Extra 5: Volumes

Light in Fog, Smoke, Clouds and Skin

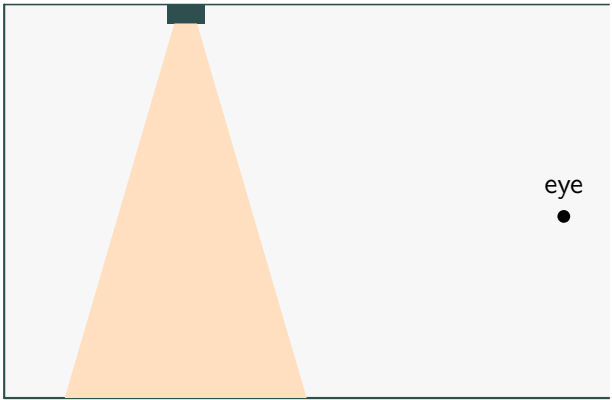
CS 453 — Computer Graphics

Fakhir Shaheen

Forman Christian University

Today: light that turns in mid-air

Extra 5: self-study, not on the exams



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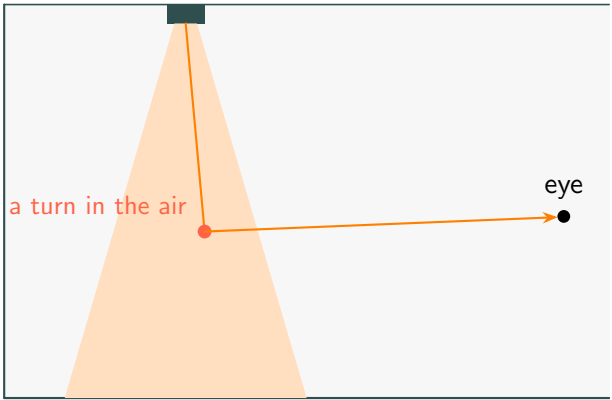

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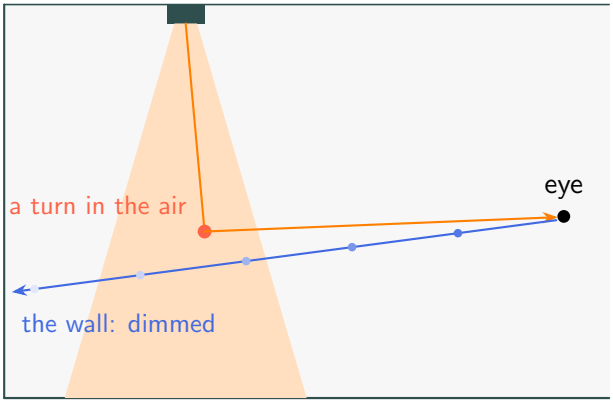

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- Far things fade, and the air in front of them glows.

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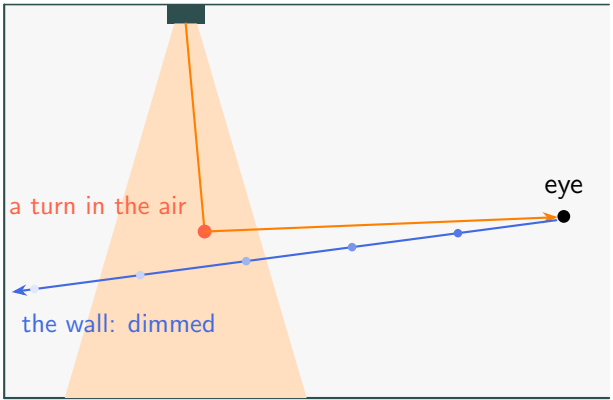

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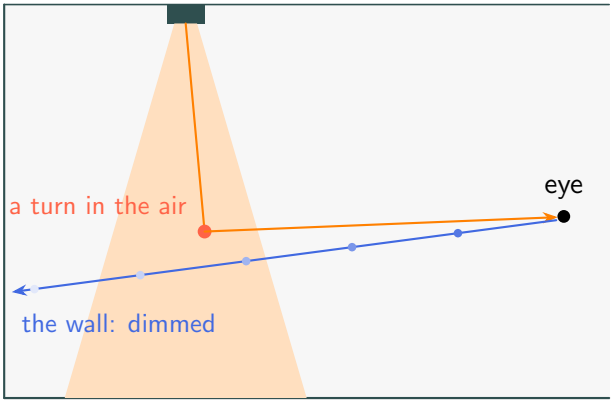
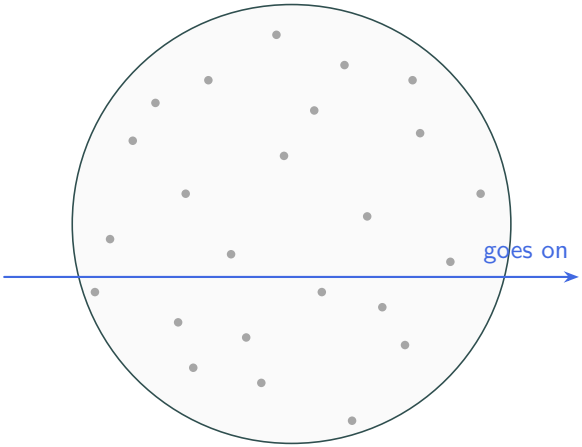


- In fog, light turns in mid-air, not only at surfaces.
- Far things fade, and the air in front of them glows.
- Today: how far a ray goes before it turns, and which way it turns.
- Then clouds, smoke and skin: uneven media, and thick ones.

A ray through fog

Chances per metre, and Beer's law grown up

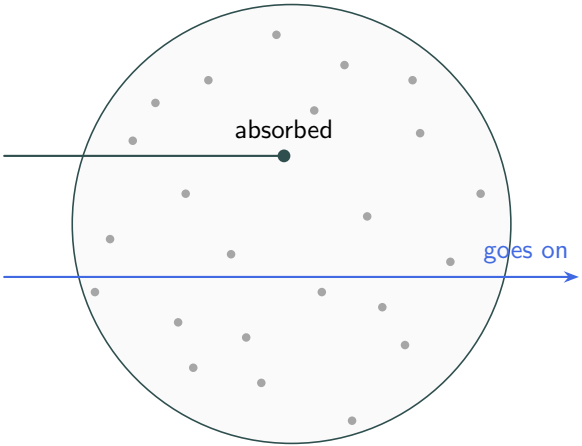
A medium: tiny particles, far too many to draw



a piece of fog, magnified

- A medium: air or water full of tiny particles (droplets, soot, cells).

A medium: tiny particles, far too many to draw



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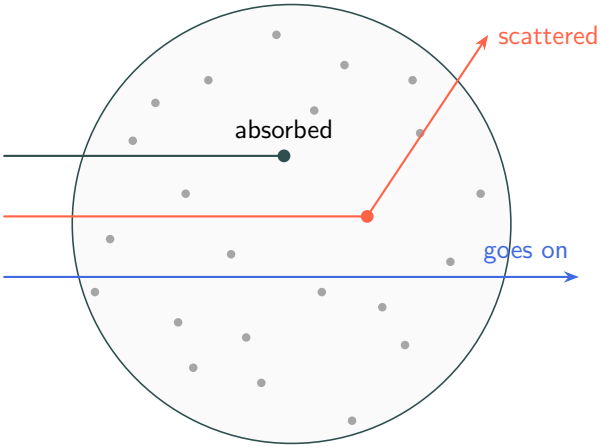

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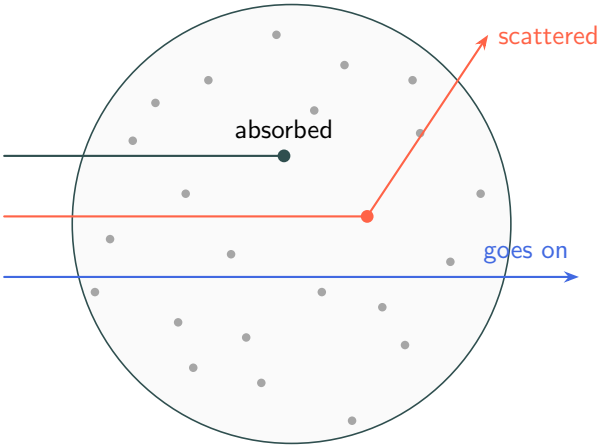
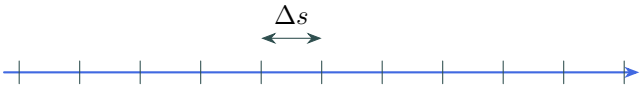


a piece of fog, magnified

- A medium: air or water full of tiny particles (droplets, soot, cells).
- A ray that hits one is absorbed, or
- it is scattered: sent on in a new direction.

- We cannot model each particle. We model the chance of a hit.

σ: the chance of a hit per metre



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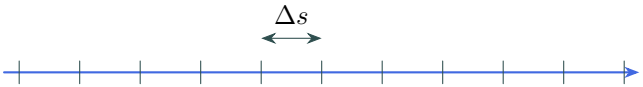
σ: the chance of a hit per metre



chance of a hit in this step: $\sigma\,\Delta s$

σ_a absorbed, per metre

σ_s scattered, per metre

- σ (sigma): hits per metre. In a short step, the chance of a hit is $\sigma\,\Delta s$.
- One σ for each fate: absorbed or scattered.

