Lecture 19: Why Whitted Is Wrong

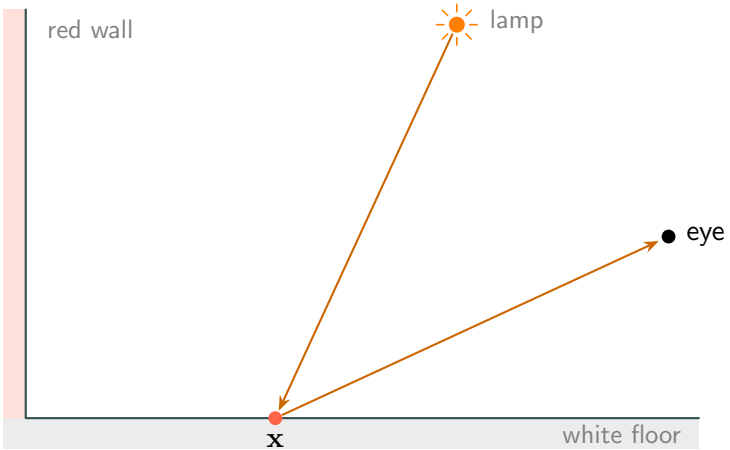
Radiance, the BRDF, and the Rendering Equation

CS 453 — Computer Graphics

Fakhir Shaheen

Forman Christian University

Today: light that bounces off a wall first



- Our tracer: lamp, surface, eye

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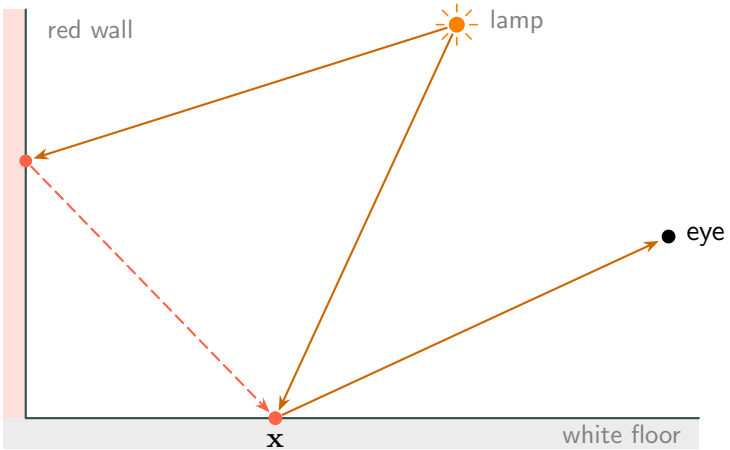
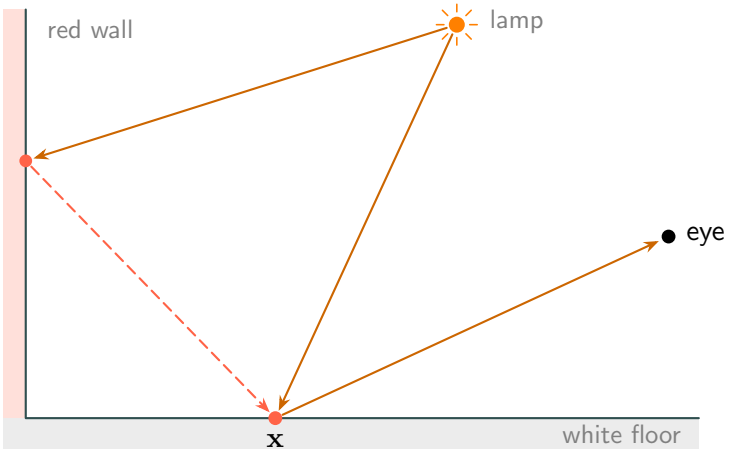


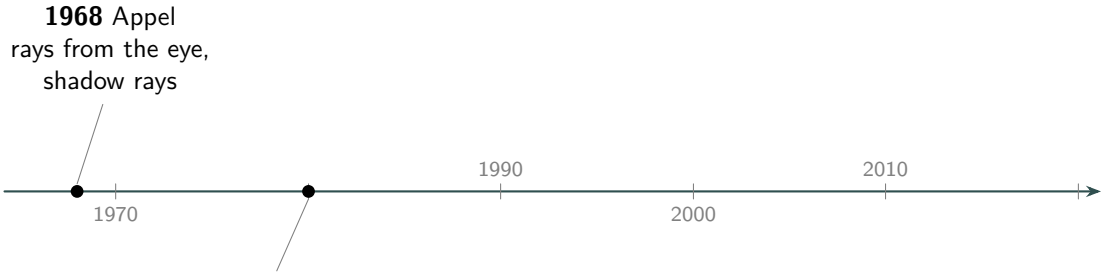
- Our tracer: lamp, surface, eye
- Missed: lamp, red wall, floor, eye

Today: light that bounces off a wall first

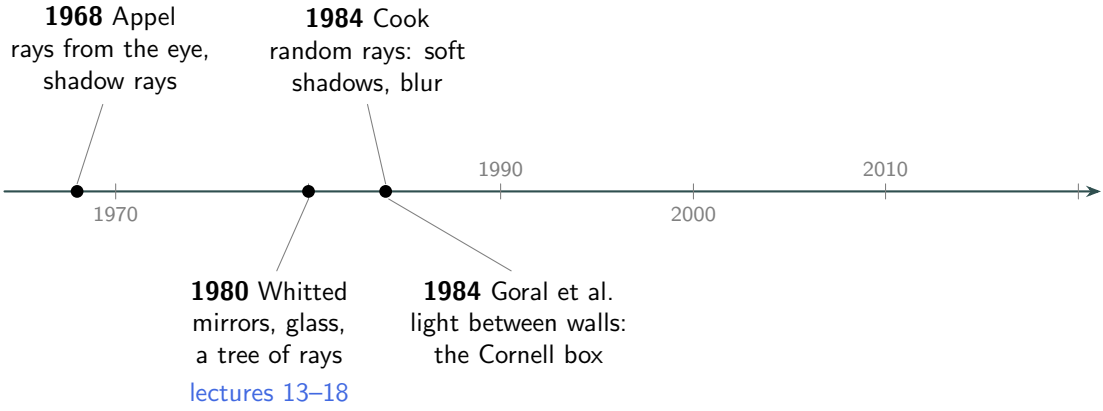


- Our tracer: lamp, surface, eye
- Missed: lamp, red wall, floor, eye
- Today: one equation for every path

Fifty years of rays, in one line

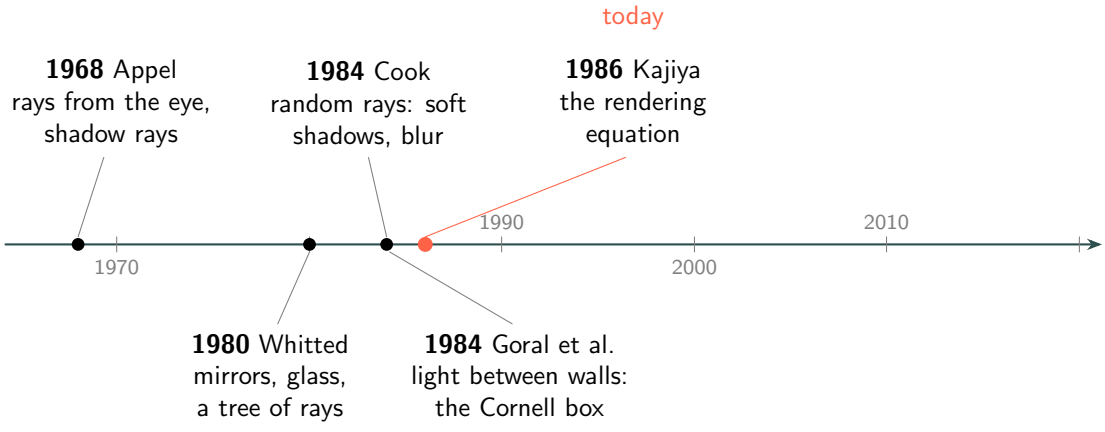


Fifty years of rays, in one line

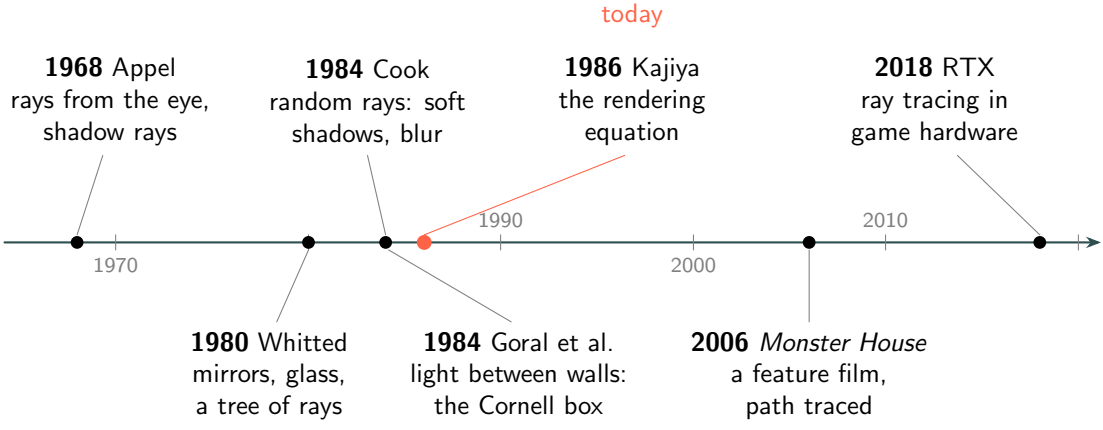


lectures 13–18

Fifty years of rays, in one line



Fifty years of rays, in one line



What Whitted's tracer misses

Light that bounces between dull surfaces

Three photographs our tracer cannot make



red card, white cast: pink

Three photographs our tracer cannot make



red card, white cast: pink



lit, with no lamp in view

photos 2, 3: aismallard, CC BY-SA 3.0, via Wikimedia Commons

Three photographs our tracer cannot make



red card, white cast: pink



lit, with no lamp in view



the closet, the corners: darker

photos 2, 3: aismallard, CC BY-SA 3.0, via Wikimedia Commons

Three photographs our tracer cannot make



red card, white cast: pink



lit, with no lamp in view

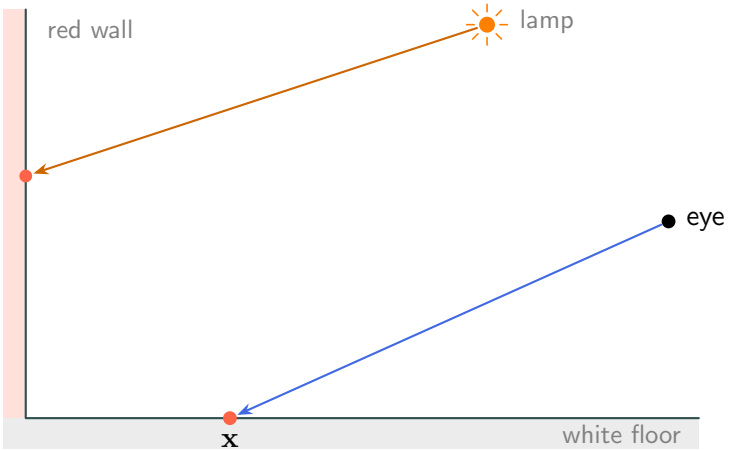


the closet, the corners: darker

photos 2, 3: aismallard, CC BY-SA 3.0, via Wikimedia Commons

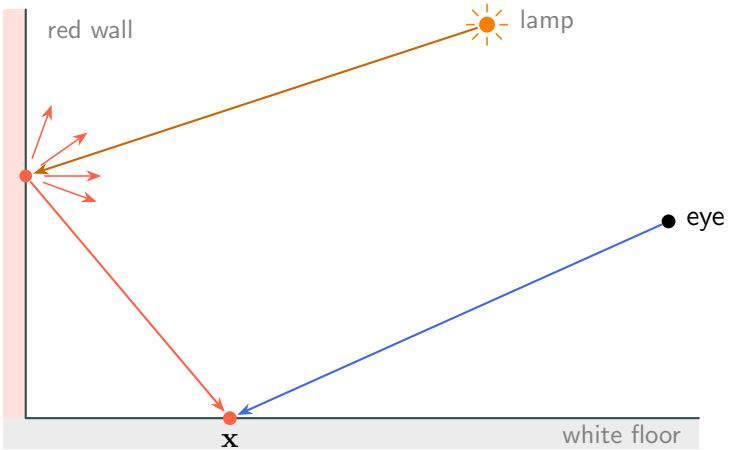
- Real light; our tracer gets each wrong

Colour bleeding: the red wall lights the floor



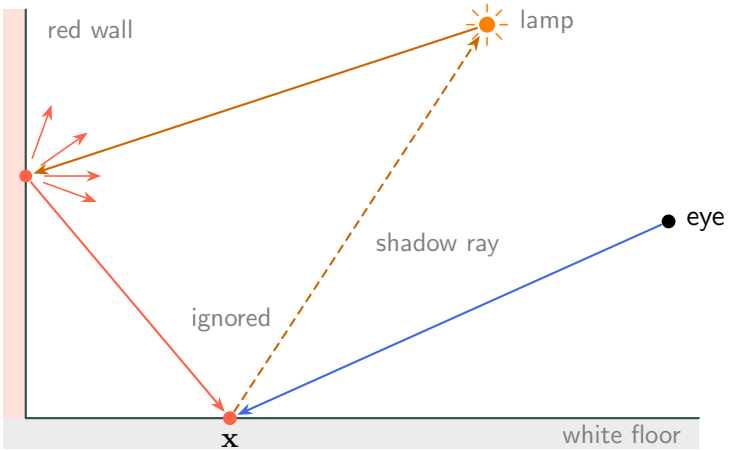
- The lamp lights the red wall

Colour bleeding: the red wall lights the floor



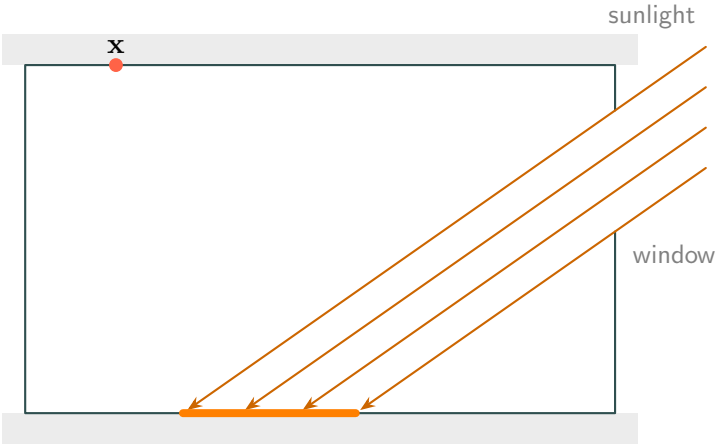
- The lamp lights the red wall
- The red wall lights the floor

Colour bleeding: the red wall lights the floor



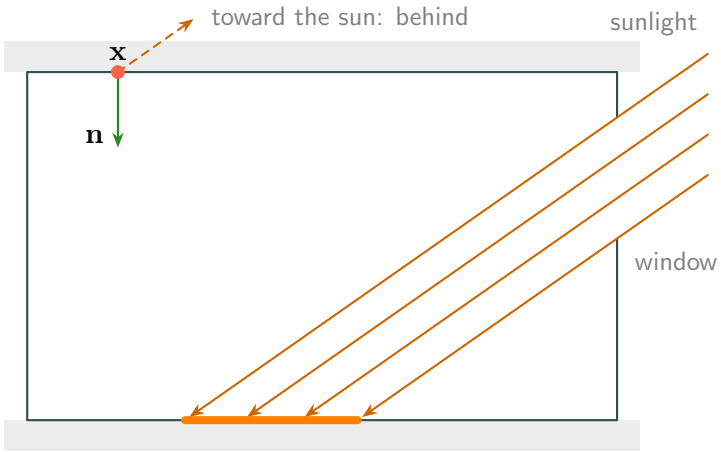
- The lamp lights the red wall
- The red wall lights the floor
- At $\mathbf{x}$, our tracer asks only the lamp

No lamp in view: the ceiling still glows



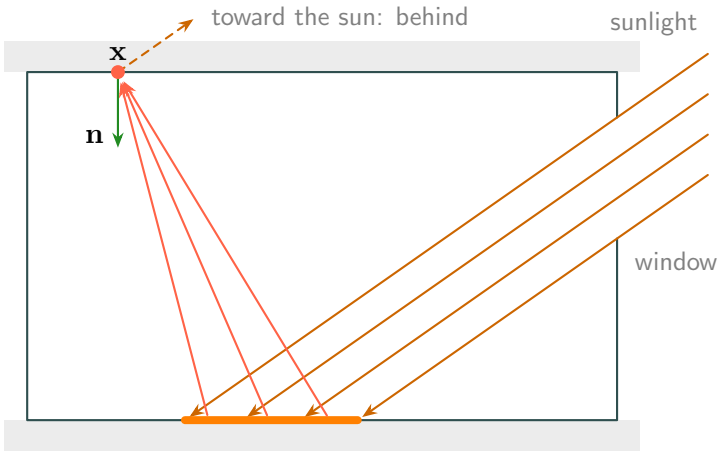
- The sun reaches only the floor

No lamp in view: the ceiling still glows



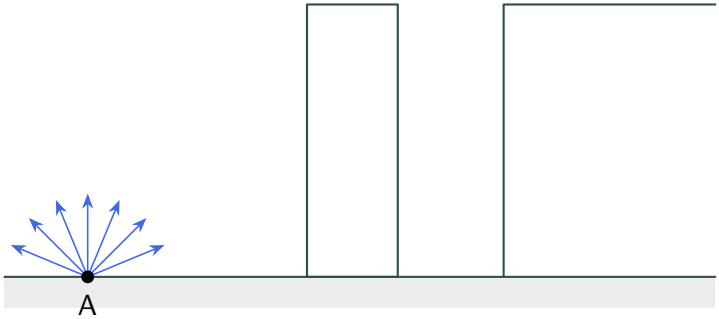
- The sun reaches only the floor
- Ceiling: no lamp in view, so black

No lamp in view: the ceiling still glows



- The sun reaches only the floor
- Ceiling: no lamp in view, so black
- Yet it glows: the floor lights it

