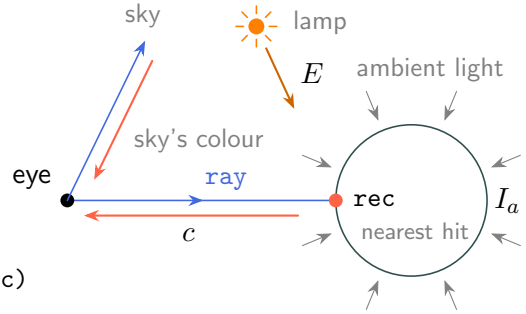

- Lecture 11's steps, written as code
- A miss is tested first, and it returns the sky's colour
- Ambient: one more light in the list
- `trace` returns c, the colour seen along the ray

Worked: the colours of two pixels

pixel $(16, 12)$: the hit $\mathbf{x}$, $\mathbf{n} \cdot \mathbf{l} = 0.8$



$(0.64,\ 0.16,\ 0.16)$ $+$



$(0.08,\ 0.02,\ 0.02)$

the lamp's share

the ambient share

pixel $(16, 4)$: a hit under the ball, $\mathbf{n} \cdot \mathbf{l} = -0.41$

$+$



$(0.08,\ 0.02,\ 0.02)$

- Try first: both pixels' colours

Worked: the colours of two pixels

pixel $(16, 12)$: the hit $\mathbf{x}$, $\mathbf{n} \cdot \mathbf{l} = 0.8$


$(0.64,\ 0.16,\ 0.16)$ $+$

$(0.08,\ 0.02,\ 0.02)$ $=$

$(0.72,\ 0.18,\ 0.18)$

the lamp's share — the ambient share — the pixel

pixel $(16, 4)$: a hit under the ball, $\mathbf{n} \cdot \mathbf{l} = -0.41$

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$\max(0,\ -0.41) = 0$

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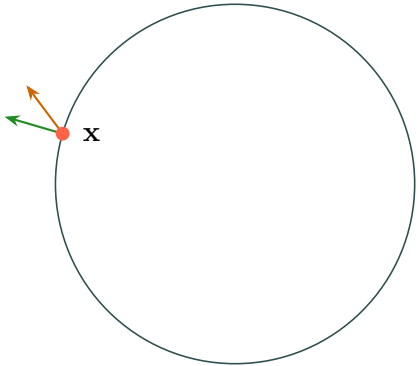
$(0.08,\ 0.02,\ 0.02)$