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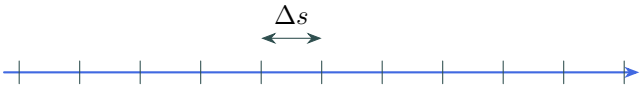

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$\sigma_t = \sigma_a + \sigma_s$ any hit, per metre

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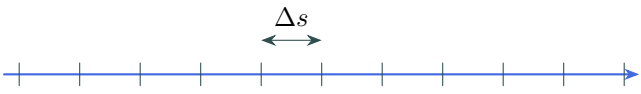

σ_a absorbed, per metre

σ_s scattered, per metre

$\sigma_t = \sigma_a + \sigma_s$ any hit, per metre

$1/\sigma_t$ the average distance between hits

(the mean free path; $\sigma_t = 0.5$ per metre: 2 metres)

- σ (sigma): hits per metre. In a short step, the chance of a hit is $\sigma\,\Delta s$.
- One σ for each fate: absorbed or scattered.
- Their sum σ_t counts every hit. It decides how far a ray gets.
- Thick fog: big σ_t, short free paths.

Beer's law, from the chances

A ray goes d metres in n steps; each step keeps it with chance $1 - \sigma_t \, d/n$.

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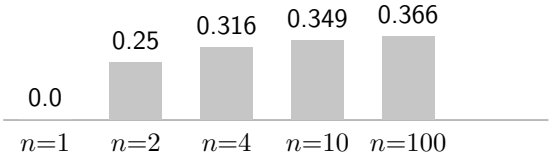

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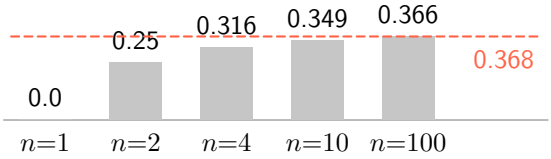
$n \to \infty$: $T(d) = e^{-\sigma_t d}$

T: the transmittance, the share that gets through



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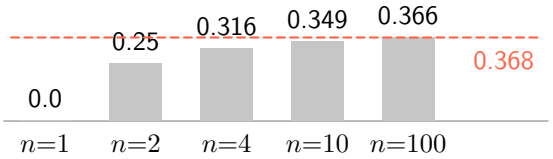
T: the transmittance, the share that gets through

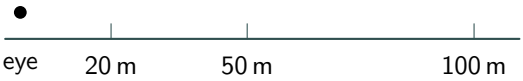
- Lecture 18's Beer's law a^d is the same: $a = e^{-\sigma}$. Its green glass, $a = 0.6$, is $\sigma = 0.51$.



Worked: how far can you see in fog?

Weather reports give *visibility*: where T has fallen to 2%. $e^{-3.9} = 0.02$, so $\sigma_t = 3.9$ / visibility

- Try first: fog with visibility 100 m. σ_t? T at 20, 50 and 100 m?



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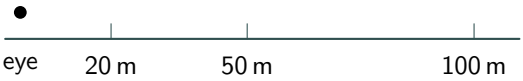
$\sigma_t = 3.9/100 = 0.039$ per metre



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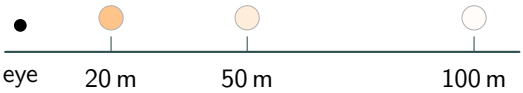
$T(20) = e^{-0.78} = 0.46 \quad T(50) = e^{-1.95} = 0.14 \quad T(100) = e^{-3.9} = 0.02$



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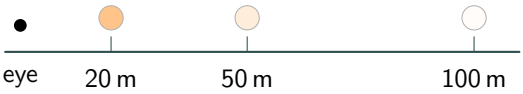
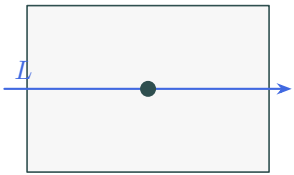


- Twice as far, the square of the share: $0.46^2 \approx 0.2$, not half.

Four things change the light along a ray

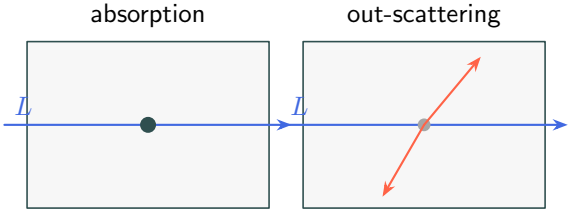
absorption



$-\sigma_a L$

smoke, ink

Four things change the light along a ray



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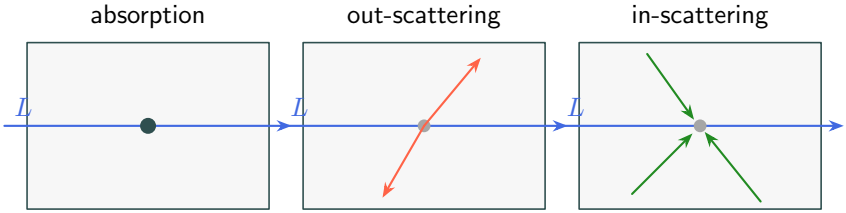

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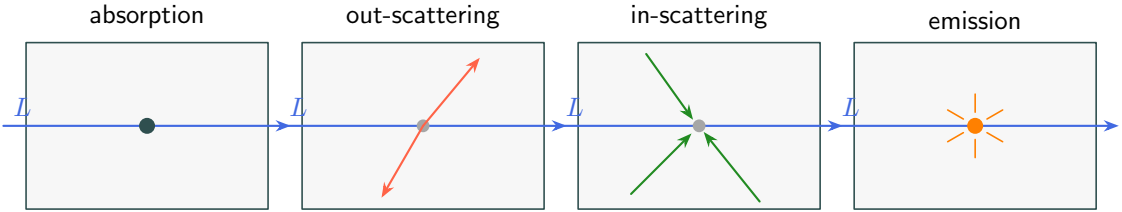

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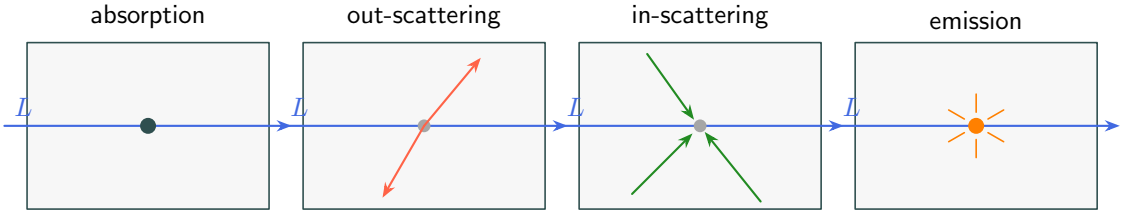

- Two take light out of the ray; two put light in.

The radiative transfer equation

How fast L changes along the ray, per metre, at distance s:

$$\frac{dL}{ds} =$$

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How fast L changes along the ray, per metre, at distance s:

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L_s: light turned into the ray

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fog: no emission; the two losses added

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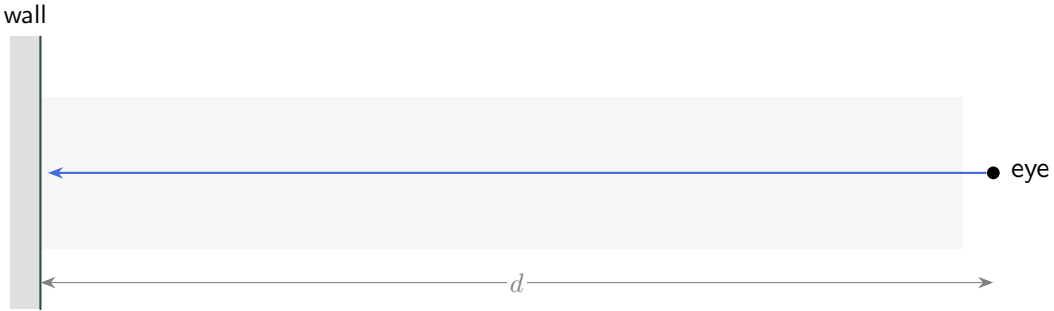
- No medium, all $\sigma = 0$: $dL/ds = 0$. Radiance does not change along a ray, as lecture 19 said.
- Only $\sigma_t L$: L falls as $e^{-\sigma_t s}$, Beer's law again.