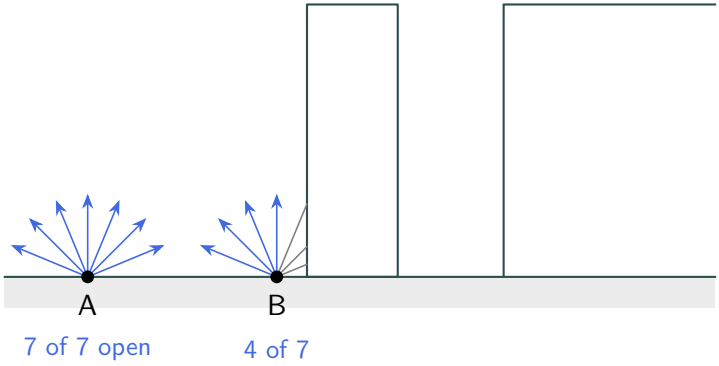
Corners are darker: fewer open directions



7 of 7 open

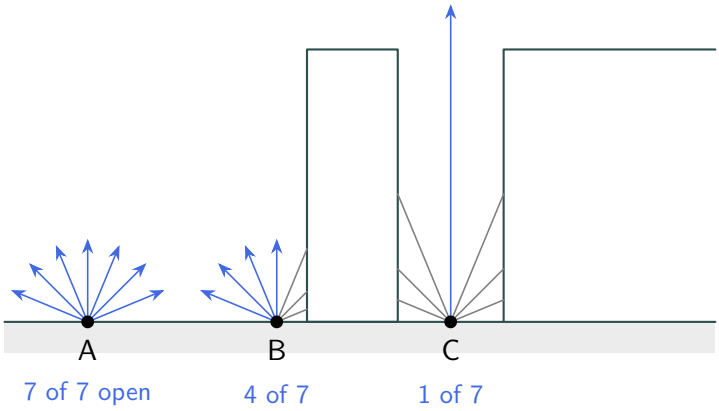
- Count the open directions

Corners are darker: fewer open directions



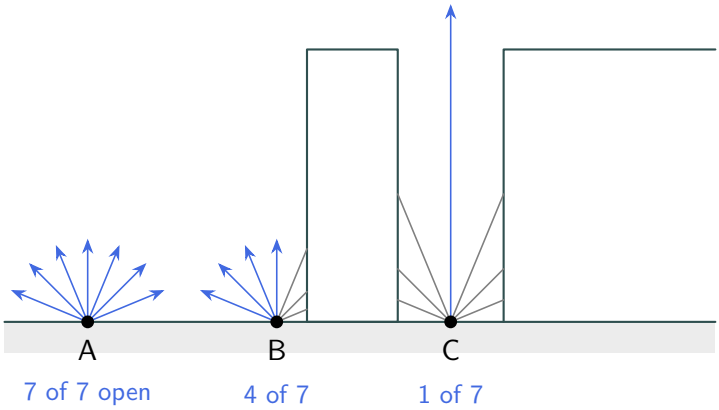
- Count the open directions

Corners are darker: fewer open directions



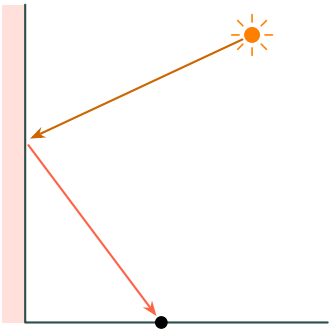
- Count the open directions
- Fewer open: less of the room's light

Corners are darker: fewer open directions

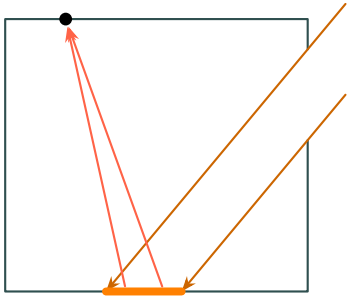


- Count the open directions
- Fewer open: less of the room's light
- Our tracer: A, B, C equally bright

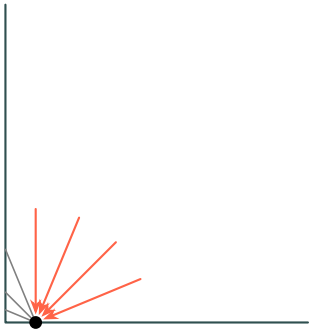
All three: light bounced between diffuse surfaces



red wall $\rightarrow$ floor

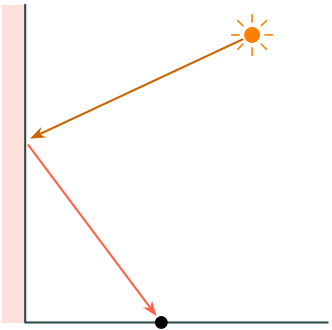


sunlit floor $\rightarrow$ ceiling

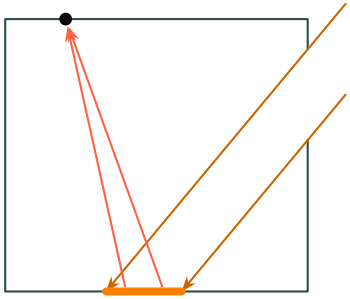


the room $\rightarrow$ floor

All three: light bounced between diffuse surfaces



red wall → floor



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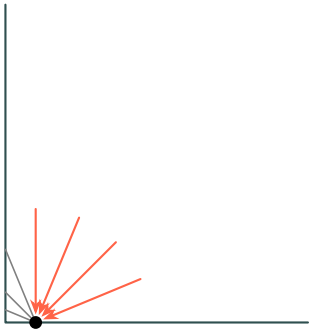
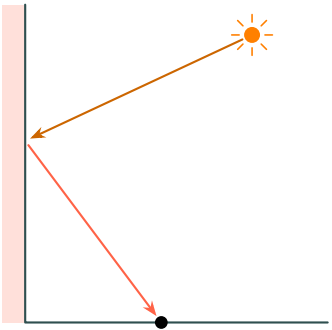


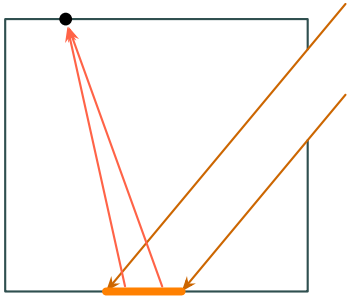
the room → floor

- Light from one dull surface to another

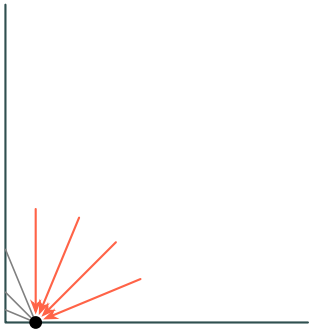
All three: light bounced between diffuse surfaces



red wall → floor



sunlit floor → ceiling

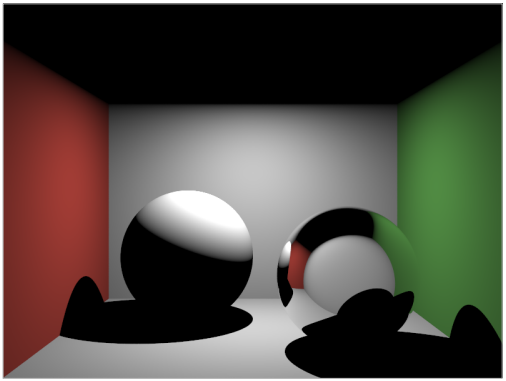


the room → floor

- Light from one dull surface to another

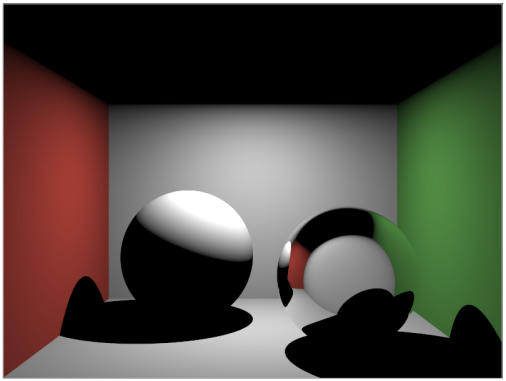
names: indirect light, global illumination

One room, rendered: the lamps only, and every bounce



our tracer: the lamps only

One room, rendered: the lamps only, and every bounce



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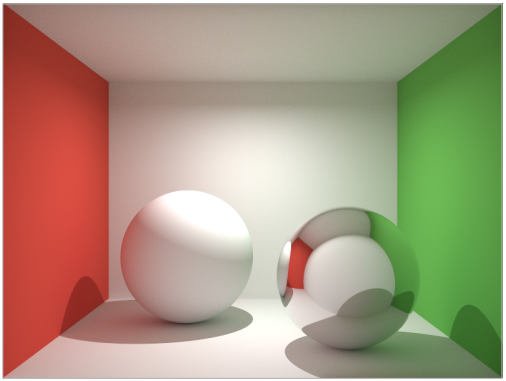
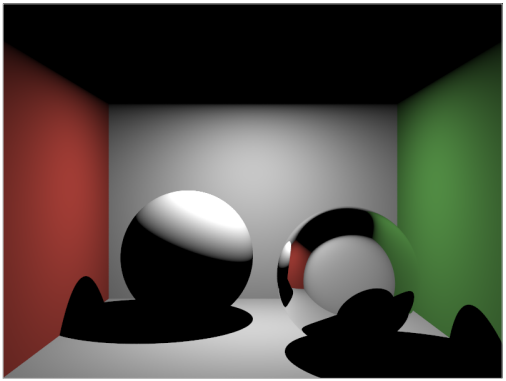


every bounce: today's equation

One room, rendered: the lamps only, and every bounce



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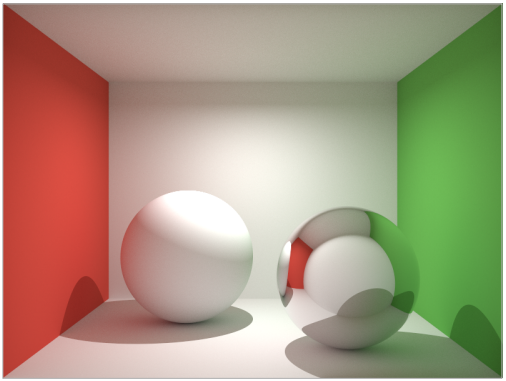
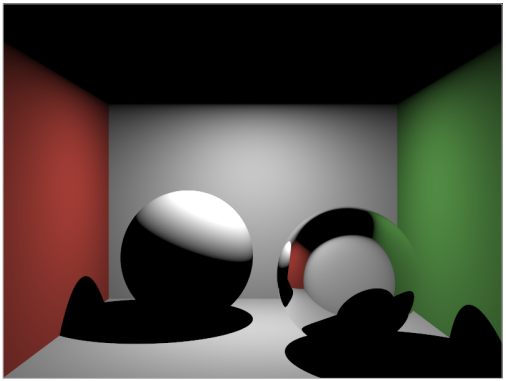


every bounce: today's equation

- White walls tinted red, green

One room, rendered: the lamps only, and every bounce



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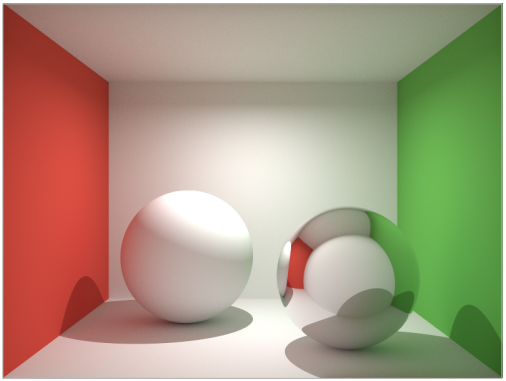
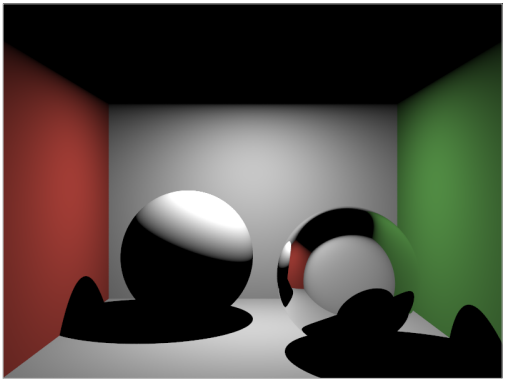


every bounce: today's equation

- White walls tinted red, green
- Ceiling lit

One room, rendered: the lamps only, and every bounce



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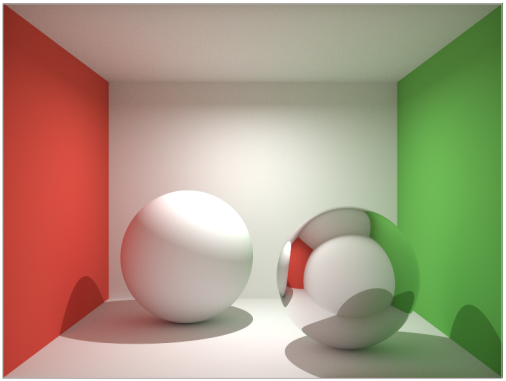
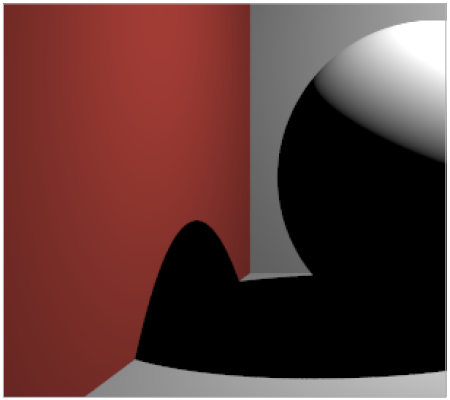


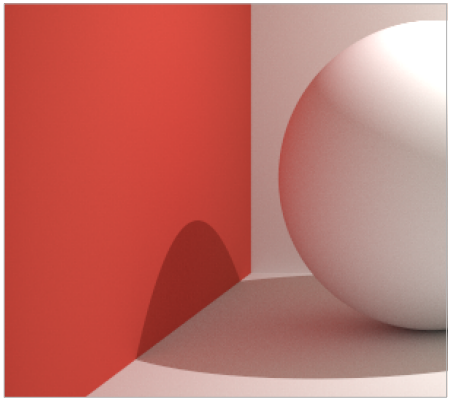
every bounce: today's equation

- White walls tinted red, green
- Ceiling lit
- Dark only where it is tight

Zoom: the red wall tints the floor and the ball



our tracer: the lamps only



every bounce

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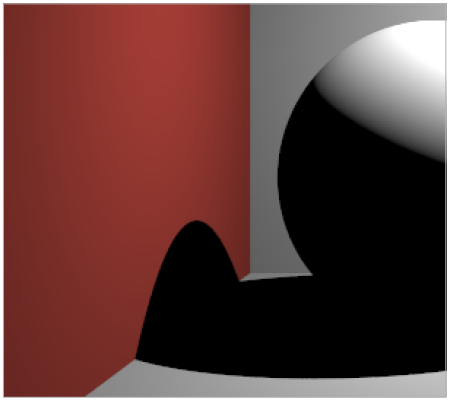
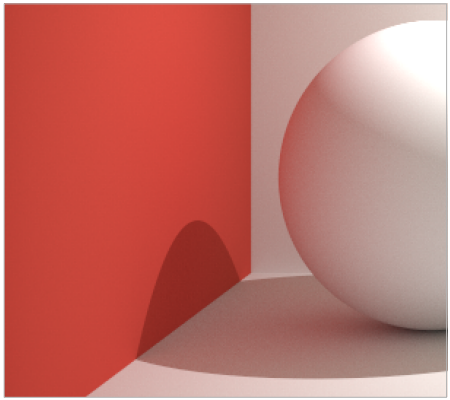


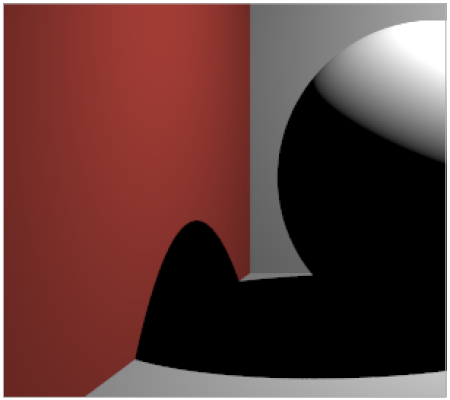
our tracer: the lamps only



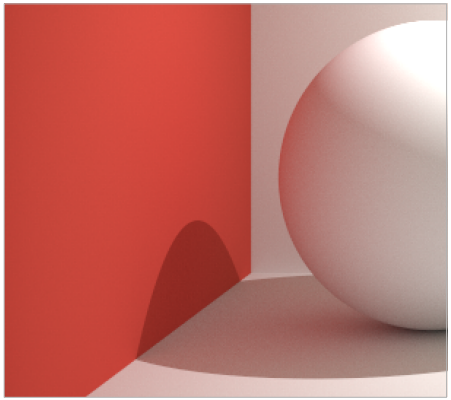
every bounce

- Lamps only: white, grey, black

Zoom: the red wall tints the floor and the ball



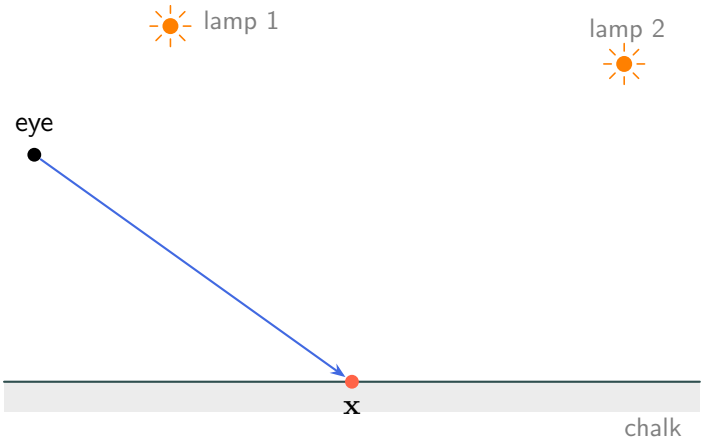
our tracer: the lamps only



every bounce

- Lamps only: white, grey, black
- Every bounce: pink, from the wall

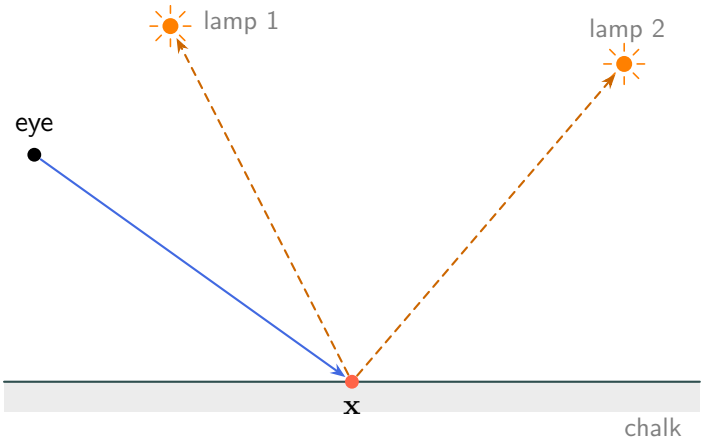
Our chalk hit: add up the lamps, then stop