$\max(0,\ -0.41) = 0$

- Try first: both pixels' colours
- Without ambient light: pure black

Around the ball, $\mathbf{n} \cdot \mathbf{l}$ falls to zero

the lamp: up and to the left

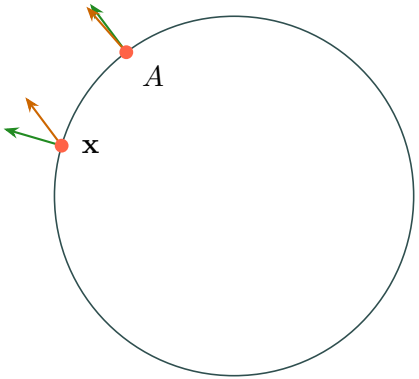


the lamp's share at four points

$\mathbf{x}$: $\mathbf{n} \cdot \mathbf{l} = 0.80$

Around the ball, $\mathbf{n} \cdot \mathbf{l}$ falls to zero

the lamp: up and to the left



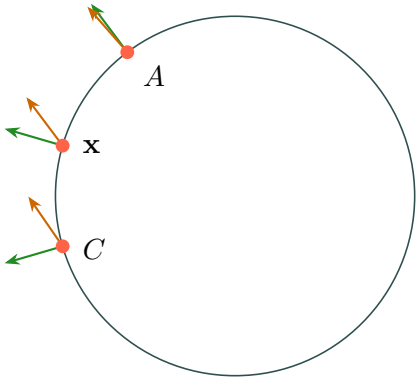
the lamp's share at four points

A: $\mathbf{n} \cdot \mathbf{l} = 1.00$

$\mathbf{x}$: $\mathbf{n} \cdot \mathbf{l} = 0.80$

Around the ball, $\mathbf{n} \cdot \mathbf{l}$ falls to zero

the lamp: up and to the left



the lamp's share at four points

A: $\mathbf{n} \cdot \mathbf{l} = 1.00$

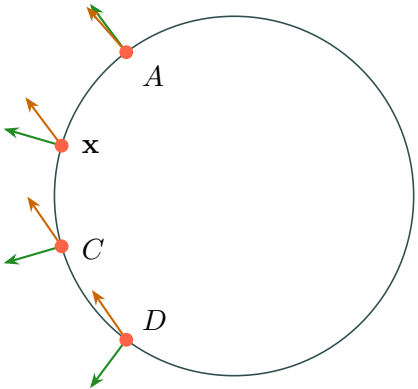
$\mathbf{x}$: $\mathbf{n} \cdot \mathbf{l} = 0.80$

C: $\mathbf{n} \cdot \mathbf{l} = 0.32$

- Turning away: darker

Around the ball, $\mathbf{n} \cdot \mathbf{l}$ falls to zero

the lamp: up and to the left

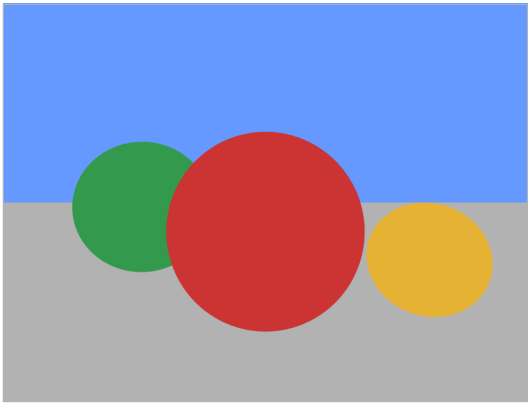


the lamp's share at four points

- A: $\mathbf{n} \cdot \mathbf{l} = 1.00$
- $\mathbf{x}$: $\mathbf{n} \cdot \mathbf{l} = 0.80$
- C: $\mathbf{n} \cdot \mathbf{l} = 0.32$
- D: $\mathbf{n} \cdot \mathbf{l} = -0.32$, so 0

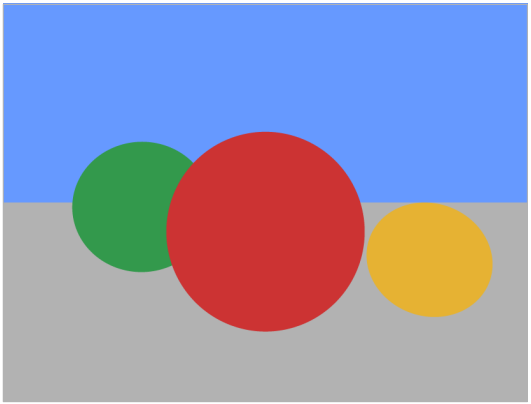
- Turning away: darker
- The fading shows the roundness