What the eye sees through fog



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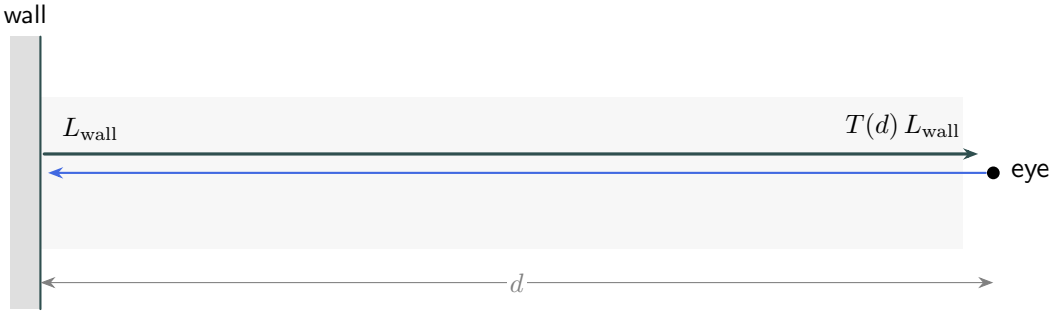

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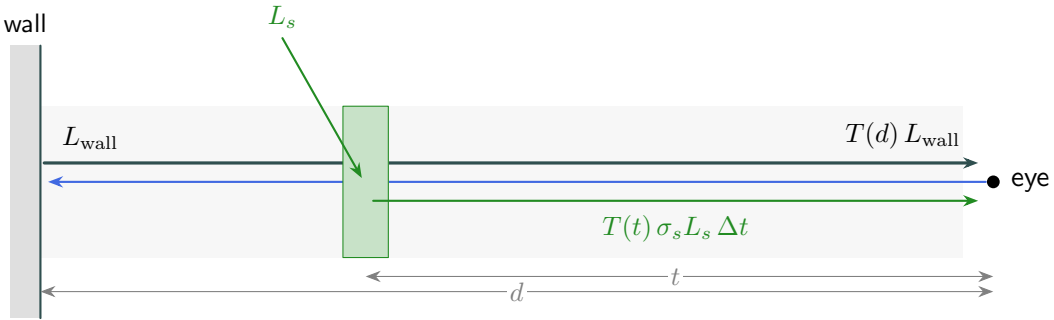

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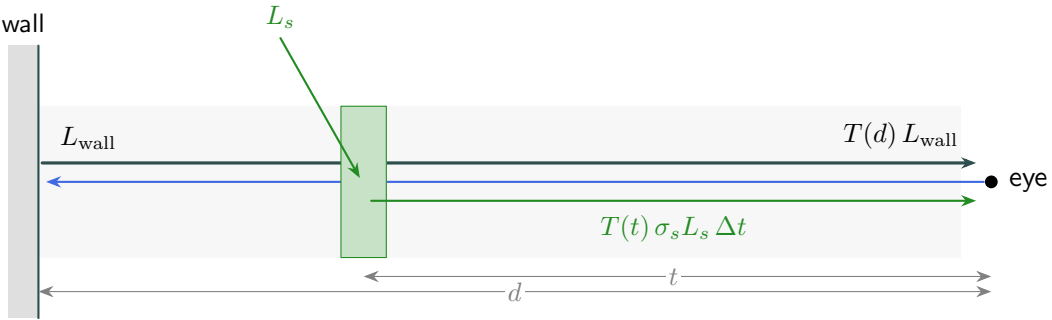
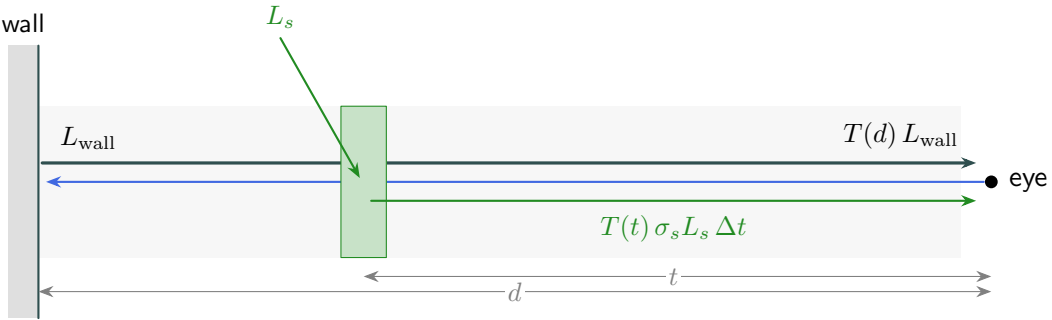


$$L = T(d)\, L_{\text{wall}} + \int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$$

t from the eye, $T(t) = e^{-\sigma_t t}$

What the eye sees through fog



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t from the eye, $T(t) = e^{-\sigma_t t}$

- Every volume renderer estimates this. Part three does it by drawing t at random.

Worked: evenly lit fog fades to its own colour

Fog with no absorption ($\sigma_s = \sigma_t$), and the same $L_s = L_{\text{fog}}$ everywhere:

- Try first: $L_{\text{wall}} = 0.2$, $L_{\text{fog}} = 0.8$, $\sigma_t = 0.5$. L at $d = 1$, 2, 4?

Worked: evenly lit fog fades to its own colour

Fog with no absorption ($\sigma_s = \sigma_t$), and the same $L_s = L_{\text{fog}}$ everywhere:

$$\int_0^d e^{-\sigma_t t}\, \sigma_t\, L_{\text{fog}}\, dt = (1 - e^{-\sigma_t d})\, L_{\text{fog}}$$

$$L = T\, L_{\text{wall}} + (1 - T)\, L_{\text{fog}}$$

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- Try first: $L_{\text{wall}} = 0.2$, $L_{\text{fog}} = 0.8$, $\sigma_t = 0.5$. L at $d = 1$, 2, 4?

$d = 1$: $T = 0.61$, $L = 0.61 \times 0.2 + 0.39 \times 0.8 = 0.44$

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$d = 1$: $T = 0.61$, $L = 0.61 \times 0.2 + 0.39 \times 0.8 = 0.44$

$d = 2$: $T = 0.37$, $L = 0.58$

$d = 4$: $T = 0.14$, $L = 0.72$



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$d = 1$: $T = 0.61$, $L = 0.61 \times 0.2 + 0.39 \times 0.8 = 0.44$

$d = 2$: $T = 0.37$, $L = 0.58$

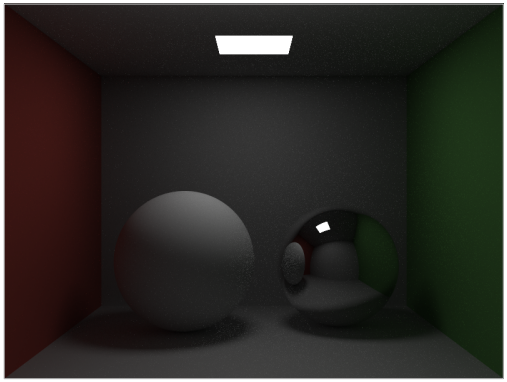
$d = 4$: $T = 0.14$, $L = 0.72$



- Old OpenGL's fog (`GL_EXP`) is exactly this blend. Real fog is not lit evenly: beams, halos, shadows.

Rendered: smoke that absorbs, fog that scatters

Extra 1's room filled with a medium; the same σ_t, the same lamp



smoke: $\sigma_t = 0.7$, all absorbed

Rendered: smoke that absorbs, fog that scatters

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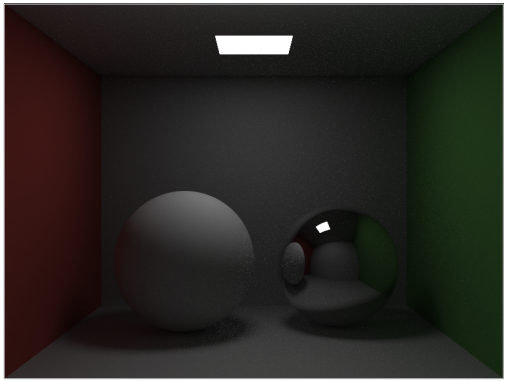
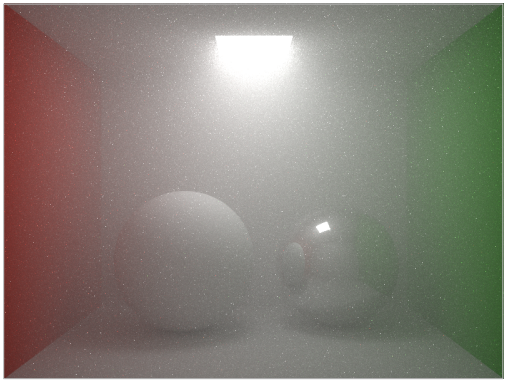