- Chalk hit: one shadow ray per lamp

```
if isinstance(material, Lambertian):  return the lamps' shares, added up
```

Our chalk hit: add up the lamps, then stop



- Chalk hit: one shadow ray per lamp
- Add the lit lamps' shares

```
if isinstance(material, Lambertian):  return the lamps' shares, added up
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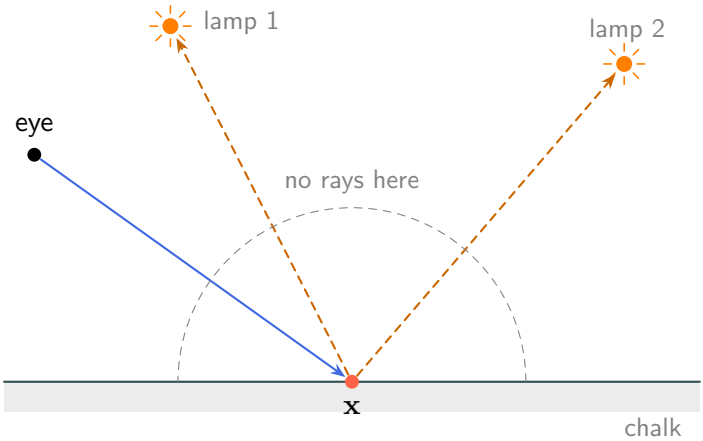
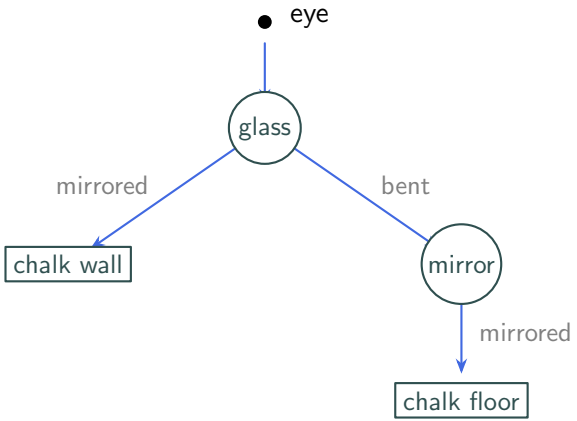
Our chalk hit: add up the lamps, then stop



- Chalk hit: one shadow ray per lamp
- Add the lit lamps' shares
- Then return: no more rays

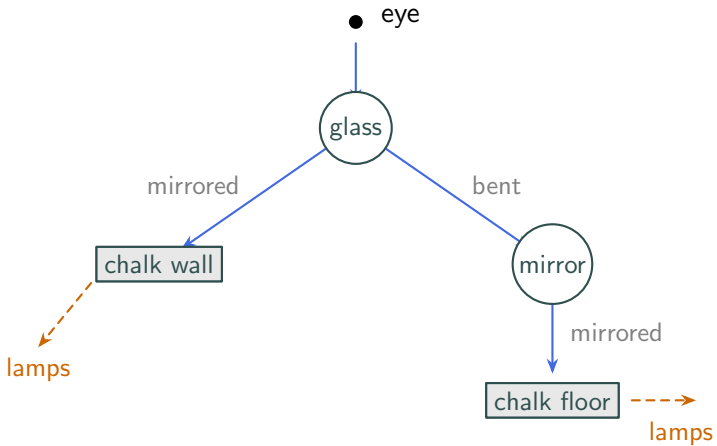
```
if isinstance(material, Lambertian):  return the lamps' shares, added up
```

A tree of rays: leaves at every chalk hit



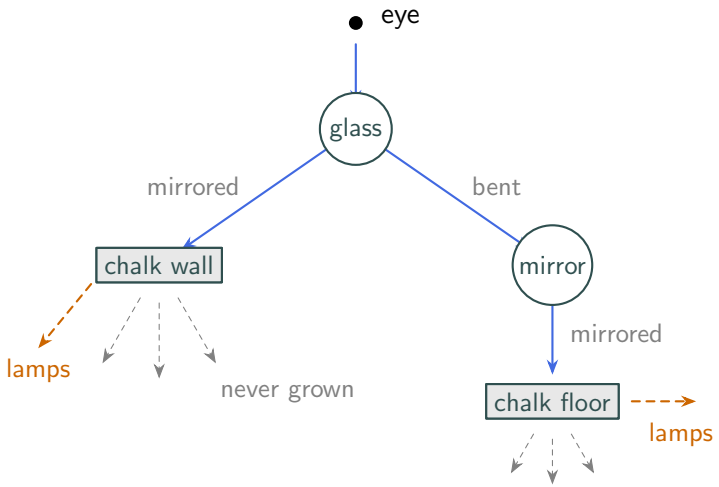
- Mirror and glass: the tree branches

A tree of rays: leaves at every chalk hit



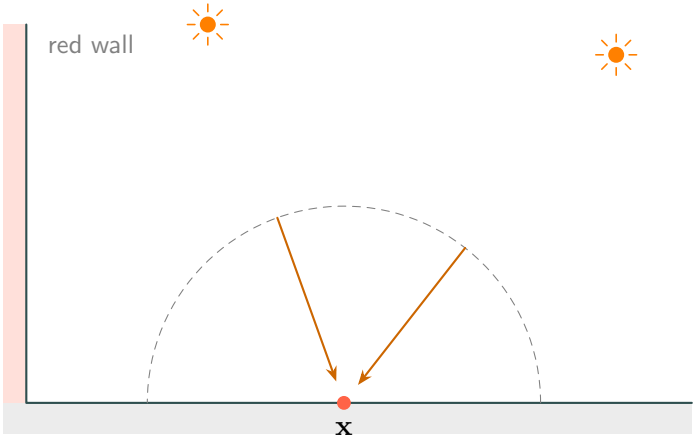
- Mirror and glass: the tree branches
- Chalk: a leaf, lamps only

A tree of rays: leaves at every chalk hit



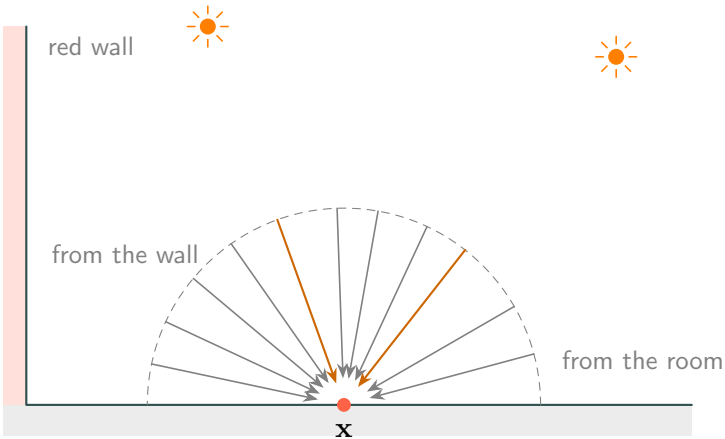
- Mirror and glass: the tree branches
- Chalk: a leaf, lamps only
- Chalk to chalk: never followed

At a leaf, light arrives from every direction



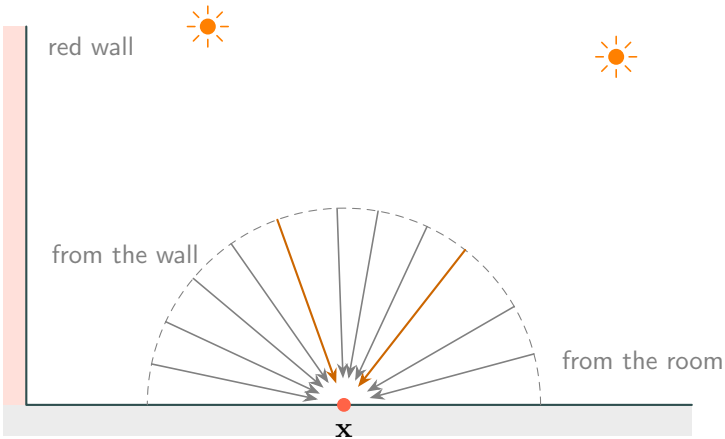
- Our tracer counts two directions

At a leaf, light arrives from every direction



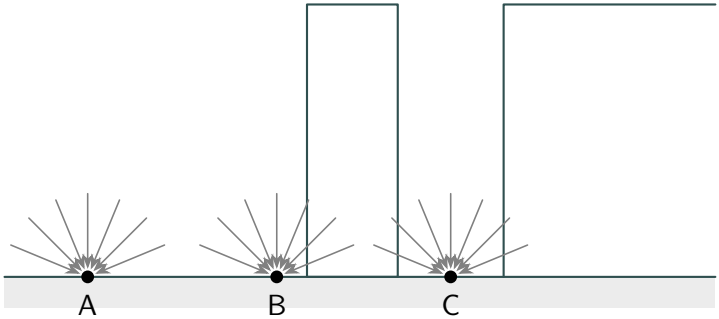
- Our tracer counts two directions
- Light comes from all the others too

At a leaf, light arrives from every direction



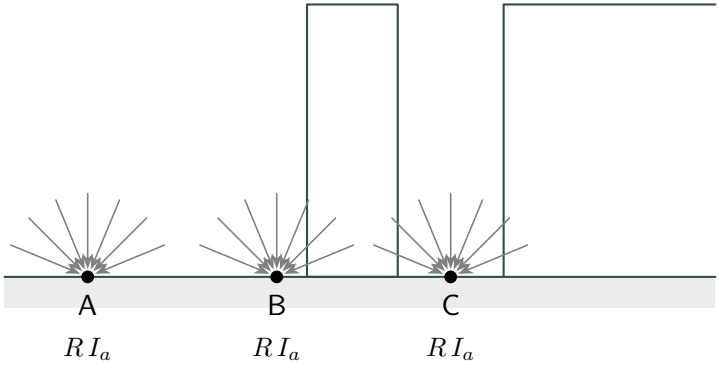
- Our tracer counts two directions
- Light comes from all the others too
- They are dropped, or guessed

Ambient light: one guess for the whole dome



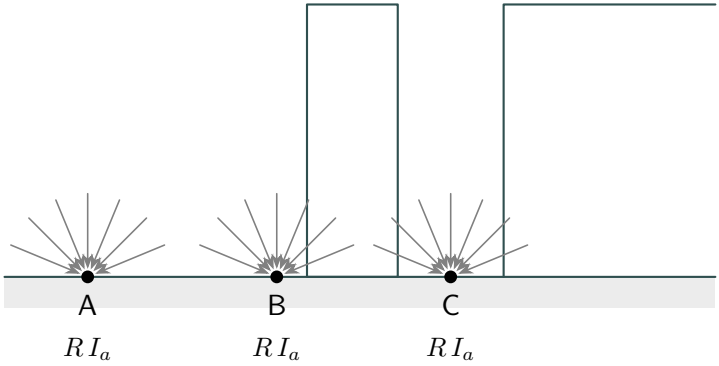
- Lecture 16: I_a from every side

Ambient light: one guess for the whole dome



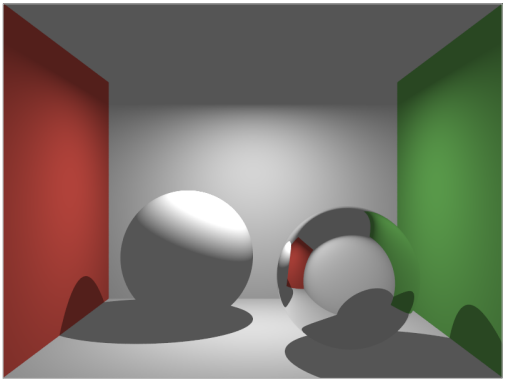
- Lecture 16: I_a from every side
- The same $R\,I_a$ at A, B, C

Ambient light: one guess for the whole dome

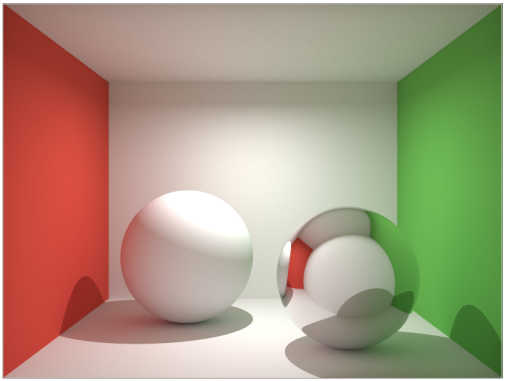


- Lecture 16: I_a from every side
- The same $R\,I_a$ at A, B, C
- No dark corners, no red tint

Rendered: ambient lifts everything by the same amount

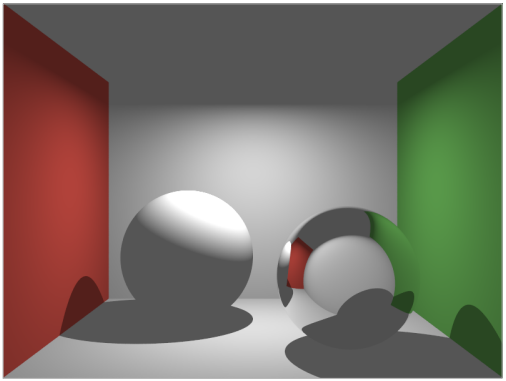


our tracer + ambient, $I_a = 0.12$

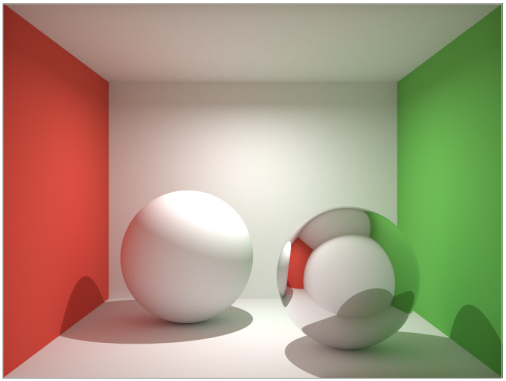


every bounce

Rendered: ambient lifts everything by the same amount



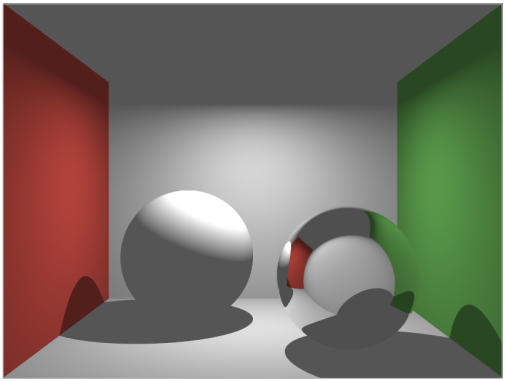
our tracer + ambient, $I_a = 0.12$



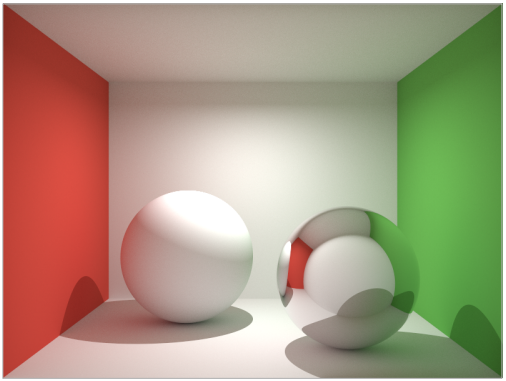
every bounce

- Ambient: flat grey shadows

Rendered: ambient lifts everything by the same amount



our tracer + ambient, $I_a = 0.12$



every bounce

- Ambient: flat grey shadows
- No tint, no dark contact

Quick check 1: can our tracer show it?

In lecture 18's ray tracer . . .	**yes or no?**
1. a ball's shadow on the floor, from the lamp	
2. the red wall, seen in a mirror ball	
3. a white floor turning pink beside the red wall	
4. the room, upside down in a glass ball	
5. the underside of a table, lit by the floor	

- Follow the light back: does it pass chalk to chalk?

Quick check 1: can our tracer show it?

In lecture 18's ray tracer . . .	**yes or no?**
1. a ball's shadow on the floor, from the lamp	yes: a shadow ray
2. the red wall, seen in a mirror ball	yes: a mirror ray
3. a white floor turning pink beside the red wall	no: chalk to chalk
4. the room, upside down in a glass ball	yes: bent rays
5. the underside of a table, lit by the floor	no: chalk to chalk

- Follow the light back: does it pass chalk to chalk?

Recap: what Whitted misses

a chalk hit: add up the lamps, then stop

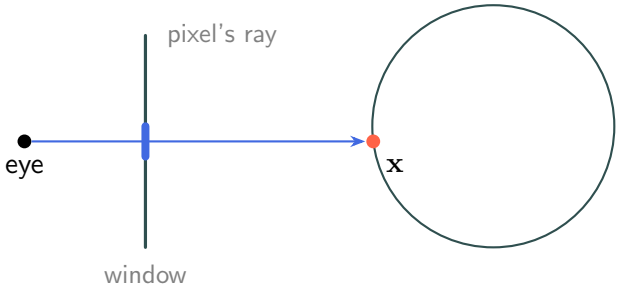
missing: light from one diffuse surface to another

colour bleeding rooms lit with no lamp in view dark corners

Radiance and the dome

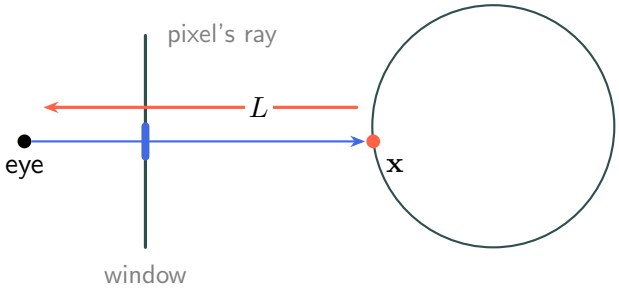
How much light arrives, from every direction

Radiance: the number a pixel wants



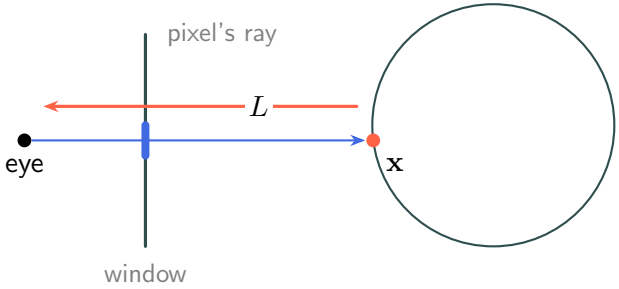
- A pixel asks: how bright, along my ray?

Radiance: the number a pixel wants



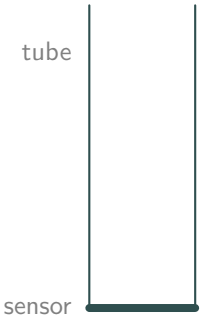
- A pixel asks: how bright, along my ray?
- The answer is called radiance, L

Radiance: the number a pixel wants

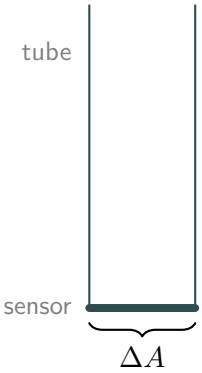


- A pixel asks: how bright, along my ray?
- The answer is called radiance, L
- Lecture 16's L_r was a radiance

Radiance: light per area, per direction

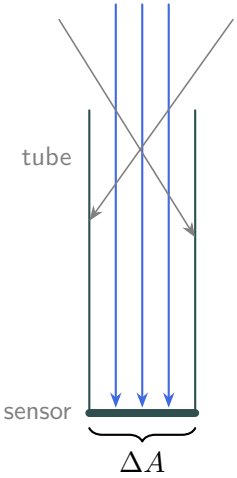


Radiance: light per area, per direction



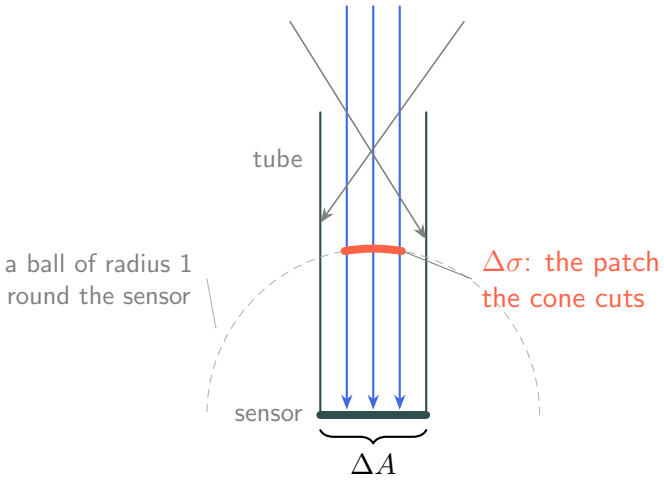
- Light through a small area ΔA

Radiance: light per area, per direction



- Light through a small area ΔA
- only from a narrow cone of directions

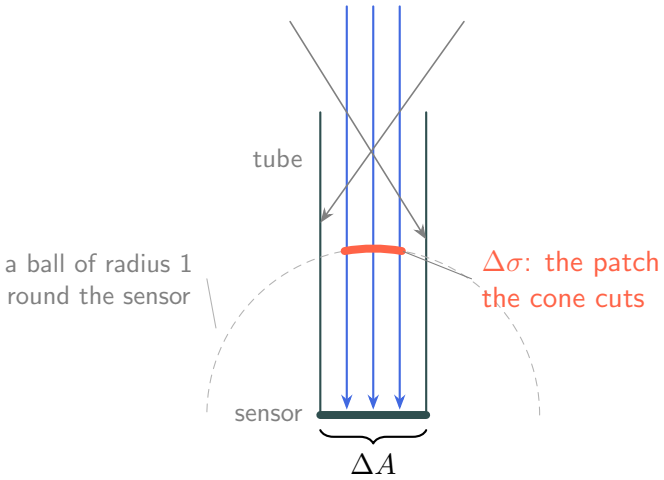
Radiance: light per area, per direction



- Light through a small area ΔA
- only from a narrow cone of directions
- The cone's width $\Delta\sigma$: the area of the patch it cuts on a ball of radius 1

lecture 16's solid angle (steradians)

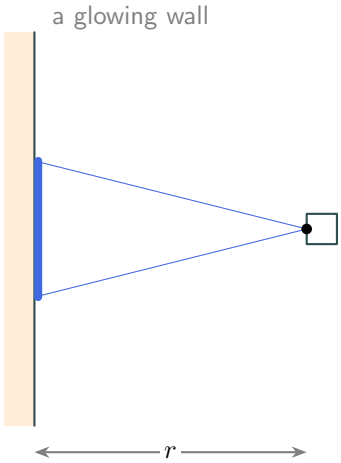
Radiance: light per area, per direction



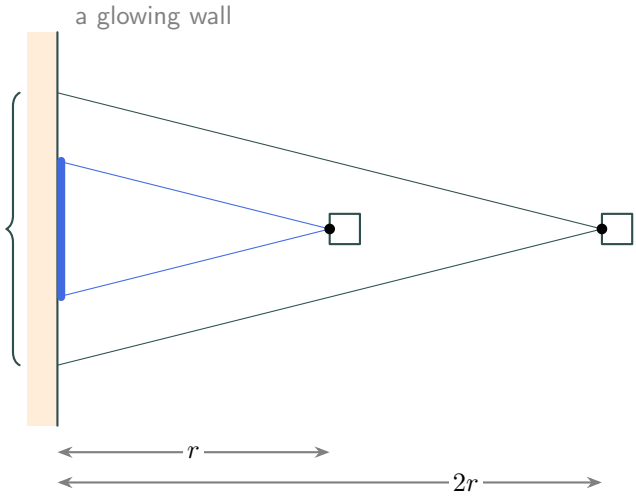
- Light through a small area ΔA
- only from a narrow cone of directions
- The cone's width $\Delta\sigma$: the area of the patch it cuts on a ball of radius 1

  lecture 16's solid angle (steradians)
- L: that light $\div$ $(\Delta A \times \Delta\sigma)$

Radiance does not fade along a ray

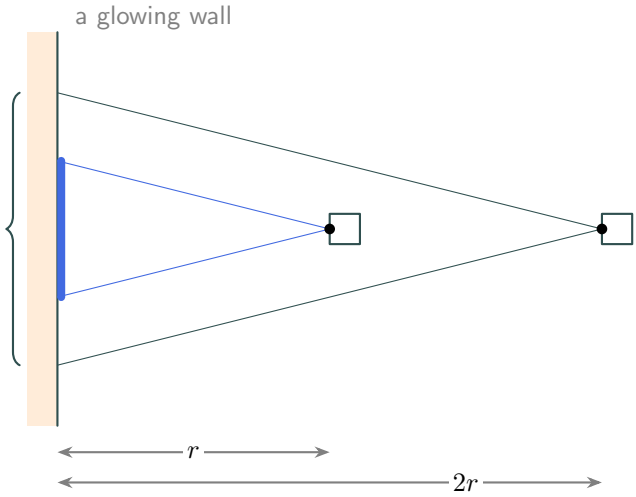


Radiance does not fade along a ray



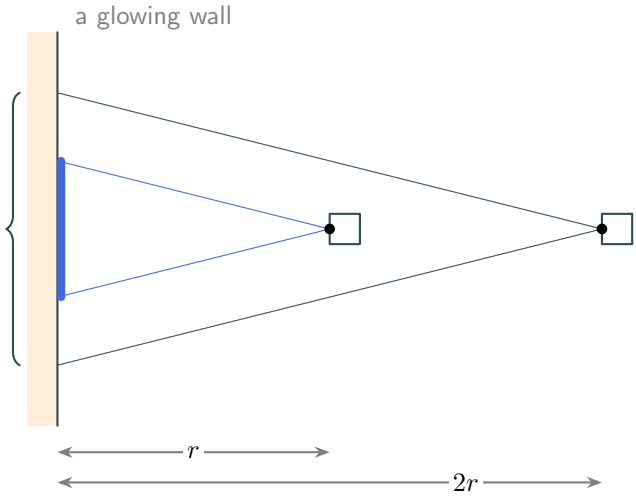
- Twice as far: 4 $\times$ the patch area

Radiance does not fade along a ray



- Twice as far: $4 \times$ the patch area
- Each bit of it: $\frac{1}{4}$ the light

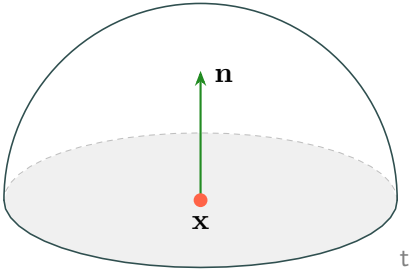
Radiance does not fade along a ray



- Twice as far: $4 \times$ the patch area
- Each bit of it: $\frac{1}{4}$ the light
- Same reading: L does not fade

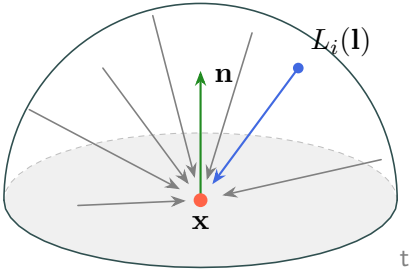
A dome of directions over every point

- Every direction $\mathbf{l}$ above $\mathbf{x}$



the floor around $\mathbf{x}$