Before and after: flat colour, then a lamp



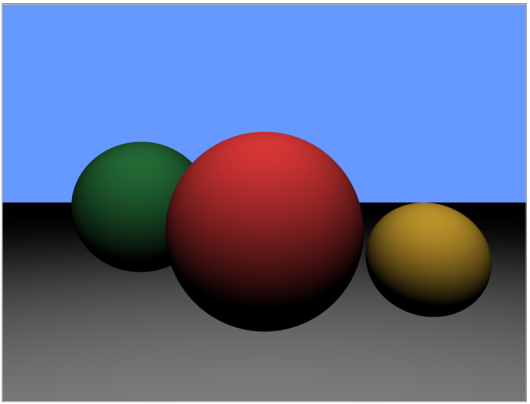
each pixel: its surface's R

Before and after: flat colour, then a lamp



each pixel: its surface's R

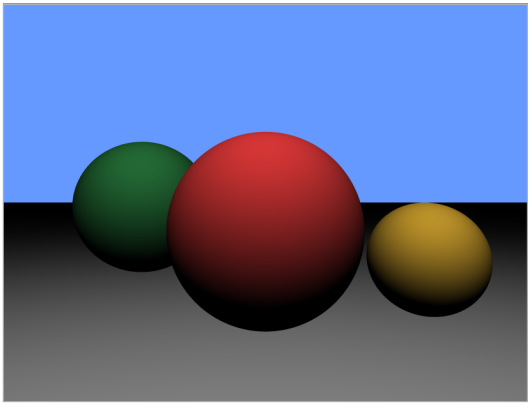
- The fading makes the balls round



each pixel: L_r from our lamp

- Black: the lamp is behind

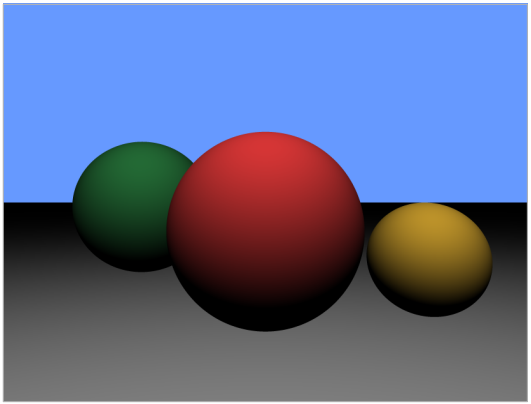
Ambient light fills in the black parts



the lamp alone

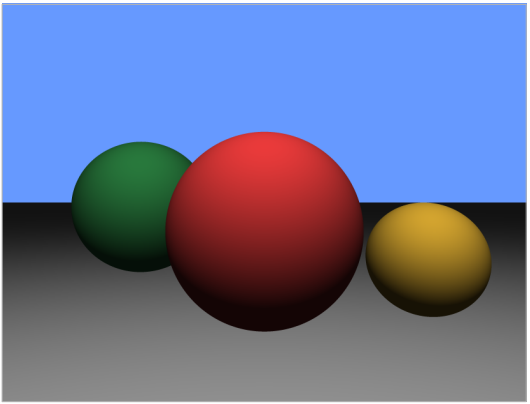
- Away from the lamp: black

Ambient light fills in the black parts



the lamp alone

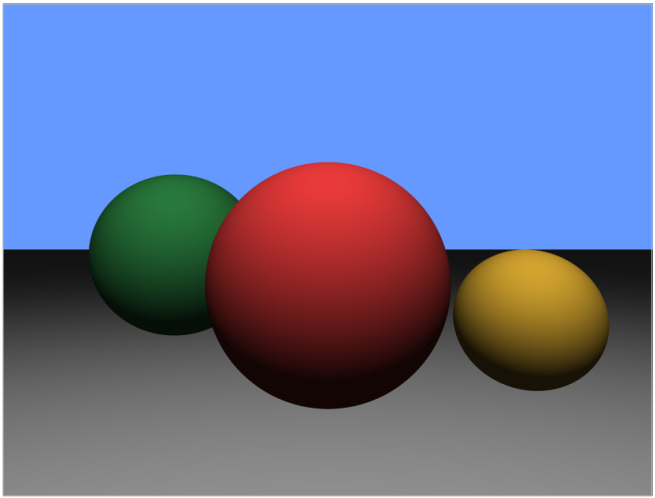
- Away from the lamp: black



the lamp and ambient light, $I_a = 0.1$

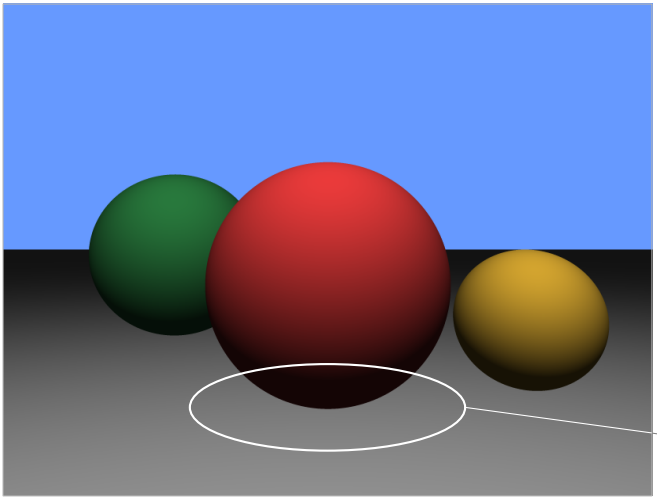
- A little light everywhere

No shadows yet: the balls seem to float



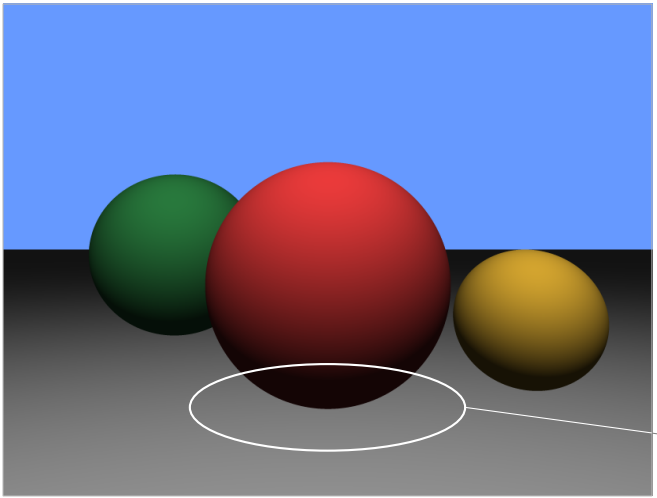
- Shading shows each ball's shape

No shadows yet: the balls seem to float



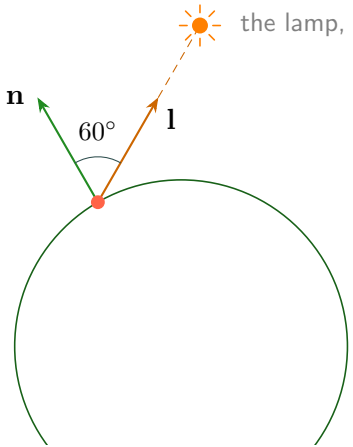
- Shading shows each ball's shape
- The floor under each ball is lit

No shadows yet: the balls seem to float



- Shading shows each ball's shape
- The floor under each ball is lit