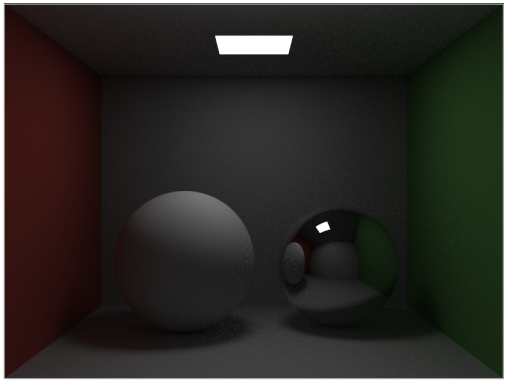
smoke: $\sigma_t = 0.7$, all absorbed



fog: $\sigma_t = 0.7$, 90% scattered

Rendered: smoke that absorbs, fog that scatters

Extra 1's room filled with a medium; the same σ_t, the same lamp



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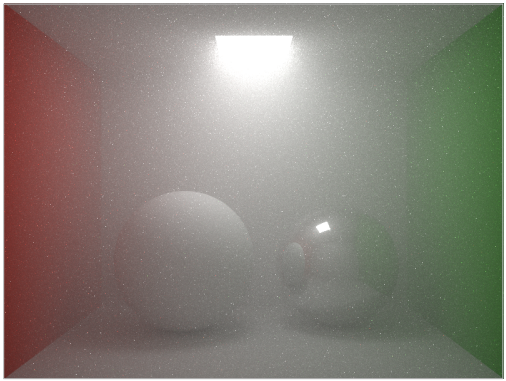
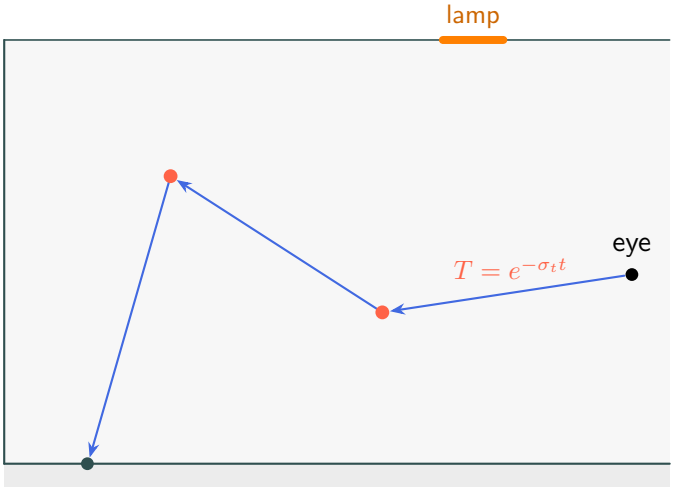


fog: $\sigma_t = 0.7$, 90% scattered

- Absorbed: far things fade to black. Scattered: they fade into a glow.

Recap: one path through fog

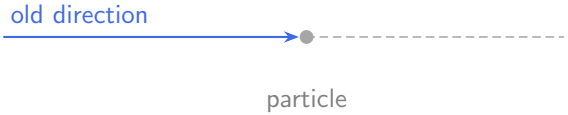


1 $L = T(d)\, L_{\text{wall}} + \int T\, \sigma_s L_s\, dt$

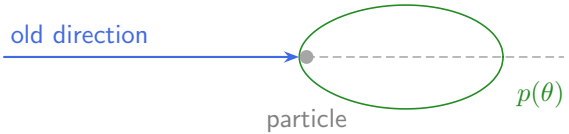
Which way it turns

Phase functions

The phase function: where a scattered ray goes

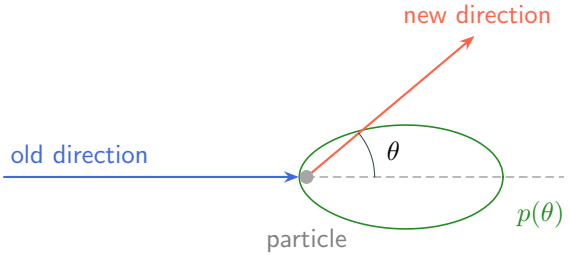


The phase function: where a scattered ray goes



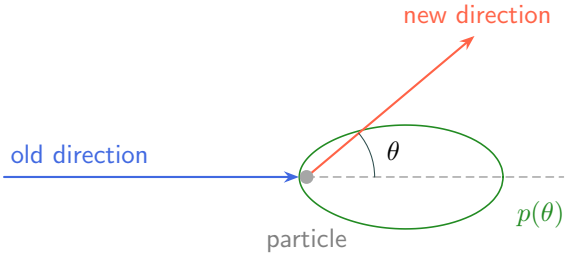
- The phase function $p(\theta)$: how likely a turn by θ is.

The phase function: where a scattered ray goes



- The phase function $p(\theta)$: how likely a turn by θ is.
- θ is measured from the old direction.

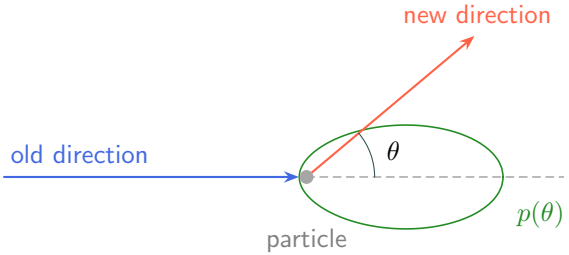
The phase function: where a scattered ray goes



- The phase function $p(\theta)$: how likely a turn by θ is.
- θ is measured from the old direction.
- Over the whole sphere of directions it adds up to 1.

$$\int_{\text{sphere}} p \, d\omega = 1$$

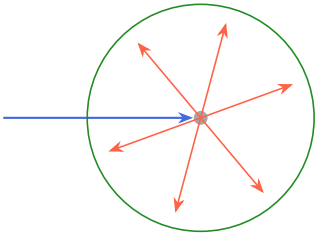
The phase function: where a scattered ray goes



- The phase function $p(\theta)$: how likely a turn by θ is.
- θ is measured from the old direction.
- Over the whole sphere of directions it adds up to 1.
- Unlike a BRDF: no normal, no cosine, all directions.

$$\int_{\text{sphere}} p \, d\omega = 1$$

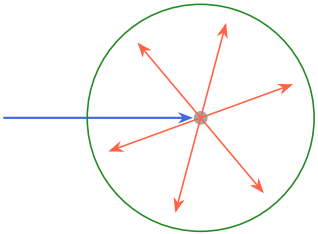
Isotropic: every way the same, $p = 1/(4\pi)$



$p = \dfrac{1}{4\pi}$ everywhere

- Isotropic: every new direction equally likely.

Isotropic: every way the same, $p = 1/(4\pi)$



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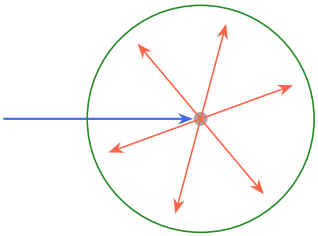

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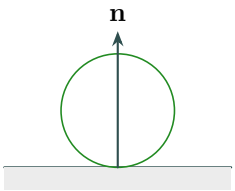
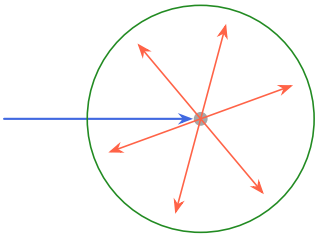


chalk: $\dfrac{R}{\pi}\cos\theta$, half sphere

- Isotropic: every new direction equally likely.
- The sphere of directions has area 4π, so $p = 1/(4\pi) = 0.080$.
- Compare chalk: a half sphere, and a cosine.

Isotropic: every way the same, $p = 1/(4\pi)$



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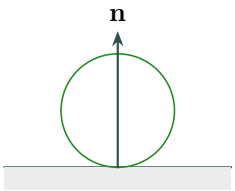


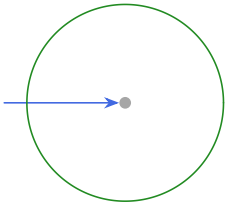
chalk: $\dfrac{R}{\pi}\cos\theta$, half sphere

- Isotropic: every new direction equally likely.
- The sphere of directions has area 4π, so $p = 1/(4\pi) = 0.080$.
- Compare chalk: a half sphere, and a cosine.
- Simple, but few real media are like this.

Henyey–Greenstein: one number, g

$$p(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}$$

$-1 < g < 1$; $g = 0$: isotropic



$g = 0$

each lobe drawn to the same length; the ray arrives from the left

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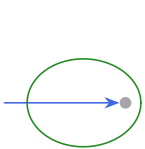
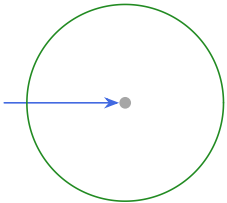


$g = -0.3$



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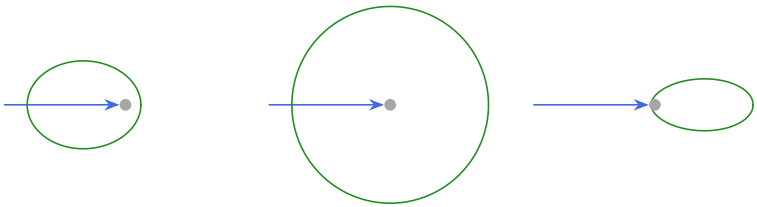