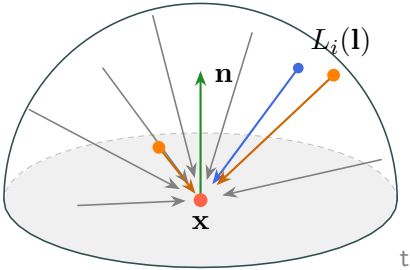
A dome of directions over every point



the floor around $\mathbf{x}$

- Every direction $\mathbf{l}$ above $\mathbf{x}$
- Each brings its own radiance $L_i(\mathbf{l})$

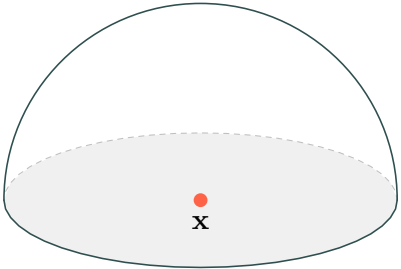
A dome of directions over every point



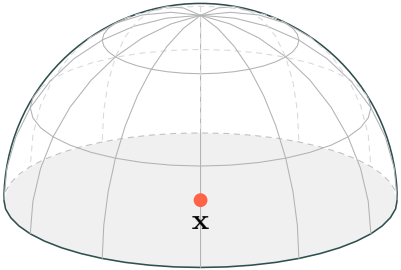
the floor around $\mathbf{x}$

- Every direction $\mathbf{l}$ above $\mathbf{x}$
- Each brings its own radiance $L_i(\mathbf{l})$
- The lamps: just a few of them

A patch of directions has a solid angle $\Delta\sigma$

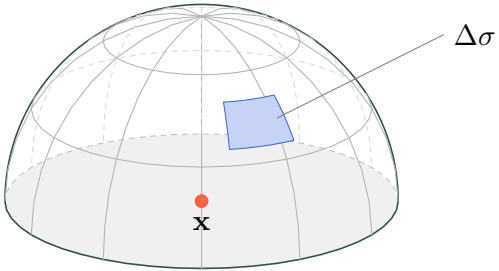


A patch of directions has a solid angle $\Delta\sigma$



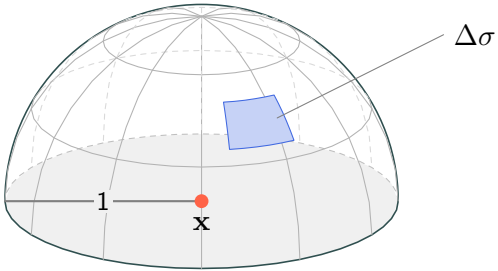
- Cut the dome into small patches

A patch of directions has a solid angle $\Delta\sigma$



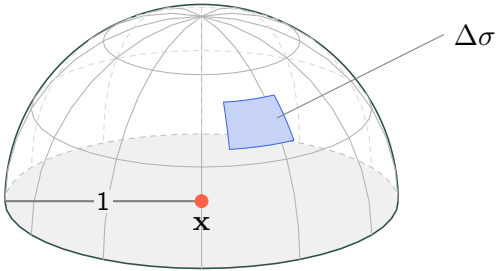
- Cut the dome into small patches
- A patch's area: its solid angle $\Delta\sigma$

A patch of directions has a solid angle $\Delta\sigma$



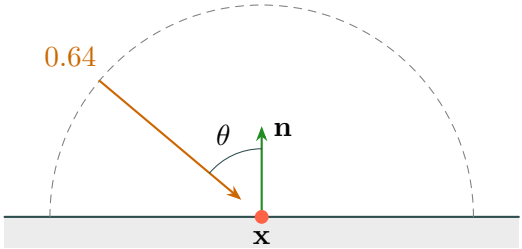
- Cut the dome into small patches
- A patch's area: its solid angle $\Delta\sigma$
- The whole dome: 2π steradians

A patch of directions has a solid angle $\Delta\sigma$



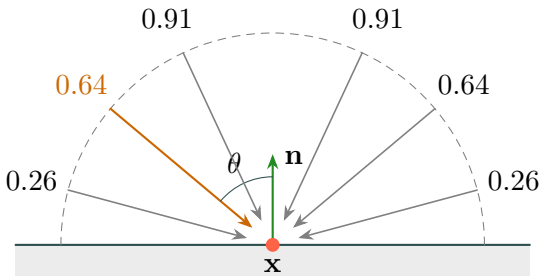
- Cut the dome into small patches
- A patch's area: its solid angle $\Delta\sigma$
- The whole dome: 2π steradians
- Think: $\Delta\sigma$ is how many rays fit in the patch. A far window: few. A black window: as many, each dark

Each direction is weighted by $\cos\theta$



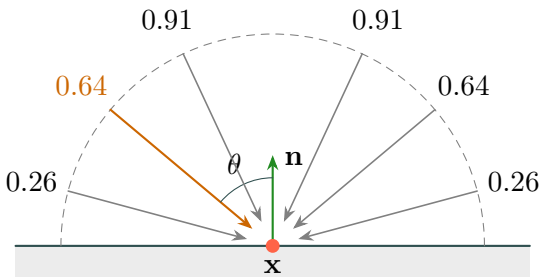
- Lecture 16: the lamp's light, times $\cos\theta$

Each direction is weighted by $\cos\theta$



- Lecture 16: the lamp's light, times $\cos\theta$
- Now: every direction, times $\cos\theta$

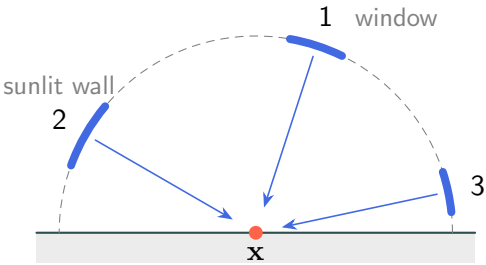
Each direction is weighted by $\cos\theta$



- Lecture 16: the lamp's light, times $\cos\theta$
- Now: every direction, times $\cos\theta$
- Overhead counts fully; grazing, barely

$$\cos\theta = \mathbf{n} \cdot \mathbf{l}$$

Worked: add the light of three patches

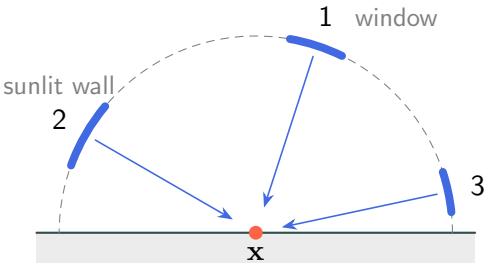


the rest of the dome: dark

patch	L_i	$\cos\theta$	$\Delta\sigma$	$L_i \cos\theta \, \Delta\sigma$
1	2	0.95	0.3	
2	1	0.5	0.4	
3	0.5	0.2	0.5	

Try first: E, the light arriving at $\mathbf{x}$

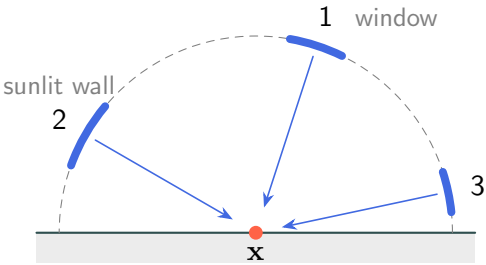
Worked: add the light of three patches



the rest of the dome: dark

patch	L_i	$\cos\theta$	$\Delta\sigma$	$L_i \cos\theta \, \Delta\sigma$
	each ray	tilt	how many rays	
1	2	0.95	0.3	0.57
2	1	0.5	0.4	
3	0.5	0.2	0.5	

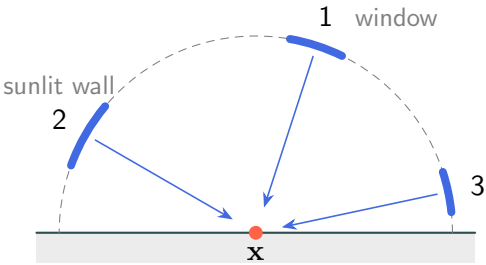
Worked: add the light of three patches



the rest of the dome: dark

patch	L_i	$\cos\theta$	$\Delta\sigma$	$L_i \cos\theta \, \Delta\sigma$
	each ray	tilt	how many rays	
1	2	0.95	0.3	0.57
2	1	0.5	0.4	0.20
3	0.5	0.2	0.5	

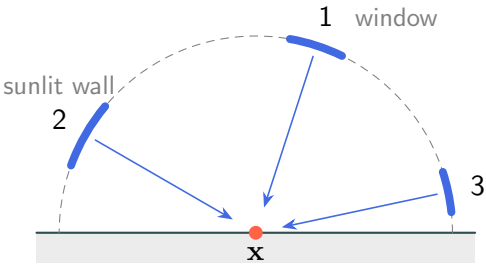
Worked: add the light of three patches



the rest of the dome: dark

patch	L_i	$\cos\theta$	$\Delta\sigma$	$L_i \cos\theta \, \Delta\sigma$
	each ray	tilt	how many rays	
1	2	0.95	0.3	0.57
2	1	0.5	0.4	0.20
3	0.5	0.2	0.5	0.05

Worked: add the light of three patches



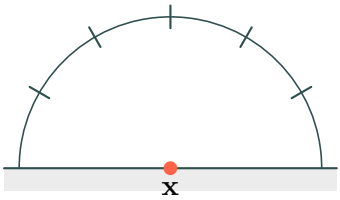
the rest of the dome: dark

patch	L_i	$\cos\theta$	$\Delta\sigma$	$L_i \cos\theta\, \Delta\sigma$
	each ray	tilt	how many rays	
1	2	0.95	0.3	0.57
2	1	0.5	0.4	0.20
3	0.5	0.2	0.5	0.05
		add up:	$E \approx$	0.82

- $E \approx \sum L_i \cos\theta_i\, \Delta\sigma_i$: lecture 16's irradiance

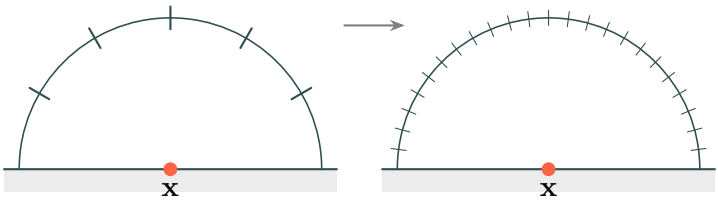
think: a patch holds 100 rays of brightness 50
each: $100 \times 50 = 5000$, then times the tilt

Smaller patches: the sum becomes an integral



$$E \approx \sum L_i \cos\theta_i \, \Delta\sigma_i$$

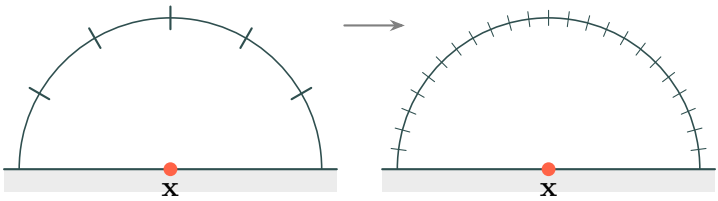
Smaller patches: the sum becomes an integral



$$E \approx \sum L_i \cos\theta_i \, \Delta\sigma_i$$

- More patches, each smaller

Smaller patches: the sum becomes an integral

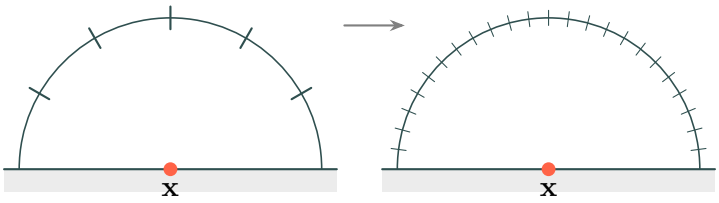


$$E \approx \sum L_i \cos\theta_i \, \Delta\sigma_i$$

$$E = \int_{\text{dome}} L_i(\mathbf{l}) \, \cos\theta \, d\sigma$$

- More patches, each smaller
- The sum becomes an integral

Smaller patches: the sum becomes an integral

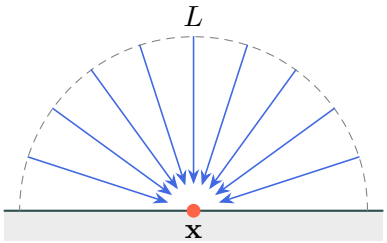


$$E \approx \sum L_i \cos\theta_i \, \Delta\sigma_i$$

$$E = \int_{\text{dome}} L_i(\mathbf{l}) \, \cos\theta \, d\sigma$$

- More patches, each smaller
- The sum becomes an integral
- Read $\int$ as: add up over the dome

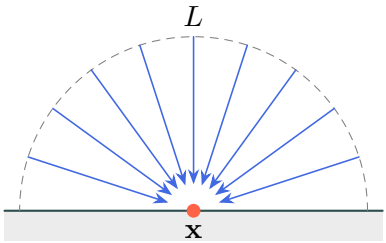
A special case: the same L from every direction gives $E = \pi L$

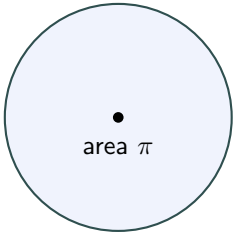


- Suppose, just for now: an evenly overcast sky, no bulb, no walls

the one integral done by hand

A special case: the same L from every direction gives $E = \pi L$





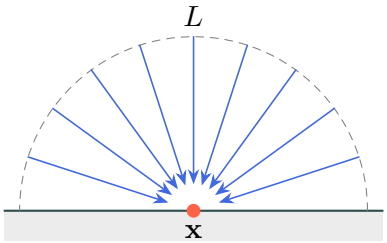
the dome, seen from straight above (lecture 16)

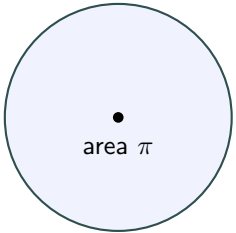
- Suppose, just for now: an evenly overcast sky, no bulb, no walls

  the one integral done by hand

- $\int \cos\theta \, d\sigma = \pi$: the disc

A special case: the same L from every direction gives $E = \pi L$





the dome, seen from straight above (lecture 16)

- Suppose, just for now: an evenly overcast sky, no bulb, no walls

  the one integral done by hand

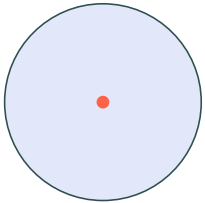
- $\int \cos\theta \, d\sigma = \pi$: the disc

- So $E = \pi L$, under that sky

  a bulb breaks the rule; its patch is handled on its own later

Worked: open floor, foot of a wall, corner

each dome seen from straight above

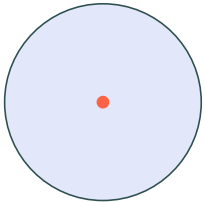


open floor

- Open part of the disc: sky

Worked: open floor, foot of a wall, corner

each dome seen from straight above



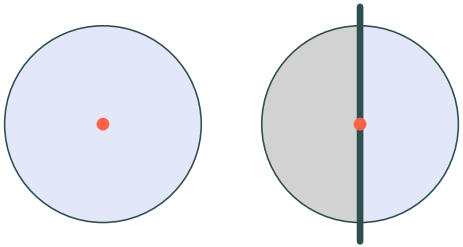
open floor

$$E = \pi L$$

- Open part of the disc: sky

Worked: open floor, foot of a wall, corner

each dome seen from straight above



open floor

$E = \pi L$

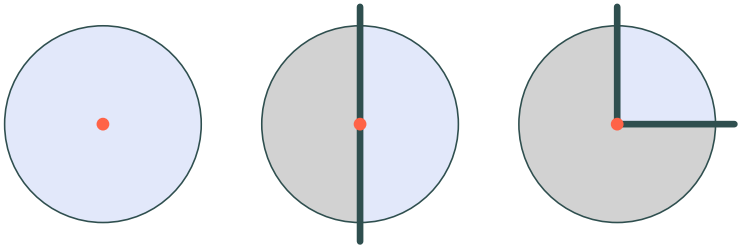
foot of a wall

$E = \pi L/2$

- Open part of the disc: sky