- Nothing ever blocks the lamp

Quick check 2: shade a green point



$R = (0.2,\ 0.6,\ 0.3), \quad \mathbf{n} \cdot \mathbf{l} = 0.5, \quad r = 10$

$I = 100\pi, \quad I_a = 0.1$

1. The pixel's colour?

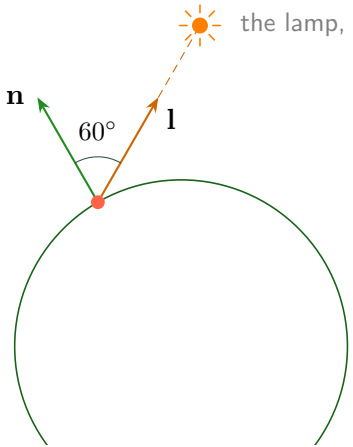
   _____

2. Lamp behind, $\mathbf{n} \cdot \mathbf{l} = -0.5$: the colour?

   _____

- Lamp's share plus ambient share

Quick check 2: shade a green point



$R = (0.2,\ 0.6,\ 0.3), \quad \mathbf{n} \cdot \mathbf{l} = 0.5, \quad r = 10$

$I = 100\pi, \quad I_a = 0.1$

1. The pixel's colour?

$(0.10,\ 0.30,\ 0.15) + (0.02,\ 0.06,\ 0.03) = (0.12,\ 0.36,\ 0$

2. Lamp behind, $\mathbf{n} \cdot \mathbf{l} = -0.5$: the colour?

$(0,\ 0,\ 0) + (0.02,\ 0.06,\ 0.03) = (0.02,\ 0.06,\ 0.03)$

- Lamp's share plus ambient share
- Lamp behind: only ambient is left

Recap: one pixel's colour

$$\text{colour} = \underbrace{R\, I_a}_{\text{the ambient share}} + \underbrace{\sum_i \frac{R}{\pi} \max(0,\ \mathbf{n} \cdot \mathbf{l}_i) \frac{I_i}{r_i^2}}_{\text{the share of point light } i}$$

The sum runs over the point lights in the list: $i = 1, 2, \ldots$ Our scene has one