$g = -0.3$ $g = 0$ $g = 0.5$

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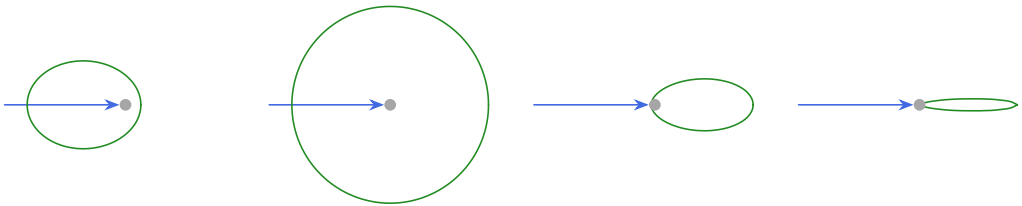

$g = -0.3$ $g = 0$ $g = 0.5$ $g = 0.85$

each lobe drawn to the same length; the ray arrives from the left

Henyey–Greenstein: one number, g

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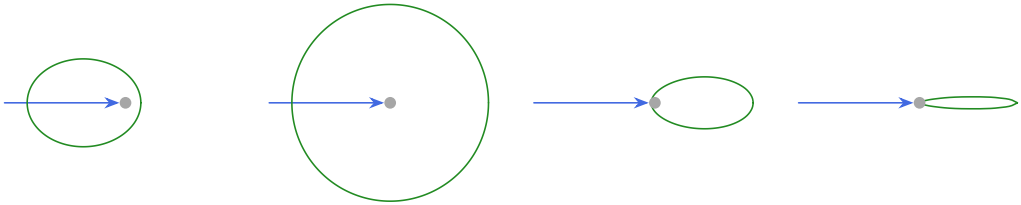
$-1 < g < 1$; $g = 0$: isotropic

- g is the average of $\cos\theta$. Clouds: $g \approx 0.85$; haze: about 0.7.



$g = -0.3$ $g = 0$ $g = 0.5$ $g = 0.85$

each lobe drawn to the same length; the ray arrives from the left

Worked: how forward is $g = 0.8$?

$$p(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}, \qquad g = 0.8; \qquad \text{isotropic: } 0.080$$

- Try first: p straight on (0°), sideways (90°), straight back (180°).

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- Try first: p straight on ($0°$), sideways ($90°$), straight back ($180°$).

$0°$: $1 + 0.64 - 1.6 = 0.04$; $0.36/0.04^{3/2} = 45$; $45/(4\pi) = 3.58$

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$180°$: $0.36/3.24^{3/2} = 0.062$; $p = 0.0049$

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against isotropic: 45 times, $\frac{1}{6}$, $\frac{1}{16}$

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against isotropic: 45 times, $\frac{1}{6}$, $\frac{1}{16}$

- A beam in such fog is bright seen from in front, faint from the side.

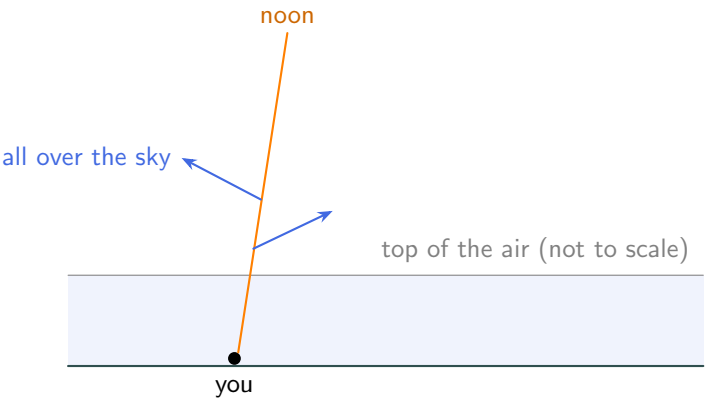
Air: molecules scatter blue the most

- Molecules are far smaller than a wavelength: σ_s grows as $1/\lambda^4$.

$p(\theta) = \frac{3}{16\pi}(1 + \cos^2\theta)$: Rayleigh



Air: molecules scatter blue the most



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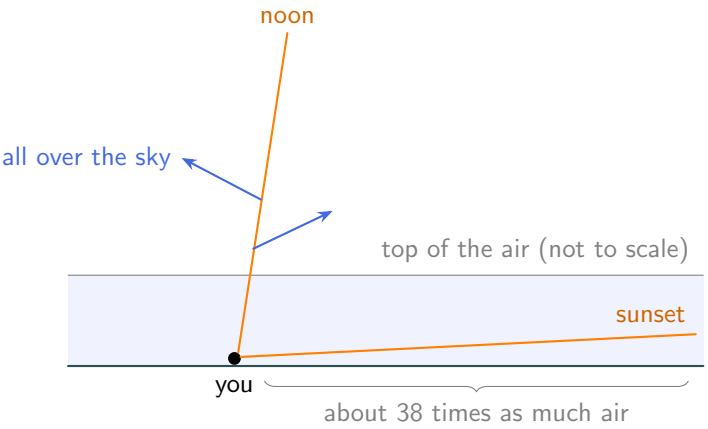

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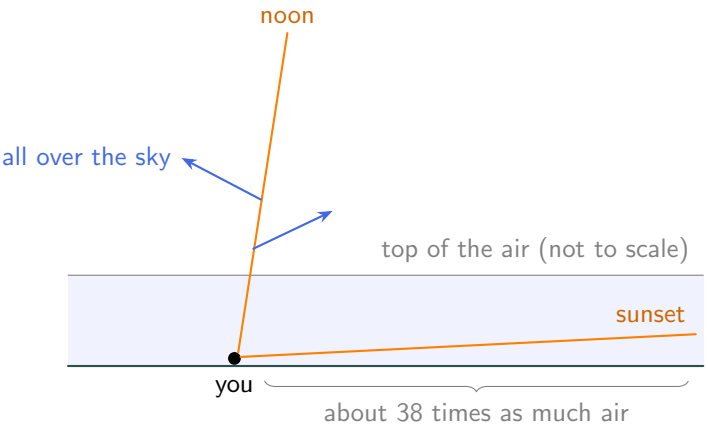

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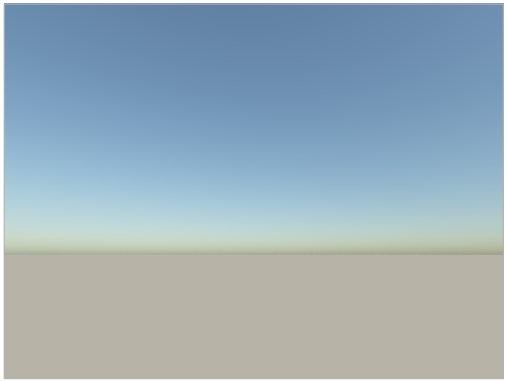
- Blue (440 nm) scatters $(680/440)^4 = 5.7$ times as much as red.
- At sunset the long path scatters the blue out of the sun's light.
- One medium, one σ_s per colour: a blue sky and a red sun.

Rendered: the sky at noon and at sunset

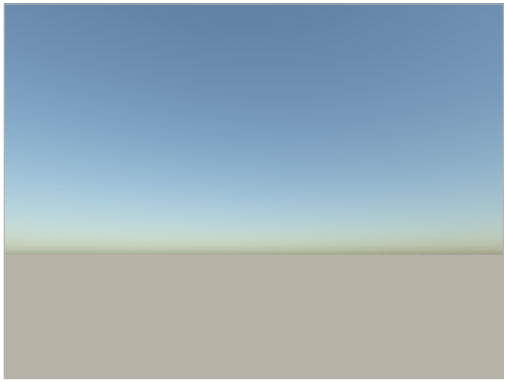
the Earth's air, light that turned once: C8's integral, summed in 24 slices per pixel



the sun 45 degrees up, behind us

Rendered: the sky at noon and at sunset

the Earth's air, light that turned once: C8's integral, summed in 24 slices per pixel



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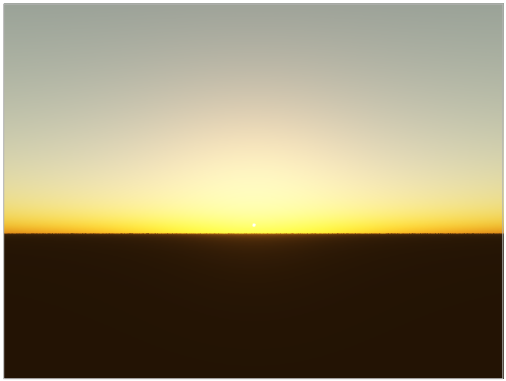
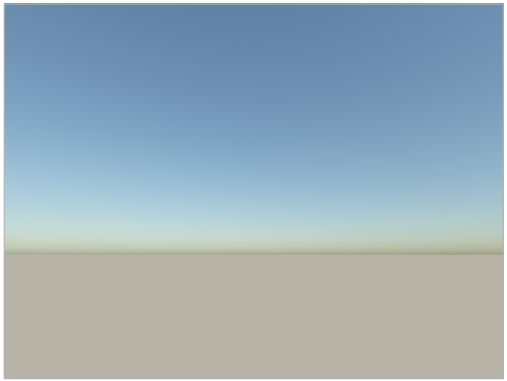


the sun 1.5 degrees up, ahead

Rendered: the sky at noon and at sunset

the Earth's air, light that turned once: C8's integral, summed in 24 slices per pixel



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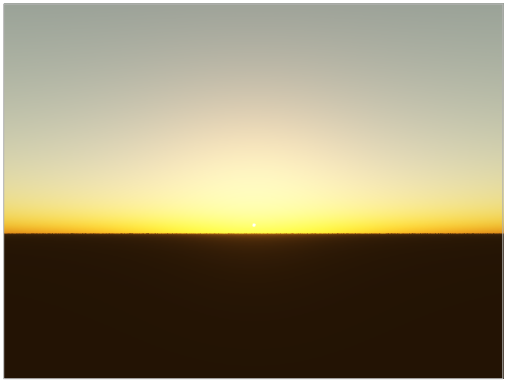

the sun 1.5 degrees up, ahead

- Nothing in these pictures is painted: one formula, the sun moved.