- A dark wall blocks the rest

Worked: open floor, foot of a wall, corner

each dome seen from straight above



open floor

$E = \pi L$

foot of a wall

$E = \pi L/2$

corner

$E = \pi L/4$

- Open part of the disc: sky
- A dark wall blocks the rest
- Corner: a quarter, so darker

Recap: light arriving from the whole dome

$$E = \int_{\text{dome}} L_i(\mathbf{l}) \cos\theta \, d\sigma$$

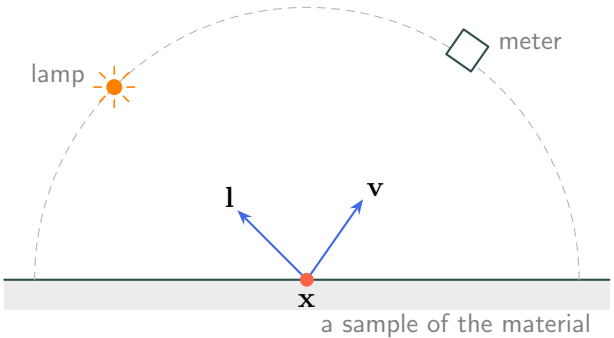
the same L everywhere: $E = \pi L$

L: radiance, light along one ray; it does not fade along the ray

The rendering equation

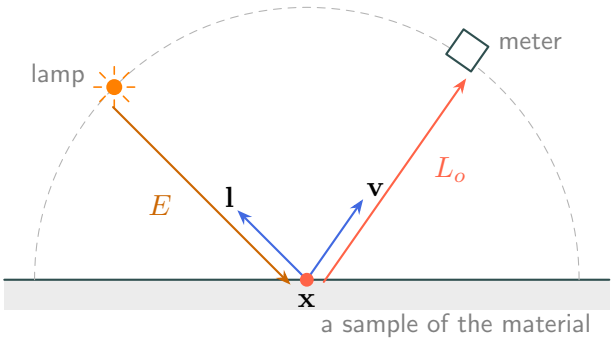
Every direction in, one direction out

The BRDF ρ: how much of $\mathbf{l}$ goes to $\mathbf{v}$



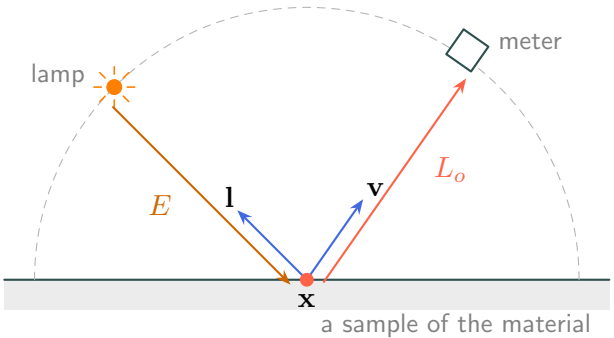
- Light from $\mathbf{l}$; a meter toward $\mathbf{v}$

The BRDF ρ: how much of $\mathbf{l}$ goes to $\mathbf{v}$



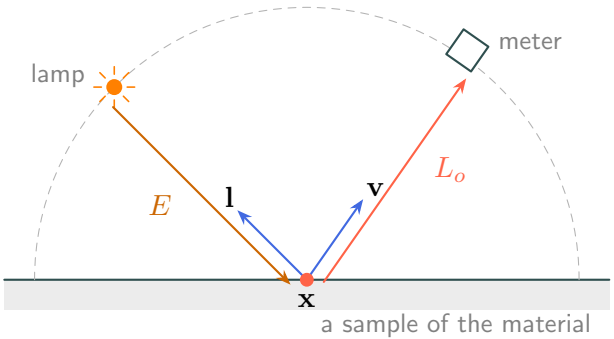
- Light from $\mathbf{l}$; a meter toward $\mathbf{v}$
- E arrives, L_o leaves toward $\mathbf{v}$

The BRDF ρ: how much of $\mathbf{l}$ goes to $\mathbf{v}$



- Light from $\mathbf{l}$; a meter toward $\mathbf{v}$
- E arrives, L_o leaves toward $\mathbf{v}$
- $\rho(\mathbf{l}, \mathbf{v}) = L_o/E$, for every pair

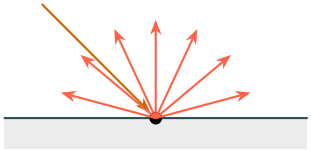
The BRDF ρ: how much of $\mathbf{l}$ goes to $\mathbf{v}$



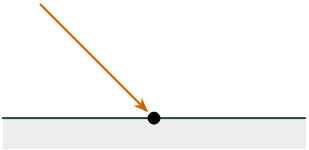
- Light from $\mathbf{l}$; a meter toward $\mathbf{v}$
- E arrives, L_o leaves toward $\mathbf{v}$
- $\rho(\mathbf{l}, \mathbf{v}) = L_o/E$, for every pair

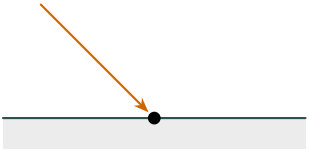
lecture 16's $k = L_r/E$ was ρ, for chalk

The shape of ρ is the material

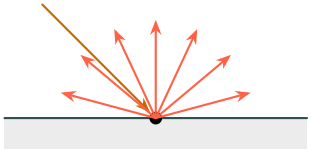


chalk
the same toward every $\mathbf{v}$

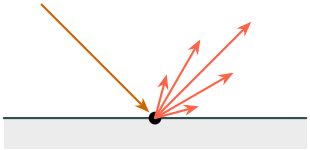




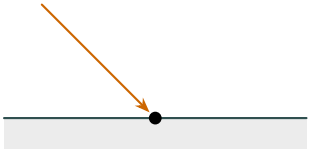
The shape of ρ is the material



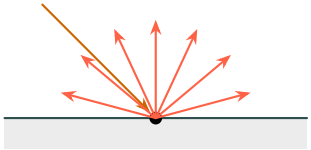
chalk
the same toward every $\mathbf{v}$



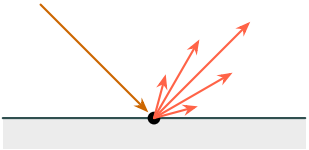
brushed metal
most near the mirror direction



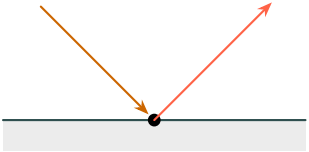
The shape of ρ is the material



chalk
the same toward every $\mathbf{v}$

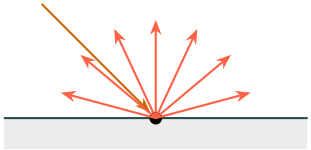


brushed metal
most near the mirror direction

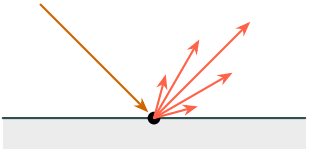


mirror
one direction only: a spike

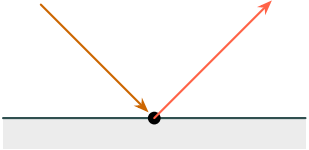
The shape of ρ is the material



chalk
the same toward every $\mathbf{v}$



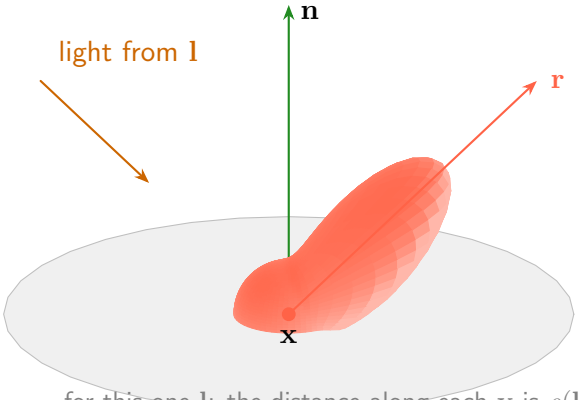
brushed metal
most near the mirror direction



mirror
one direction only: a spike

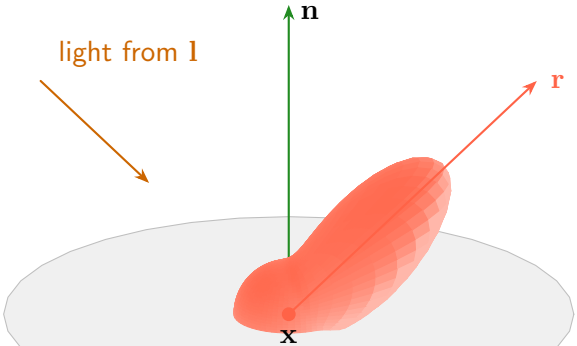
- Our tracer follows spikes exactly, spreads never

ρ pictured, and ρ measured



for this one $\mathbf{l}$: the distance along each $\mathbf{v}$ is $\rho(\mathbf{l}, \mathbf{v})$
the lobe sits around the mirror direction $\mathbf{r}$

ρ pictured, and ρ measured



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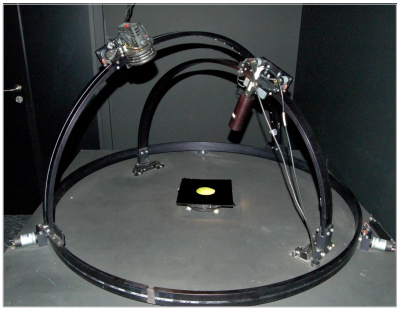
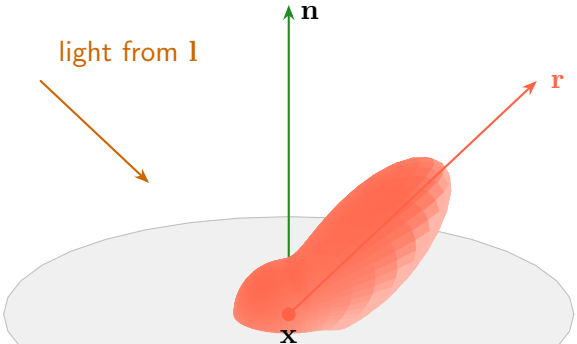
the lobe sits around the mirror direction $\mathbf{r}$

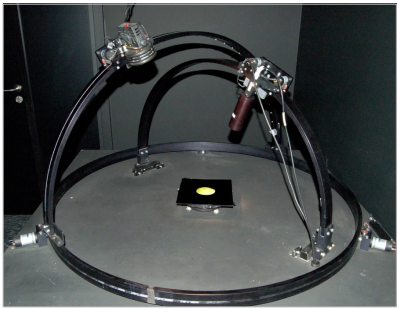


a gonioreflectometer: a lamp on one arc,
a camera on the other, the sample below

ρ pictured, and ρ measured



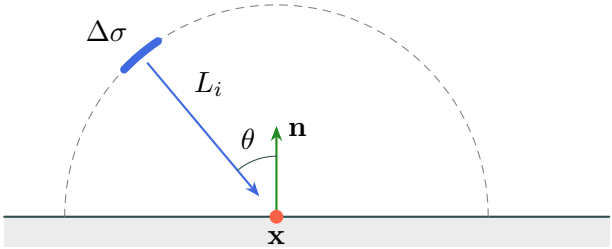
for this one $\mathbf{l}$: the distance along each $\mathbf{v}$ is $\rho(\mathbf{l}, \mathbf{v})$
the lobe sits around the mirror direction $\mathbf{r}$



a gonioreflectometer: a lamp on one arc,
a camera on the other, the sample below

- Two directions, two angles each: ρ is a 4D function; move $\mathbf{l}$ and the slice changes

One patch of the dome: its share of L_o

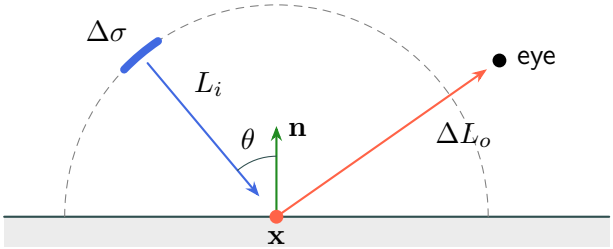


- Light arriving from the patch:

$$\Delta E = L_i \cos\theta \, \Delta\sigma$$

each ray $\times$ tilt $\times$ how many rays

One patch of the dome: its share of L_o



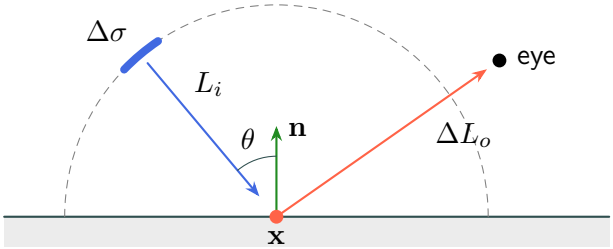
- Light arriving from the patch:

  $$\Delta E = L_i \cos\theta \, \Delta\sigma$$

  each ray $\times$ tilt $\times$ how many rays
- ρ turns it into light toward $\mathbf{v}$:

  $$\Delta L_o = \rho(\mathbf{l}, \mathbf{v}) \, \Delta E$$

One patch of the dome: its share of L_o



- Light arriving from the patch:

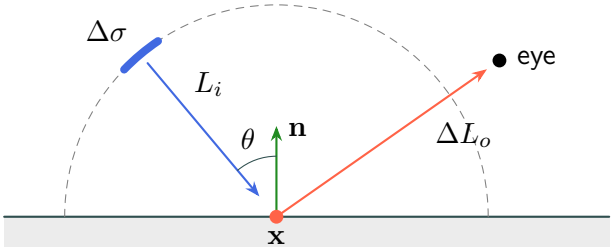
  $$\Delta E = L_i \cos\theta \, \Delta\sigma$$

  each ray $\times$ tilt $\times$ how many rays
- ρ turns it into light toward $\mathbf{v}$:

  $$\Delta L_o = \rho(\mathbf{l}, \mathbf{v}) \, \Delta E$$

  $$\Delta L_o = \rho(\mathbf{l}, \mathbf{v}) \, L_i \cos\theta \, \Delta\sigma$$

One patch of the dome: its share of L_o



- Light arriving from the patch:

$$\Delta E = L_i \cos\theta \, \Delta\sigma$$

each ray $\times$ tilt $\times$ how many rays

- ρ turns it into light toward $\mathbf{v}$:

$$\Delta L_o = \rho(\mathbf{l}, \mathbf{v}) \, \Delta E$$

$$\Delta L_o = \rho(\mathbf{l}, \mathbf{v}) \, L_i \cos\theta \, \Delta\sigma$$

- Every patch adds a share like this

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

$=$ the light $\mathbf{x}$ gives off by itself (a lamp; zero for chalk)

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

$=$ the light $\mathbf{x}$ gives off by itself (a lamp; zero for chalk)

$+$ add up, over every direction $\mathbf{l}$ of the dome:

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

$=$ the light $\mathbf{x}$ gives off by itself (a lamp; zero for chalk)

$+$ add up, over every direction $\mathbf{l}$ of the dome:

how much of the light from $\mathbf{l}$ the material sends to the eye

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

$=$ the light $\mathbf{x}$ gives off by itself (a lamp; zero for chalk)

$+$ add up, over every direction $\mathbf{l}$ of the dome:

how much of the light from $\mathbf{l}$ the material sends to the eye

$\times$ the light arriving from $\mathbf{l}$

The rendering equation, in words

the light leaving $\mathbf{x}$ toward the eye

$=$ the light $\mathbf{x}$ gives off by itself (a lamp; zero for chalk)

$+$ add up, over every direction $\mathbf{l}$ of the dome:

how much of the light from $\mathbf{l}$ the material sends to the eye

$\times$ the light arriving from $\mathbf{l}$

$\times$ how steeply it arrives (the tilt: $\cos\theta$)

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v})$$

leaving $\mathbf{x}$
toward the eye

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v})$$

leaving $\mathbf{x}$ toward the eye

given off by itself

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \quad d\sigma$$

leaving $\mathbf{x}$ toward the eye

given off by itself

add up over the dome

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \, d\sigma$$

leaving $\mathbf{x}$ toward the eye

given off by itself

material: the BRDF

add up over the dome

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \; L_i(\mathbf{x}, \mathbf{l}) \; d\sigma$$

$L_o(\mathbf{x}, \mathbf{v})$	$L_e(\mathbf{x}, \mathbf{v})$	$\rho(\mathbf{l}, \mathbf{v})$	$L_i(\mathbf{x}, \mathbf{l})$
leaving $\mathbf{x}$ toward the eye	given off by itself	material: the BRDF	arriving from $\mathbf{l}$

add up over the dome

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \; L_i(\mathbf{x}, \mathbf{l}) \; \cos\theta \; d\sigma$$

$L_o(\mathbf{x}, \mathbf{v})$	$L_e(\mathbf{x}, \mathbf{v})$	$\int_{\text{dome}}$	$\rho(\mathbf{l}, \mathbf{v})$	$L_i(\mathbf{x}, \mathbf{l})$	$\cos\theta$	$d\sigma$
leaving $\mathbf{x}$ toward the eye	given off by itself	add up over the dome	material: the BRDF	arriving from $\mathbf{l}$	tilt: $\mathbf{n} \cdot \mathbf{l}$	

The rendering equation, in symbols

$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \; L_i(\mathbf{x}, \mathbf{l}) \; \cos\theta \; d\sigma$$

$L_o(\mathbf{x}, \mathbf{v})$	$L_e(\mathbf{x}, \mathbf{v})$	$\rho(\mathbf{l}, \mathbf{v})$	$L_i(\mathbf{x}, \mathbf{l})$	$\cos\theta$
leaving $\mathbf{x}$ toward the eye	given off by itself	material: the BRDF	arriving from $\mathbf{l}$	tilt: $\mathbf{n} \cdot \mathbf{l}$

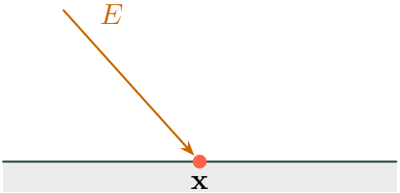
add up over the dome

- Kajiya, 1986: the rendering equation

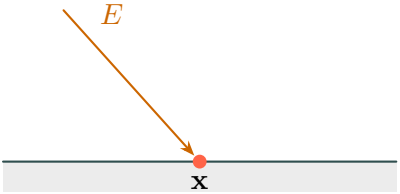
  textbook (14.4): the same without L_e; it writes L_s, L_f, $\mathbf{k}_o$, $\mathbf{k}_i$ for our L_o, L_i, $\mathbf{v}$, $\mathbf{l}$

Energy fixes it: chalk's $\rho = R/\pi$

1. In: E.



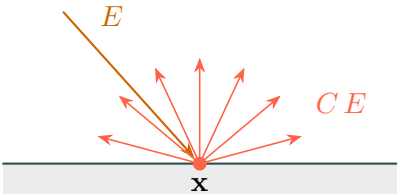
Energy fixes it: chalk's $\rho = R/\pi$



1. In: E.

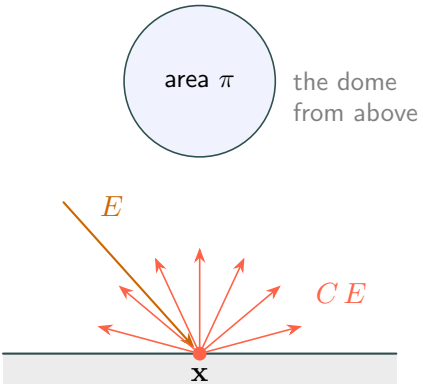
2. Out, all together: RE
   albedo R: the fraction sent back (lecture 16)

Energy fixes it: chalk's $\rho = R/\pi$



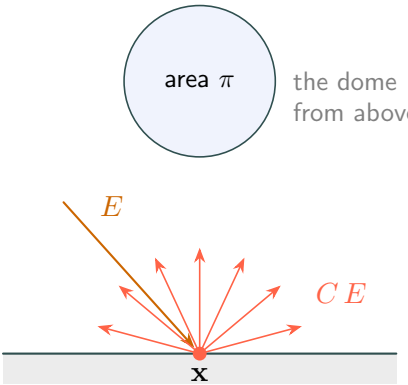
1. In: E.
2. Out, all together: $R\,E$
   albedo R: the fraction sent back (lecture 16)
3. Toward each $\mathbf{v}$: $L_o = C\,E$
   chalk: the same toward every $\mathbf{v}$, one number C

Energy fixes it: chalk's $\rho = R/\pi$



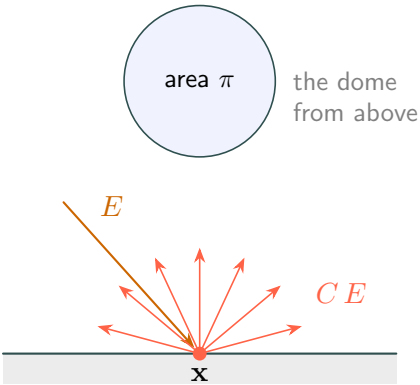
1. In: E.
2. Out, all together: $R\,E$
   albedo R: the fraction sent back (lecture 16)
3. Toward each $\mathbf{v}$: $L_o = C\,E$
   chalk: the same toward every $\mathbf{v}$, one number C
4. All out: $C\,E \int \cos\theta \, d\sigma \;=\; C\,E\,\pi$
   a slanted way out counts $\cos\theta$ too (lecture 16)

Energy fixes it: chalk's $\rho = R/\pi$



1. In: E.
2. Out, all together: $R\,E$
   albedo R: the fraction sent back (lecture 16)
3. Toward each $\mathbf{v}$: $L_o = C\,E$
   chalk: the same toward every $\mathbf{v}$, one number C
4. All out: $C\,E \int \cos\theta \, d\sigma \;=\; C\,E\,\pi$
   a slanted way out counts $\cos\theta$ too (lecture 16)
5. $C\,E\,\pi = R\,E$, so $\rho = \dfrac{R}{\pi}$

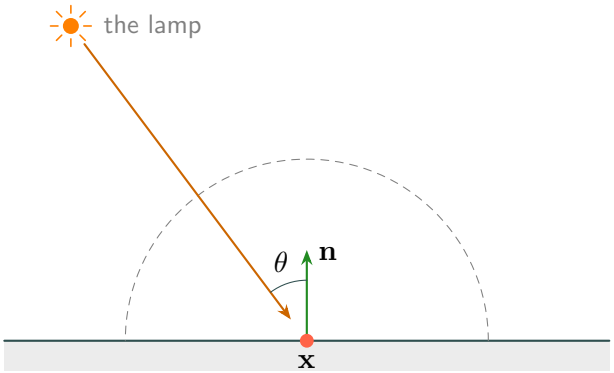
Energy fixes it: chalk's $\rho = R/\pi$



1. In: E.
2. Out, all together: $R\,E$
   albedo R: the fraction sent back (lecture 16)
3. Toward each $\mathbf{v}$: $L_o = C\,E$
   chalk: the same toward every $\mathbf{v}$, one number C
4. All out: $C\,E \int \cos\theta \, d\sigma \;=\; C\,E\,\pi$
   a slanted way out counts $\cos\theta$ too (lecture 16)
5. $C\,E\,\pi = R\,E$, so $\rho = \dfrac{R}{\pi}$

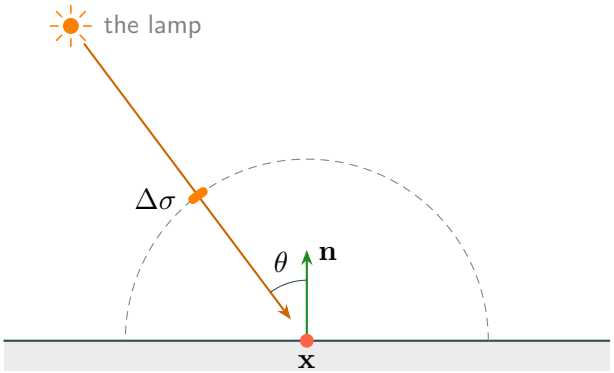
lecture 16's $k = R/\pi$: chalk's BRDF

A point lamp: one tiny patch of the dome



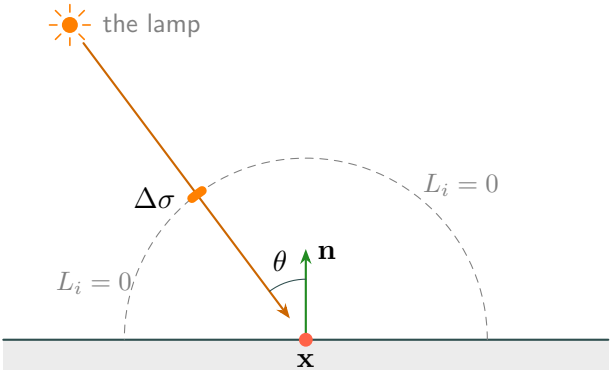
- Lecture 16's world: one lamp, black sky

A point lamp: one tiny patch of the dome



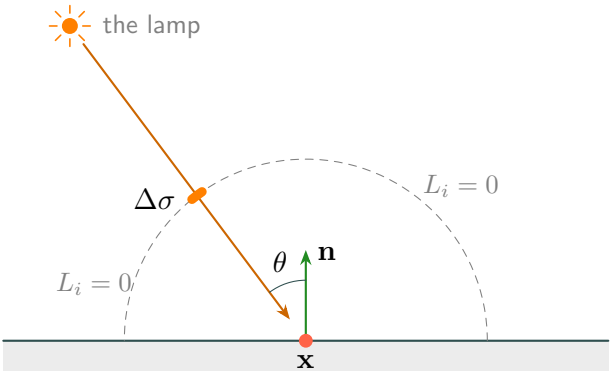
- Lecture 16's world: one lamp, black sky
- The lamp fills one tiny patch

A point lamp: one tiny patch of the dome



- Lecture 16's world: one lamp, black sky
- The lamp fills one tiny patch
- Everywhere else $L_i = 0$

A point lamp: one tiny patch of the dome

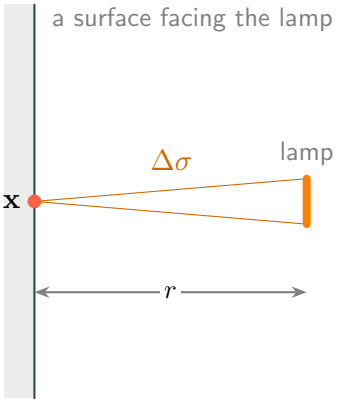


- Lecture 16's world: one lamp, black sky
- The lamp fills one tiny patch
- Everywhere else $L_i = 0$

$$L_o = \rho\, L_i \cos\theta\, \Delta\sigma$$

the integral keeps one patch

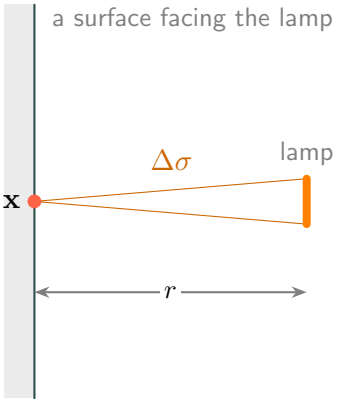
The lamp's patch brings $L_i \, \Delta\sigma = I/r^2$



- Facing the lamp, lecture 16:

  $E = I/r^2$

The lamp's patch brings $L_i \, \Delta\sigma = I/r^2$



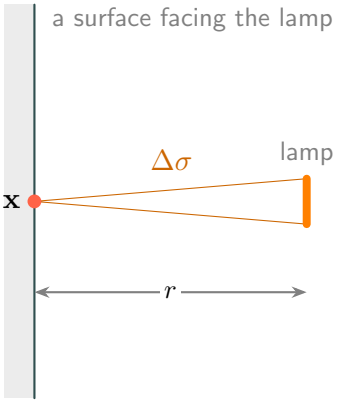
- Facing the lamp, lecture 16:

  $E = I/r^2$

- The same, from its patch:

  $E = L_i \times 1 \times \Delta\sigma$

The lamp's patch brings $L_i \, \Delta\sigma = I/r^2$



- Facing the lamp, lecture 16:

  $E = I/r^2$

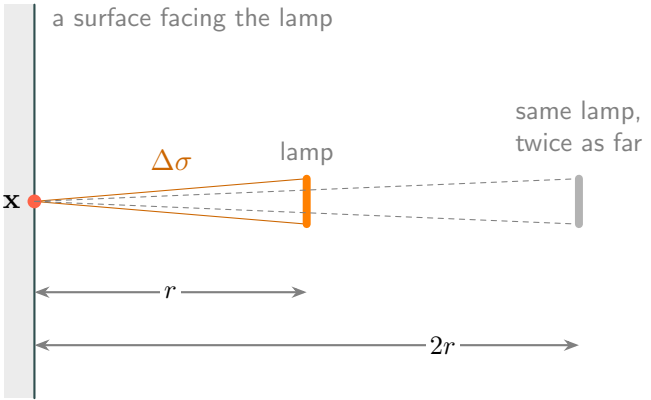
- The same, from its patch:

  $E = L_i \times 1 \times \Delta\sigma$

- So: $L_i \, \Delta\sigma = I/r^2$

  only the product is ever needed

The lamp's patch brings $L_i \, \Delta\sigma = I/r^2$



- Facing the lamp, lecture 16:

  $E = I/r^2$

- The same, from its patch:

  $E = L_i \times 1 \times \Delta\sigma$

- So: $L_i \, \Delta\sigma = I/r^2$

  only the product is ever needed

- Farther: same L_i, smaller patch

The integral collapses to lecture 16's formula

$$L_o = L_e + \int_{\text{dome}} \rho \, L_i \cos\theta \, d\sigma$$

the rendering equation

The integral collapses to lecture 16's formula

$$L_o = L_e + \int_{\text{dome}} \rho\, L_i \cos\theta \, d\sigma$$ the rendering equation