Summary

1 How much light arrives

irradiance $E = \max(0,\ \mathbf{n} \cdot \mathbf{l})\,\dfrac{I}{r^2}$

twice as far: a quarter; tilted $60°$: half

2 What a Lambertian surface sends back

$$L_r = \frac{R}{\pi}\,E$$

the same for every eye; one R per channel

3 In code: lights and materials

`light.illuminate`: E, then $k\,E$

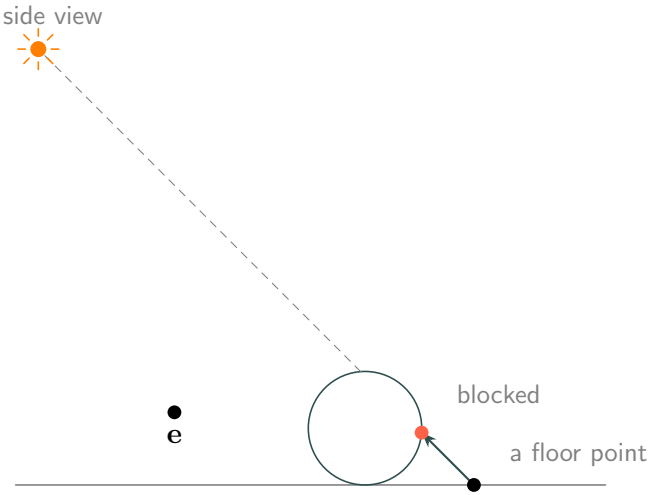
`material.evaluate`: $k = R/\pi$

4 A pixel: the sum of the shares

colour $= R\,I_a + \sum_i k\,E_i$

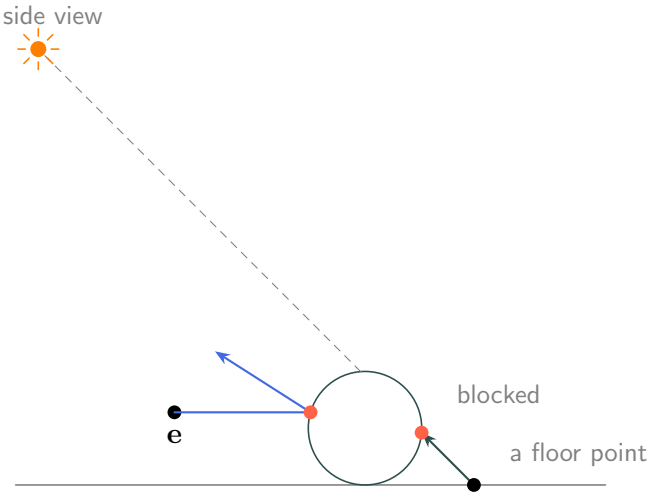
one term per lamp; no shadows yet

Next lecture: shadows and mirrors



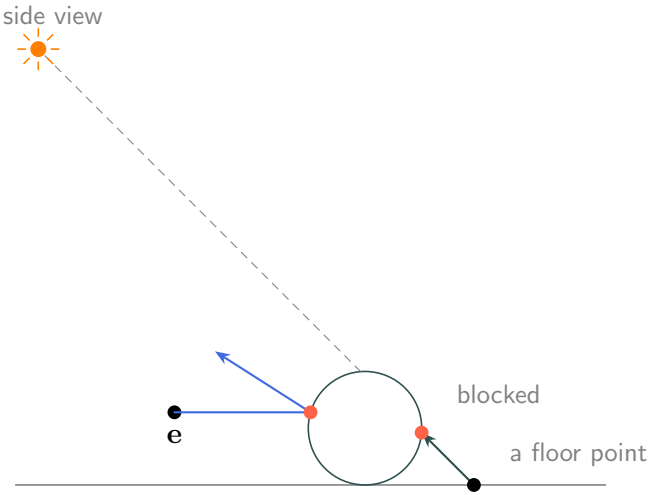
- Is the lamp blocked? A shadow ray

Next lecture: shadows and mirrors



- Is the lamp blocked? A shadow ray
- A mirror: the ray bounces on

Next lecture: shadows and mirrors



- Is the lamp blocked? A shadow ray
- A mirror: the ray bounces on
- And the shiny top layer

Exit Ticket

Today's question *(2 minutes)*

A lamp with $I = 50\pi$ is 5 away from a point of a surface. There, $\mathbf{n} \cdot \mathbf{l} = 0.8$.

(a) Find the irradiance E at the point.

(b) The surface is Lambertian, with reflectance $R = (0.5, 0.25, 0.1)$. Find L_r, the colour the eye sees.

CS 453 — Exit Ticket #16

Name: _____

Roll #: _____ Date: _____

Answer:

Hand it to me on your way out.
No name, no attendance.