Picking a direction with the density p

$$\cos\theta = \frac{1}{2g}\left(1 + g^2 - \left(\frac{1-g^2}{1-g+2g\,\xi_1}\right)^2\right)$$

then turn round the old direction by $\phi = 2\pi\,\xi_2$



$g = 0.8$

Picking a direction with the density p

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then turn round the old direction by $\phi = 2\pi\,\xi_2$

$g = 0.8$, $\xi_1 = 0.5$:

$$\frac{0.36}{1.0} = 0.36; \quad \cos\theta = \frac{1.64 - 0.13}{1.6} = 0.944; \quad \theta = 19^\circ$$



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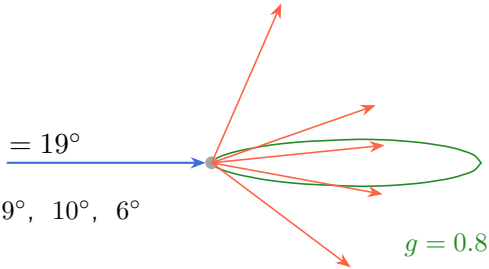
ξ_1:	0.1,	0.25,	0.5,	0.75,	0.9	θ:	66°,	37°,	19°,	10°,	6°



- Half of all turns are smaller than 19°.

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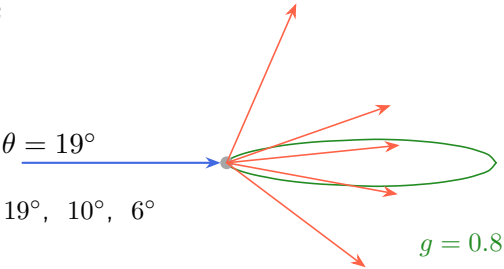
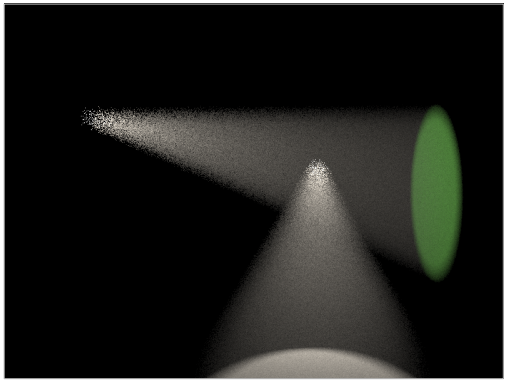
ξ_1: 0.1, 0.25, 0.5, 0.75, 0.9 θ: 66°, 37°, 19°, 10°, 6°



- Half of all turns are smaller than 19°.
- Picked with density p, divided by p: p cancels, as lecture 20's cosine did.

Rendered: two beams, $g = 0$ and $g = 0.8$

one beam across the room, one down toward us; only light that turned once



$g = 0$: isotropic

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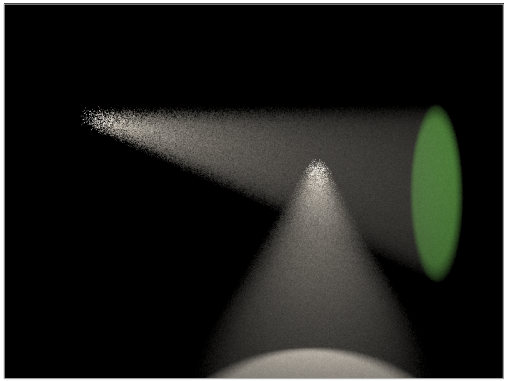
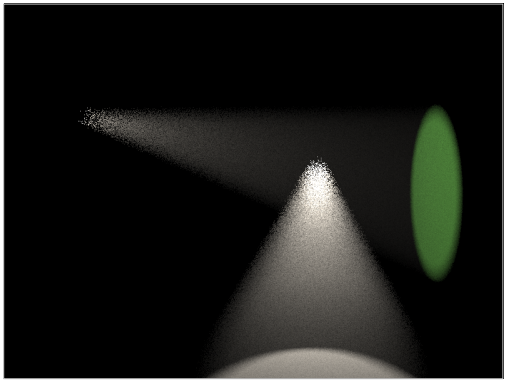


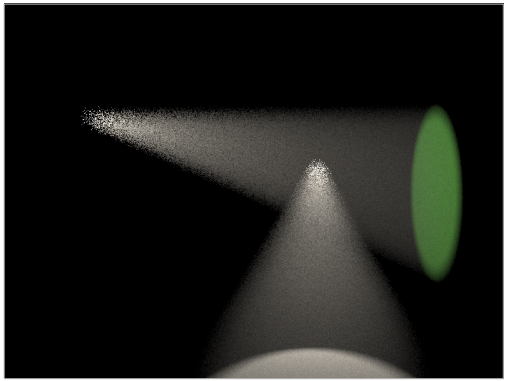
$g = 0$: isotropic



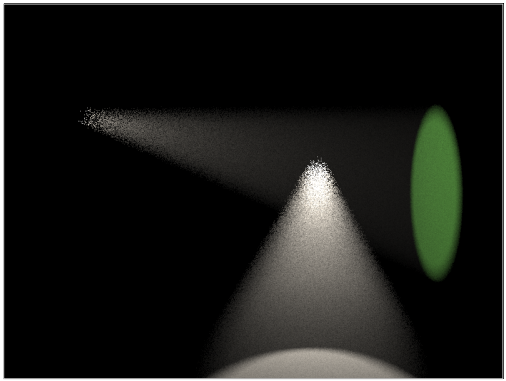
$g = 0.8$: forward

Rendered: two beams, $g = 0$ and $g = 0.8$

one beam across the room, one down toward us; only light that turned once



$g = 0$: isotropic



$g = 0.8$: forward

- With $g = 0.8$ the beam toward us brightens; the beam across fades.

Quick check 1: fog and phase

$\sigma_t = \sigma_a + \sigma_s$; $T = e^{-\sigma_t d}$; $a = e^{-\sigma}$; **isotropic** $p = 1/(4\pi)$

1. $\sigma_t = 0.25$ per metre. The average distance between hits?
2. The same fog: how much light gets through 8 metres?
3. Lecture 18's glass leaves $a = 0.9$ per unit. Its σ?
4. $g = -0.5$: does light go mostly forward or mostly back?

- σ_t says how far a ray goes; the phase function says where it turns.

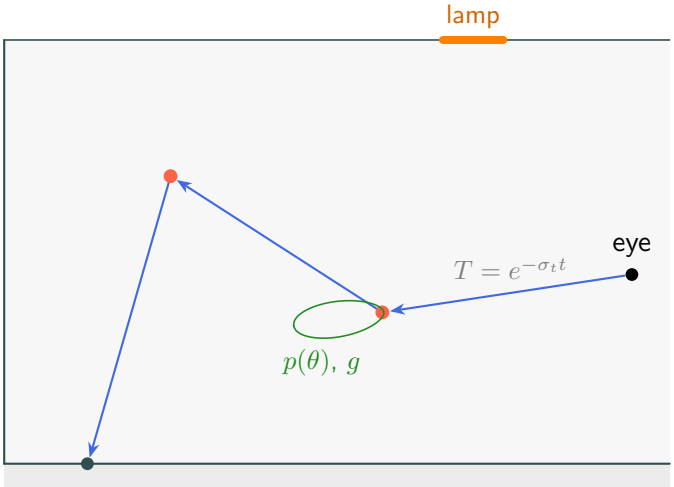
Quick check 1: fog and phase

$\sigma_t = \sigma_a + \sigma_s$; $T = e^{-\sigma_t d}$; $a = e^{-\sigma}$; **isotropic** $p = 1/(4\pi)$

1. $\sigma_t = 0.25$ per metre. The average distance between hits?	4 metres
2. The same fog: how much light gets through 8 metres?	$e^{-2} = 0.14$
3. Lecture 18's glass leaves $a = 0.9$ per unit. Its σ?	0.105
4. $g = -0.5$: does light go mostly forward or mostly back?	back

- σ_t says how far a ray goes; the phase function says where it turns.