$$= 0 + \rho\, L_i \cos\theta \, \Delta\sigma$$ chalk gives off nothing; one patch

The integral collapses to lecture 16's formula

$$L_o = L_e + \int_{\text{dome}} \rho\, L_i \cos\theta \, d\sigma$$ the rendering equation

$$= 0 + \rho\, L_i \cos\theta \, \Delta\sigma$$ chalk gives off nothing; one patch

$$= \rho\, (\mathbf{n} \cdot \mathbf{l})\, (L_i \, \Delta\sigma)$$ reorder; $\cos\theta = \mathbf{n} \cdot \mathbf{l}$

The integral collapses to lecture 16's formula

$$L_o = L_e + \int_{\text{dome}} \rho\, L_i \cos\theta \, d\sigma$$ the rendering equation

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$$= \rho\, (\mathbf{n} \cdot \mathbf{l})\, (L_i\, \Delta\sigma)$$ reorder; $\cos\theta = \mathbf{n} \cdot \mathbf{l}$

$$= \frac{R}{\pi}\, (\mathbf{n} \cdot \mathbf{l})\, \frac{I}{r^2}$$ chalk: $\rho = R/\pi$; the lamp: I/r^2

The integral collapses to lecture 16's formula

$$L_o = L_e + \int_{\text{dome}} \rho\, L_i \cos\theta \, d\sigma$$ the rendering equation

$$= 0 + \rho\, L_i \cos\theta \, \Delta\sigma$$ chalk gives off nothing; one patch

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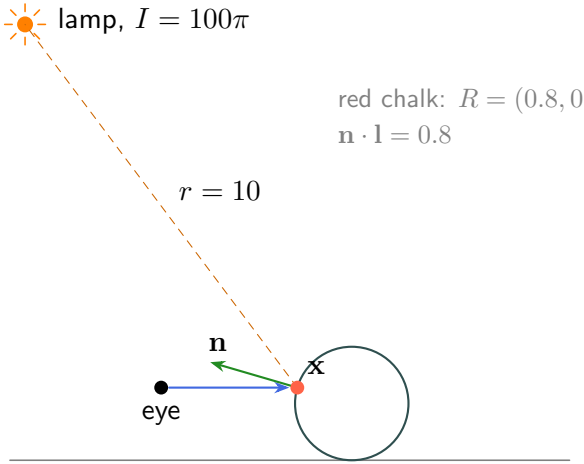
$$= \frac{R}{\pi}\, (\mathbf{n} \cdot \mathbf{l})\, \frac{I}{r^2}$$ chalk: $\rho = R/\pi$; the lamp: I/r^2

lecture 16, exactly — several lamps: one patch each, added: our loop over the lamps

Worked: lecture 16's hit, as a rendering equation

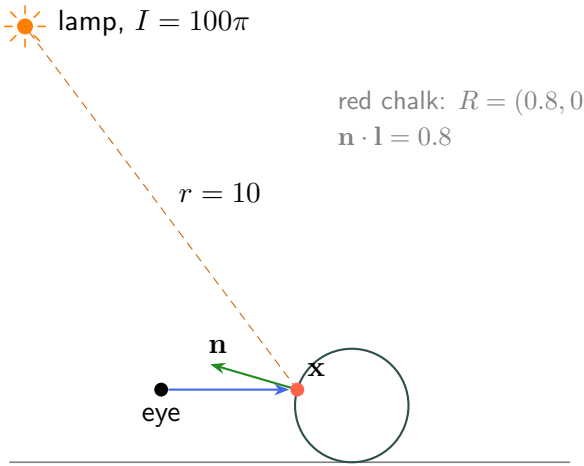


Try first: L_o at $\mathbf{x}$

red chalk: $R = (0.8, 0.2, 0.2)$

$\mathbf{n} \cdot \mathbf{l} = 0.8$

Worked: lecture 16's hit, as a rendering equation



$$L_i \, \Delta\sigma = I/r^2 = 100\pi/100 = \pi$$

red chalk: $R = (0.8, 0.2, 0.2)$

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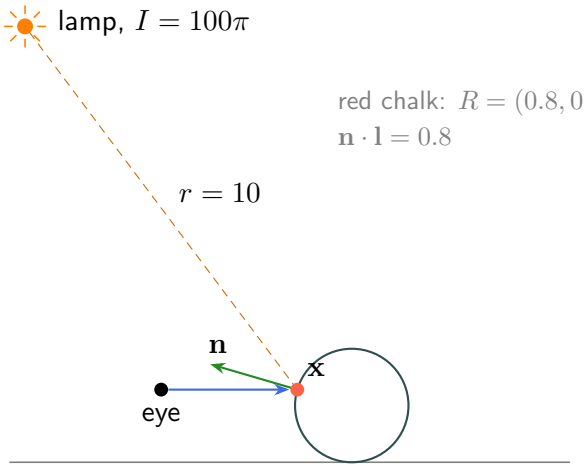
Worked: lecture 16's hit, as a rendering equation



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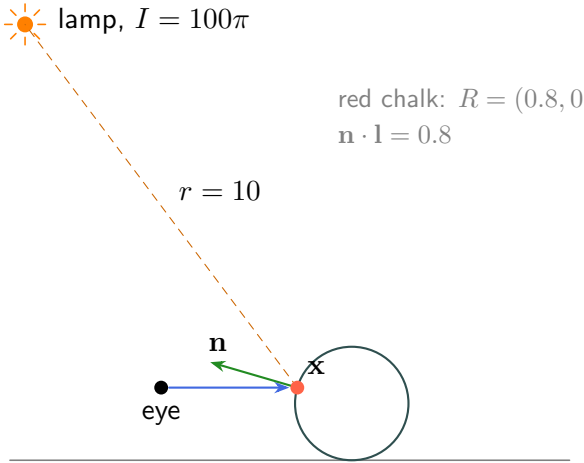
$$\rho = R/\pi = (0.8, 0.2, 0.2)/\pi$$

Worked: lecture 16's hit, as a rendering equation



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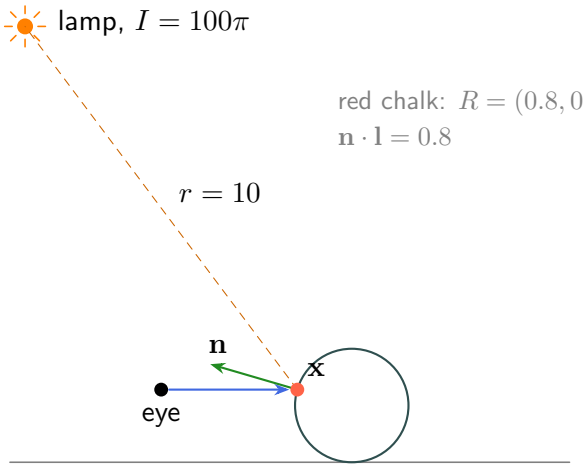
$\mathbf{n} \cdot \mathbf{l} = 0.8$

$$L_i \, \Delta\sigma = I/r^2 = 100\pi/100 = \pi$$

$$\rho = R/\pi = (0.8, 0.2, 0.2)/\pi$$

$$L_o = \frac{R}{\pi} \times 0.8 \times \pi = 0.8\, R$$

Worked: lecture 16's hit, as a rendering equation



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$\mathbf{n} \cdot \mathbf{l} = 0.8$

$$L_i \, \Delta\sigma = I/r^2 = 100\pi/100 = \pi$$

$$\rho = R/\pi = (0.8, 0.2, 0.2)/\pi$$

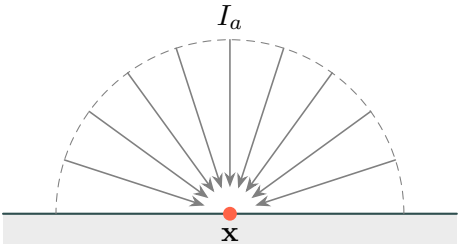
$$L_o = \frac{R}{\pi} \times 0.8 \times \pi = 0.8\, R$$

$L_o = (0.64,\ 0.16,\ 0.16)$

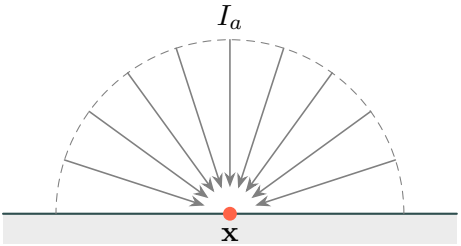
lecture 16's answer for pixel $(16, 12)$

Ambient: the same L_i from every direction

- The guess: $L_i = I_a$ from every direction



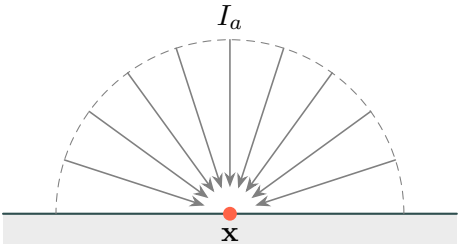
Ambient: the same L_i from every direction



- The guess: $L_i = I_a$ from every direction

$$L_o = \int_{\text{dome}} \frac{R}{\pi} I_a \cos\theta \, d\sigma$$

Ambient: the same L_i from every direction



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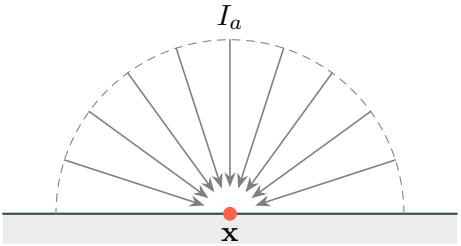
$$= \frac{R}{\pi} I_a \int \cos\theta \, d\sigma = \frac{R}{\pi} I_a \, \pi$$

Ambient: the same L_i from every direction



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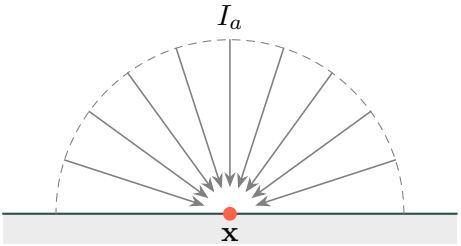
$$= R \, I_a$$ lecture 16's ambient term

Ambient: the same L_i from every direction



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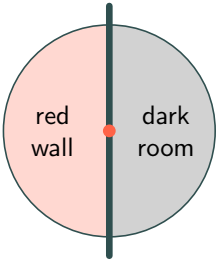
$$= \frac{R}{\pi} I_a \int \cos\theta \, d\sigma = \frac{R}{\pi} I_a \, \pi$$

$$= R \, I_a$$ lecture 16's ambient term

- Wrong where walls block or tint the light

Worked: the floor at a red wall's foot

Try first: the wall's part of L_o

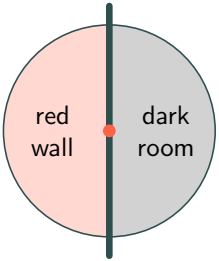


the dome of $\mathbf{x}$,
seen from above

white chalk floor, $R = 0.8$; wall: $L = (0.5, 0.1, 0.1)$

Worked: the floor at a red wall's foot

$$E_{\text{wall}} = L \times \frac{\pi}{2}$$ half the disc



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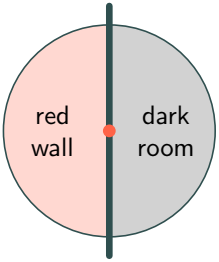


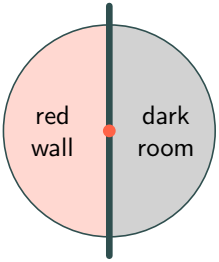
the dome of $\mathbf{x}$,
seen from above

$$E_{\text{wall}} = L \times \frac{\pi}{2} \qquad \text{half the disc}$$

$$L_o = \frac{0.8}{\pi} \times L \times \frac{\pi}{2} = 0.4\,L = (0.20,\ 0.04,\ 0.04)$$

white chalk floor, $R = 0.8$; wall: $L = (0.5, 0.1, 0.1)$

Worked: the floor at a red wall's foot



the dome of $\mathbf{x}$,
seen from above

$$E_{\text{wall}} = L \times \frac{\pi}{2}$$ half the disc

$$L_o = \frac{0.8}{\pi} \times L \times \frac{\pi}{2} = 0.4\,L = (0.20,\ 0.04,\ 0.04)$$

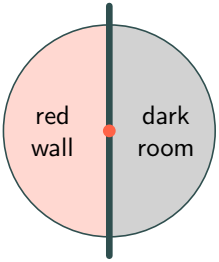
lamp's part + wall's part =

0.64 each $(0.20, 0.04, 0.04)$ $(0.84, 0.68, 0.68)$

lamp: $\frac{0.8}{\pi} \times 0.8 \times \pi = 0.64$, the last worked slide's lamp, white floor

white chalk floor, $R = 0.8$; wall: $L = (0.5, 0.1, 0.1)$

Worked: the floor at a red wall's foot



the dome of $\mathbf{x}$,
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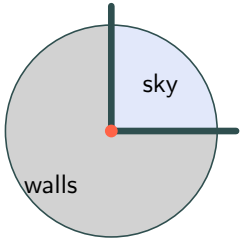
$(0.84, 0.68, 0.68)$

lamp: $\frac{0.8}{\pi} \times 0.8 \times \pi = 0.64$, the last worked slide's lamp, white floor

- The white floor turns pink

white chalk floor, $R = 0.8$; wall: $L = (0.5, 0.1, 0.1)$

Quick check 2: a grey floor in a corner



the corner's dome,
seen from above

Grey chalk floor, $R = 0.5$. Sky: $L = 1$. Walls: $L = 0.2$.

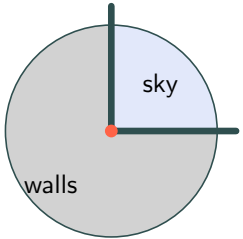
1. E at the corner?

2. L_o toward the eye?

3. What does ambient with $I_a = 1$ give?

- Each part of the disc: its L times its area

Quick check 2: a grey floor in a corner



the corner's dome,
seen from above

Grey chalk floor, $R = 0.5$. Sky: $L = 1$. Walls: $L = 0.2$.

1. E at the corner?
   $1 \times \frac{\pi}{4} + 0.2 \times \frac{3\pi}{4} = 0.4\,\pi \approx 1.26$
2. L_o toward the eye?
   $\frac{0.5}{\pi} \times 0.4\,\pi = 0.2$
3. What does ambient with $I_a = 1$ give?