Recap: one path through fog

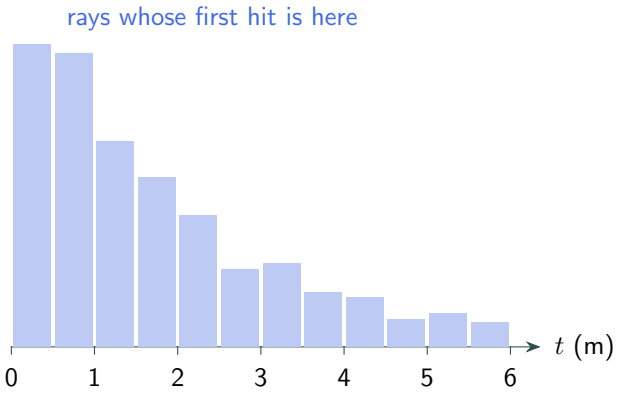


1. $L = T(d)\, L_{\text{wall}} + \int T\, \sigma_s L_s\, dt$
2. $p(\theta)$: Henyey–Greenstein, g

Free flight

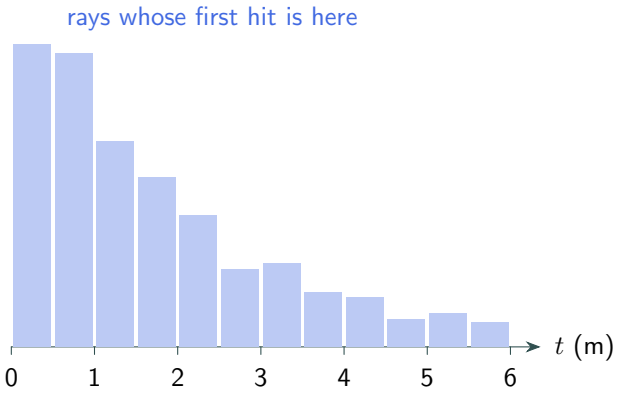
Draw where the path turns, and trace on

Where does the path turn? Count the first hits



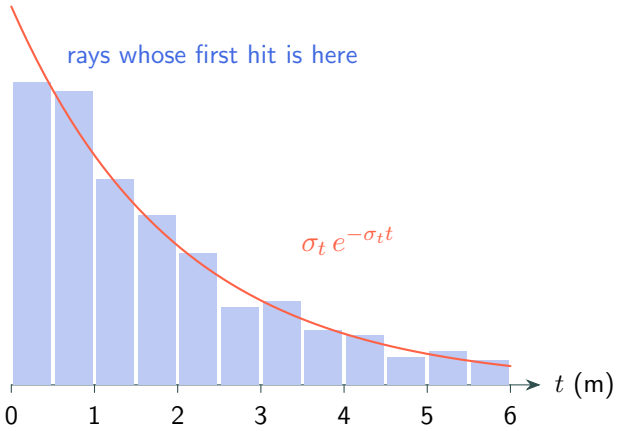
- 1000 rays marched in 1 cm steps, $\sigma_t = 0.5$: where each first hits.

Where does the path turn? Count the first hits



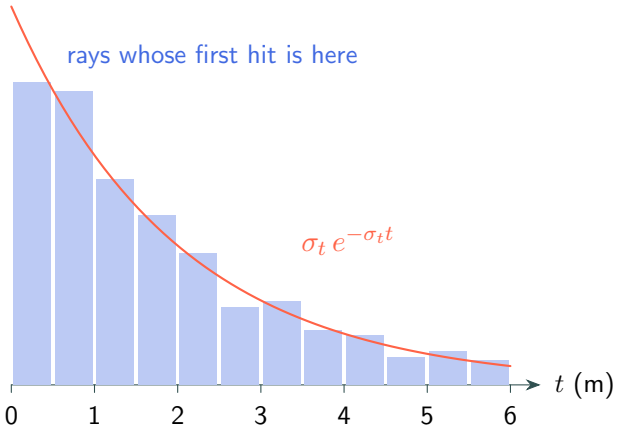
- 1000 rays marched in 1 cm steps, $\sigma_t = 0.5$: where each first hits.
- Early hits are common: every metre takes its share of the rays still going.

Where does the path turn? Count the first hits



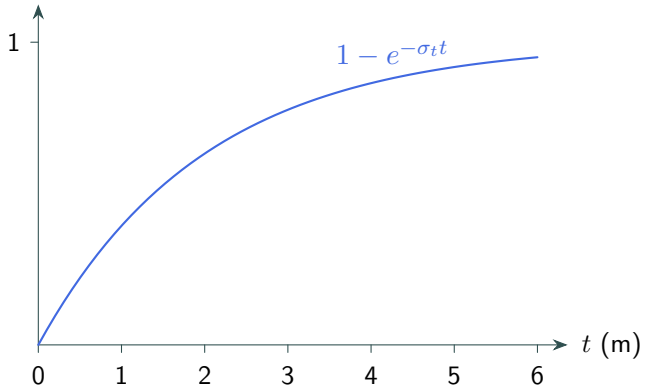
- 1000 rays marched in 1 cm steps, $\sigma_t = 0.5$: where each first hits.
- Early hits are common: every metre takes its share of the rays still going.
- Density of the first hit: survive to t, $T(t)$, then hit there, σ_t.

Where does the path turn? Count the first hits



- 1000 rays marched in 1 cm steps, $\sigma_t = 0.5$: where each first hits.
- Early hits are common: every metre takes its share of the rays still going.
- Density of the first hit: survive to t, $T(t)$, then hit there, σ_t.
- Marching costs 200 coin flips for 2 metres. We want one draw.

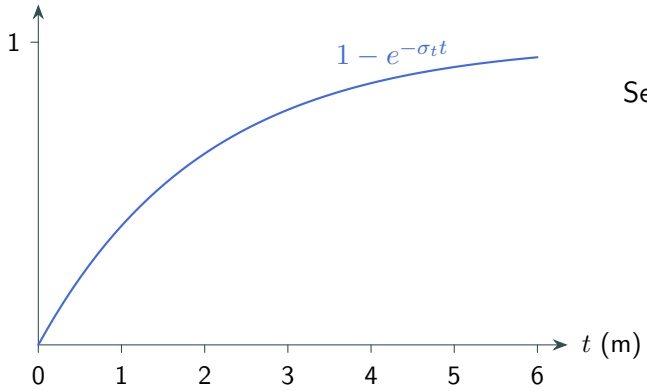
Draw the distance in one go



Share of first hits before t:

$$1 - T(t) = 1 - e^{-\sigma_t t}$$

Draw the distance in one go



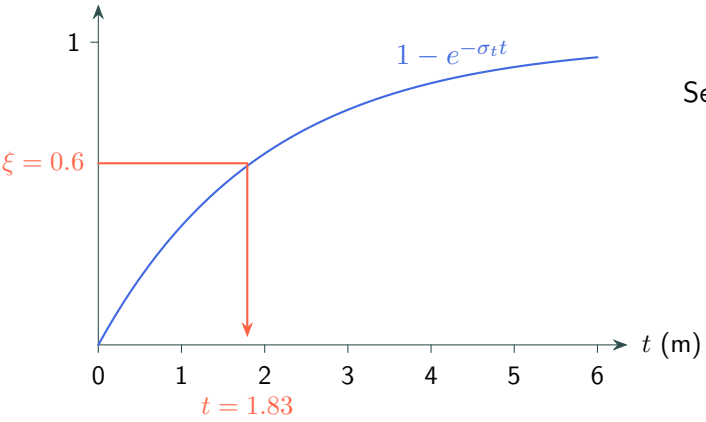
Share of first hits before t:

$$1 - T(t) = 1 - e^{-\sigma_t t}$$

Set it to a random ξ:

$$e^{-\sigma_t t} = 1 - \xi$$

Draw the distance in one go



Share of first hits before t:

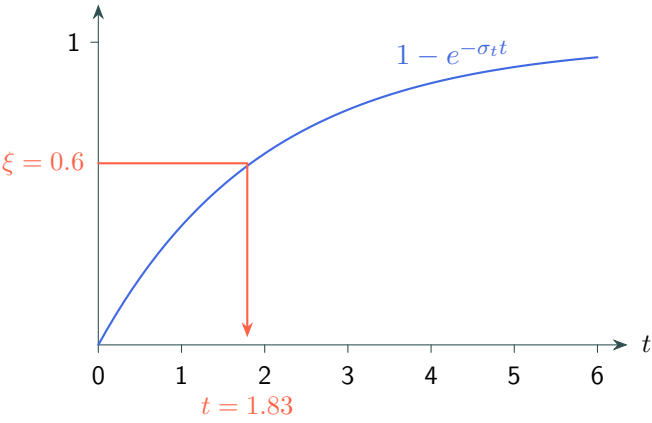
$$1 - T(t) = 1 - e^{-\sigma_t t}$$

Set it to a random ξ:

$$e^{-\sigma_t t} = 1 - \xi$$

$$t = -\frac{\ln(1-\xi)}{\sigma_t}$$

Draw the distance in one go



Share of first hits before t:

$$1 - T(t) = 1 - e^{-\sigma_t t}$$

Set it to a random ξ:

$$e^{-\sigma_t t} = 1 - \xi$$

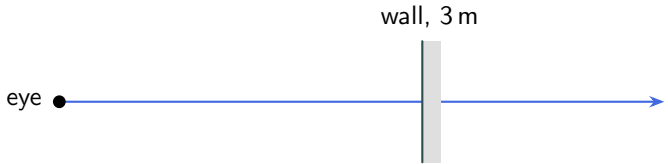
$$t = -\frac{\ln(1-\xi)}{\sigma_t}$$

- One random number gives a distance with exactly the marching's chances.