   $R\,I_a = 0.5$: two and a half times too bright

- Each part of the disc: its L times its area

Recap: the rendering equation

$$L_o(\mathbf{x}, \mathbf{v}) \;=\; L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v})\, L_i(\mathbf{x}, \mathbf{l})\, \cos\theta \; d\sigma$$

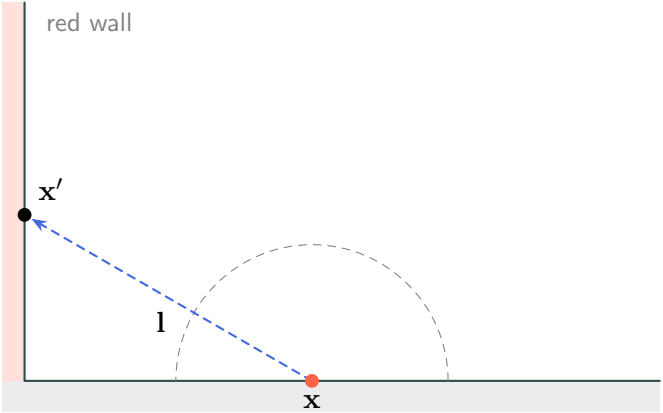
chalk: $\rho = R/\pi$ one lamp: $\frac{R}{\pi}(\mathbf{n} \cdot \mathbf{l})\frac{I}{r^2}$ the same I_a everywhere: $R\, I_a$

lecture 16's formulas are this equation with most of the dome left dark

An equation that calls itself

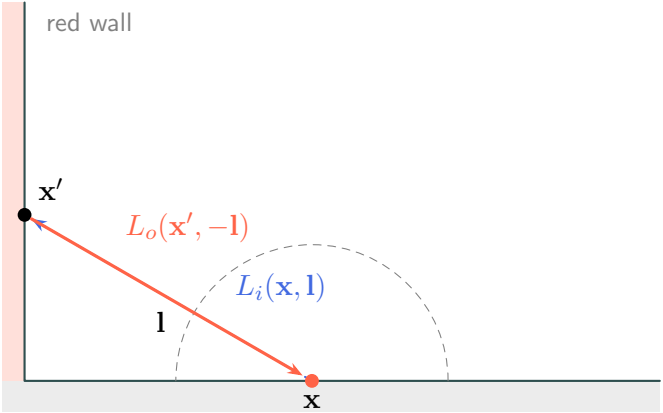
Incoming light is someone else's outgoing light

Light arriving at $\mathbf{x}$ left some other point



- From $\mathbf{x}$ along $\mathbf{l}$: first hit $\mathbf{x}'$

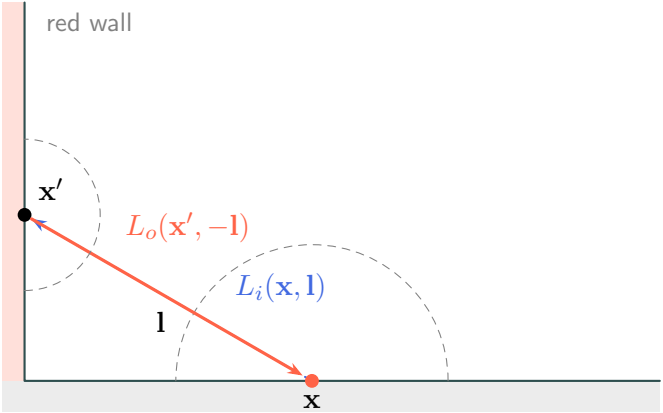
Light arriving at $\mathbf{x}$ left some other point



- From $\mathbf{x}$ along $\mathbf{l}$: first hit $\mathbf{x}'$
- $L_i(\mathbf{x}, \mathbf{l}) = L_o(\mathbf{x}', -\mathbf{l})$

radiance does not fade on the way

Light arriving at $\mathbf{x}$ left some other point



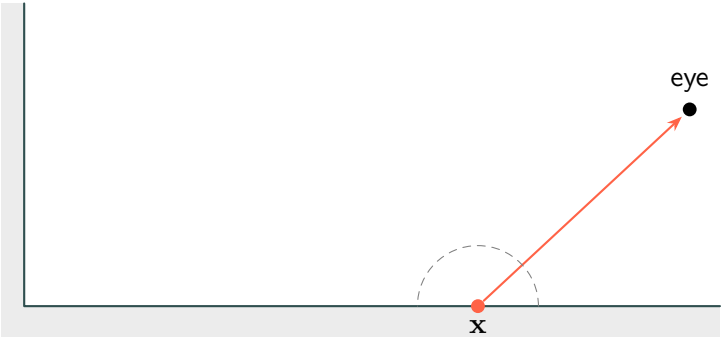
- From $\mathbf{x}$ along $\mathbf{l}$: first hit $\mathbf{x}'$
- $L_i(\mathbf{x}, \mathbf{l}) = L_o(\mathbf{x}', -\mathbf{l})$

radiance does not fade on the way

- $\mathbf{x}'$ has its own equation

L_o on both sides: the equation is recursive

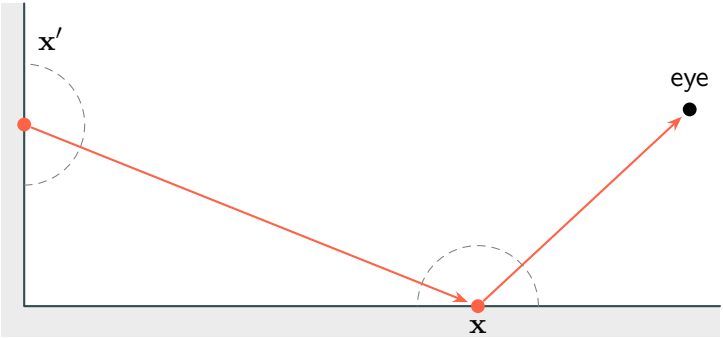
$$L_o(\mathbf{x}, \mathbf{v}) \;=\; L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v})\; L_o(\mathbf{x}', -\mathbf{l})\; \cos\theta \; d\sigma$$



- The unknown L_o is on both sides

L_o on both sides: the equation is recursive

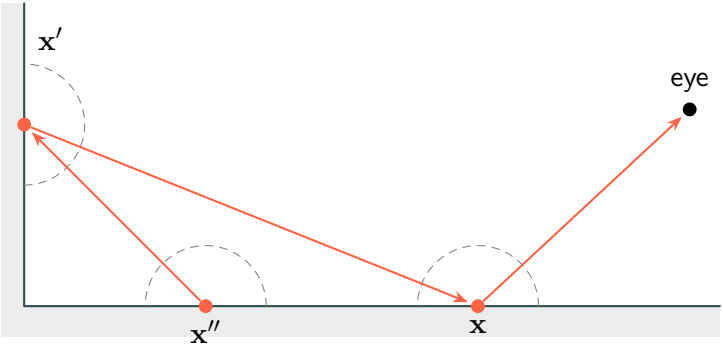
$$L_o(\mathbf{x}, \mathbf{v}) = L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v})\, L_o(\mathbf{x}', -\mathbf{l})\, \cos\theta\, d\sigma$$



- The unknown L_o is on both sides
- To know $\mathbf{x}$, first know $\mathbf{x}'$

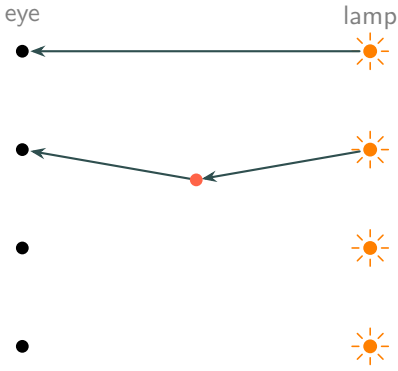
L_o on both sides: the equation is recursive

$$L_o(\mathbf{x}, \mathbf{v}) \;=\; L_e(\mathbf{x}, \mathbf{v}) + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v})\; L_o(\mathbf{x}', -\mathbf{l})\; \cos\theta \; d\sigma$$



- The unknown L_o is on both sides
- To know $\mathbf{x}$, first know $\mathbf{x}'$
- $\mathbf{x}'$ needs $\mathbf{x}''$, and so on

Unrolled: one more bounce in every term

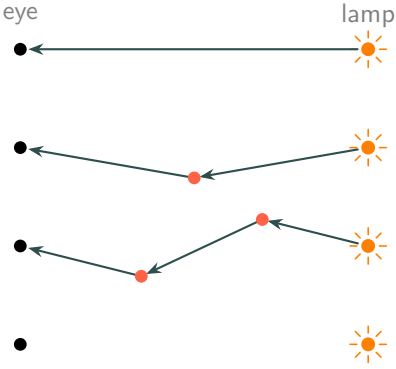


the lamp, seen directly (L_e)

bounce 0: direct light

- a red dot: a diffuse surface on the way

Unrolled: one more bounce in every term



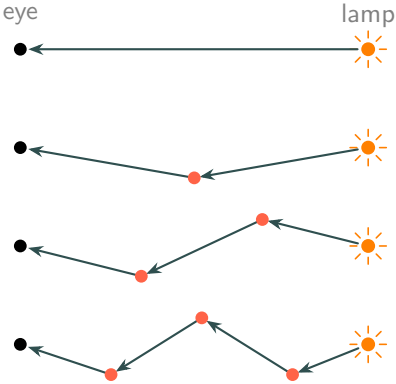
the lamp, seen directly (L_e)

bounce 0: direct light

bounce 1: colour bleeding

- a red dot: a diffuse surface on the way

Unrolled: one more bounce in every term



the lamp, seen directly (L_e)

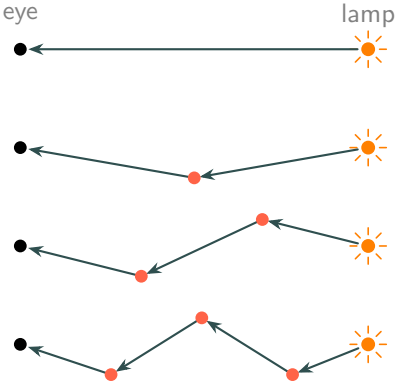
bounce 0: direct light

bounce 1: colour bleeding

bounce 2, 3, ...

- a red dot: a diffuse surface on the way

Unrolled: one more bounce in every term



the lamp, seen directly (L_e)

bounce 0: direct light

our tracer
(and mirror, glass spikes)

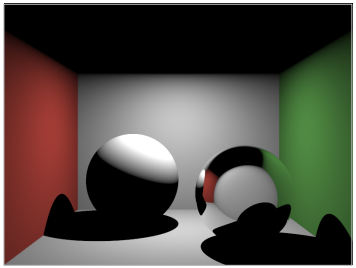
bounce 1: colour bleeding

bounce 2, 3, . . .

missing

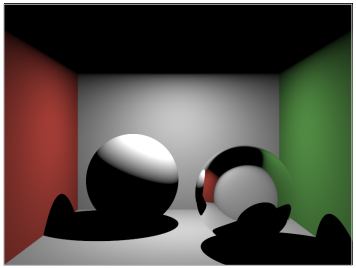
- a red dot: a diffuse surface on the way

Rendered: bounce 0, bounce 1, every bounce

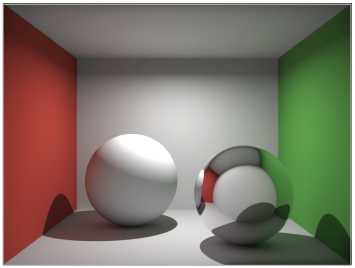


bounce 0
our tracer

Rendered: bounce 0, bounce 1, every bounce

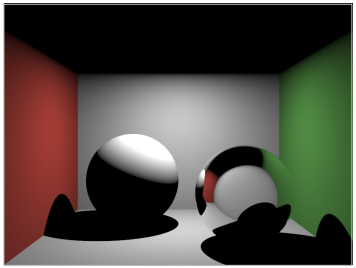


bounce 0
our tracer

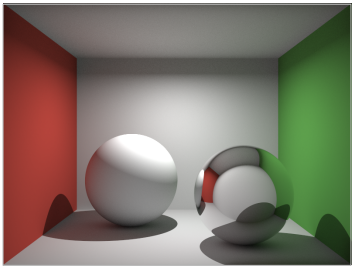


up to bounce 1
the first missing term

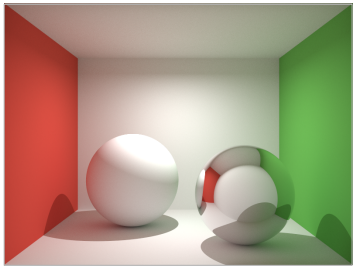
Rendered: bounce 0, bounce 1, every bounce



bounce 0
our tracer

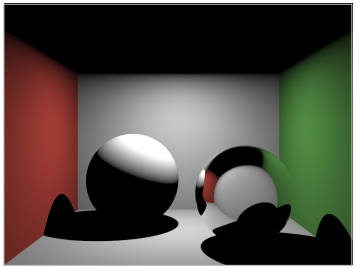


up to bounce 1
the first missing term

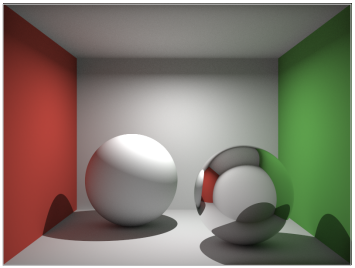


every bounce
the rendering equation

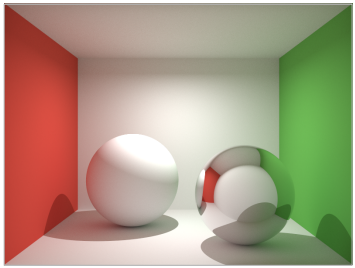
Rendered: bounce 0, bounce 1, every bounce



bounce 0
our tracer



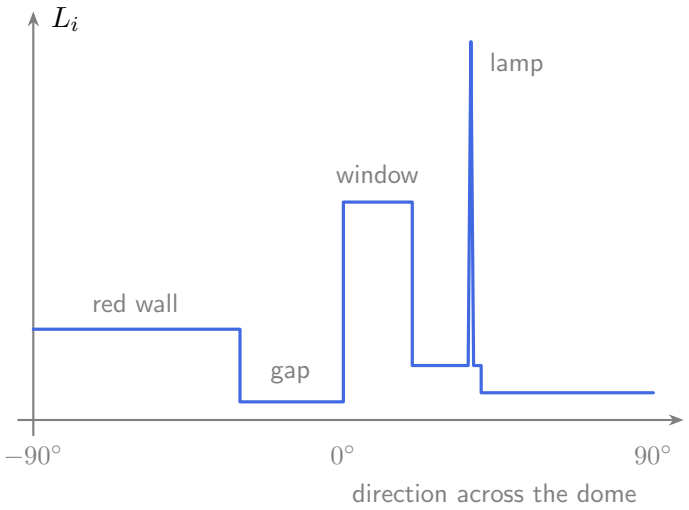
up to bounce 1
the first missing term



every bounce
the rendering equation

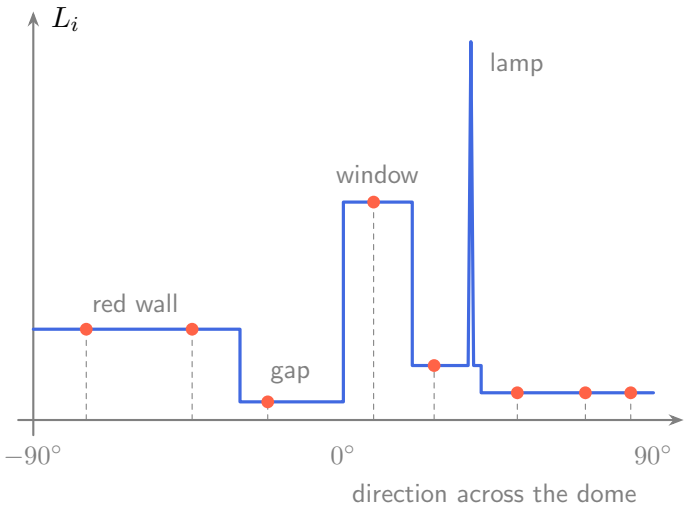
- Each bounce adds less, but together they add a lot

No closed form: the integral must be estimated



- L_i: jagged, new at every point
- No formula integrates it

No closed form: the integral must be estimated



- L_i: jagged, new at every point
- No formula integrates it
- Lecture 20: try random directions

Summary

1 What Whitted misses

chalk hit: lamps, then stop

bounce 0 of the rendering equation

no light from diffuse to diffuse

2 Radiance and the dome

$$E = \int_{\text{dome}} L_i \cos\theta \, d\sigma \qquad \text{even sky: } E = \pi L$$

L: light along one ray; it does not fade

3 The rendering equation

$$L_o(\mathbf{x}, \mathbf{v}) = L_e + \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \, L_i(\mathbf{x}, \mathbf{l}) \cos\theta \, d\sigma$$

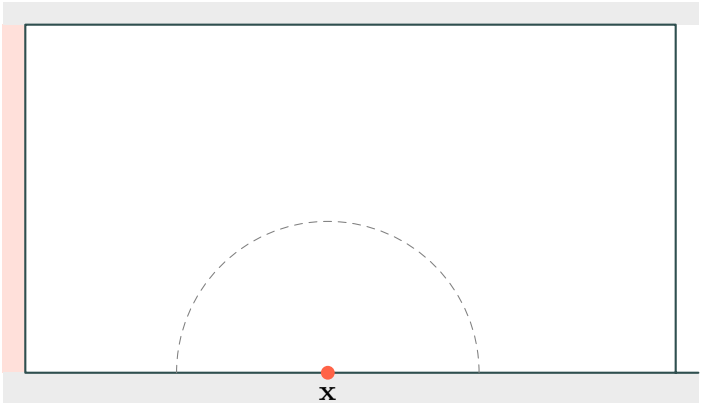
chalk: $\rho = R/\pi$; lamps and ambient: special cases

4 It calls itself

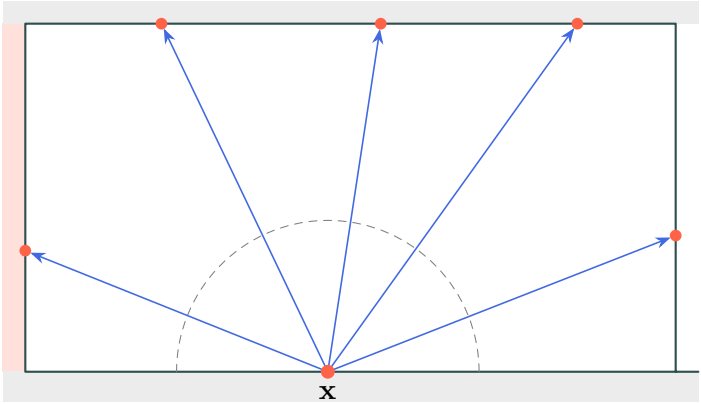
$$L_i(\mathbf{x}, \mathbf{l}) = L_o(\mathbf{x}', -\mathbf{l})$$

no closed form:
estimate it (lecture 20)

Next lecture: Monte Carlo and path tracing

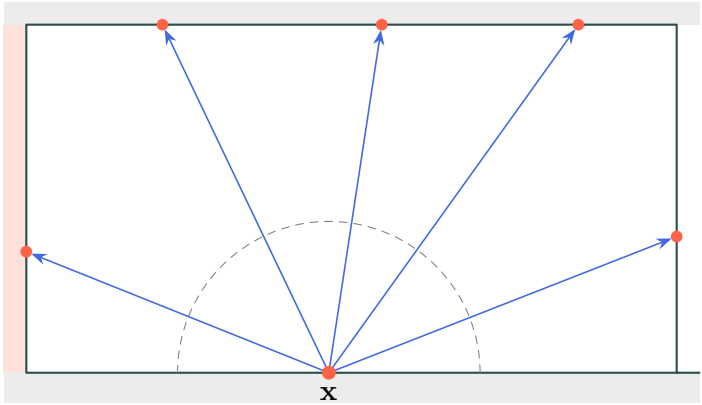


Next lecture: Monte Carlo and path tracing



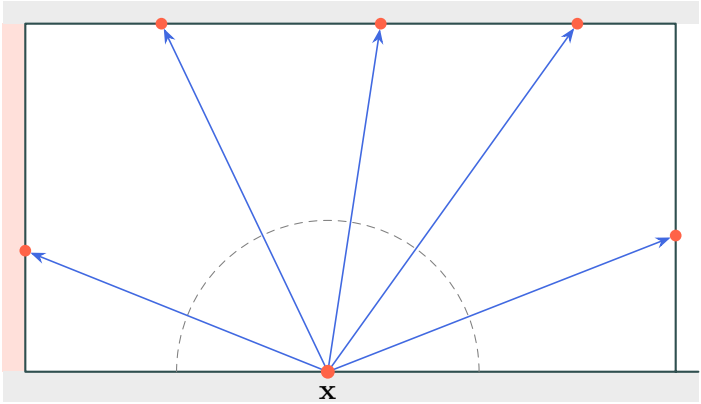
- Random directions over the dome

Next lecture: Monte Carlo and path tracing



- Random directions over the dome
- Average what they bring back

Next lecture: Monte Carlo and path tracing



- Random directions over the dome
- Average what they bring back
- Path tracing: `trace`, one random bounce

Exit Ticket

Today's question *(2 minutes)*

A chalk floor point ($R = 0.5$) stands at the foot of a tall, black wall. Everywhere else its dome sees sky of radiance $L = 2$.

(a) Find the irradiance E.

(b) Find L_o, the radiance toward the eye.

CS 453 — Exit Ticket #19

Name: _____

Roll #: _____ Date: _____

Answer:

Hand it to me on your way out.
No name, no attendance.