Worked: three draws in fog

$\sigma_t = 0.5$ per metre; a wall $d = 3$ metres ahead; $t = -\ln(1-\xi)/\sigma_t$

- Try first: $\xi = 0.2,\ 0.6,\ 0.9$. Where does each path turn?



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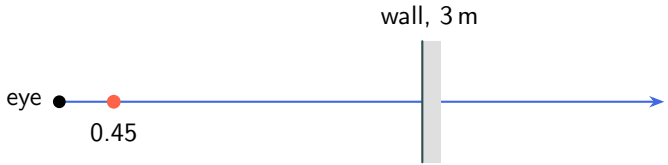
$\xi = 0.2$: $-\ln 0.8/0.5 = 0.45$ a turn



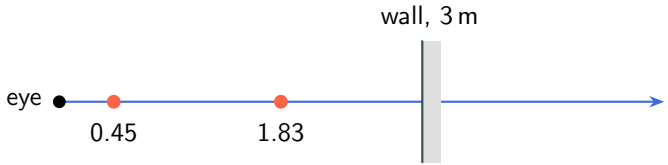
Worked: three draws in fog

$\sigma_t = 0.5$ per metre; a wall $d = 3$ metres ahead; $t = -\ln(1-\xi)/\sigma_t$

- Try first: $\xi = 0.2,\ 0.6,\ 0.9$. Where does each path turn?

$\xi = 0.2$: $-\ln 0.8/0.5 = 0.45$ a turn

$\xi = 0.6$: $-\ln 0.4/0.5 = 1.83$ a turn



Worked: three draws in fog

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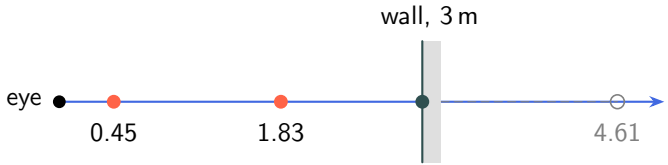
$\xi = 0.2$: $-\ln 0.8/0.5 = 0.45$ a turn

$\xi = 0.6$: $-\ln 0.4/0.5 = 1.83$ a turn

$\xi = 0.9$: $-\ln 0.1/0.5 = 4.61$ past the wall: the wall



Worked: three draws in fog

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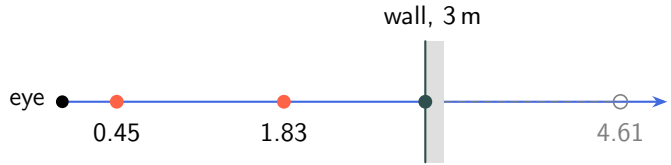
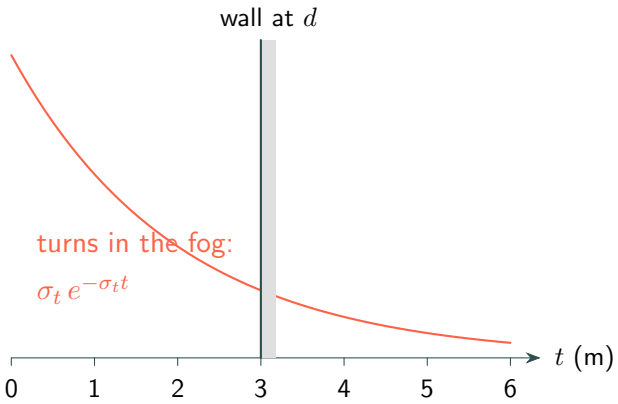


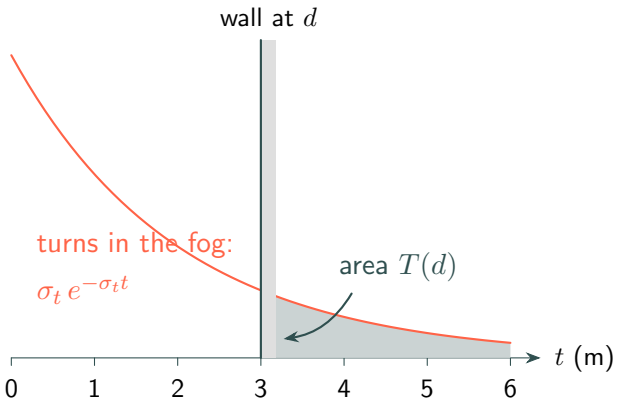
- How often is the draw past the wall? $T(3) = e^{-1.5} = 0.22$.

Past the surface? Then the surface it is



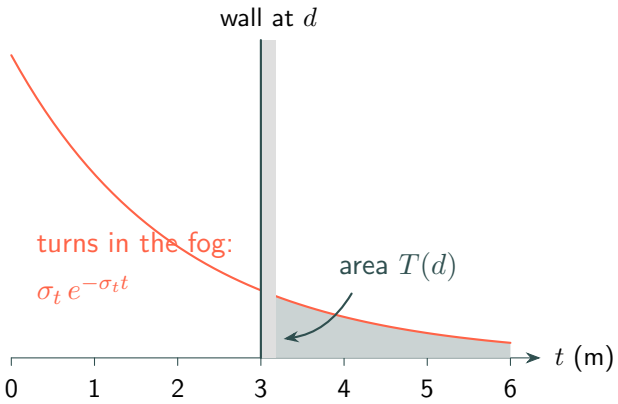
- A draw before the wall: the path turns in the fog.

Past the surface? Then the surface it is



- A draw before the wall: the path turns in the fog.
- A draw past the wall: no hit before it. That happens with chance $T(d)$.

Past the surface? Then the surface it is



- A draw before the wall: the path turns in the fog.
- A draw past the wall: no hit before it. That happens with chance $T(d)$.
- One draw chooses: a turn in the fog, or the surface.

Beer's law cancels

C8: $L = T(d)\, L_{\text{wall}} + \int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$

One draw; count what it is worth, divided by the chance of drawing it.

a turn at t:

the wall:

Beer's law cancels

C8: $L = T(d)\, L_{\text{wall}} + \displaystyle\int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$

One draw; count what it is worth, divided by the chance of drawing it.

a turn at t: $\dfrac{T(t)\, \sigma_s\, L_s(t)}{\sigma_t\, T(t)} = \dfrac{\sigma_s}{\sigma_t}\, L_s(t)$

the wall:

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the wall: $\dfrac{T(d)\, L_{\text{wall}}}{T(d)} = L_{\text{wall}}$

Beer's law cancels

C8: $L = T(d)\, L_{\text{wall}} + \displaystyle\int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$

One draw; count what it is worth, divided by the chance of drawing it.

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the wall: $\dfrac{T(d)\, L_{\text{wall}}}{T(d)} = L_{\text{wall}}$

- T cancels both times. The draws do the dimming: far turns and far walls are rarer.

Beer's law cancels

C8: $L = T(d)\, L_{\text{wall}} + \int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$

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the wall: $\dfrac{T(d)\, L_{\text{wall}}}{T(d)} = L_{\text{wall}}$

- T cancels both times. The draws do the dimming: far turns and far walls are rarer.
- Left at a turn: $\alpha = \sigma_s/\sigma_t$, the medium's albedo.

Beer's law cancels

C8: $L = T(d)\, L_{\text{wall}} + \displaystyle\int_0^d T(t)\, \sigma_s\, L_s(t)\, dt$

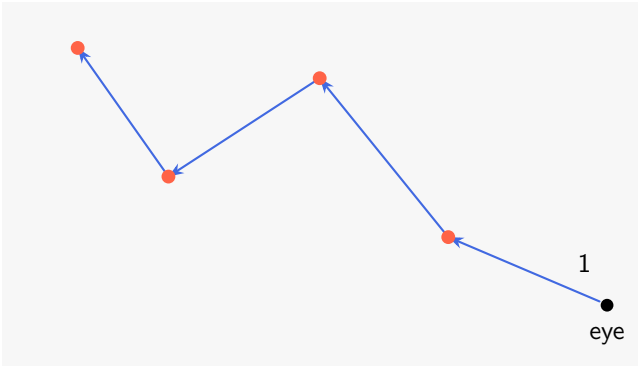
One draw; count what it is worth, divided by the chance of drawing it.

a turn at t: $$\frac{T(t)\, \sigma_s\, L_s(t)}{\sigma_t\, T(t)} = \frac{\sigma_s}{\sigma_t}\, L_s(t)$$

the wall: $$\frac{T(d)\, L_{\text{wall}}}{T(d)} = L_{\text{wall}}$$

- T cancels both times. The draws do the dimming: far turns and far walls are rarer.
- Left at a turn: $\alpha = \sigma_s/\sigma_t$, the medium's albedo.
- The code multiplies by no T at all.

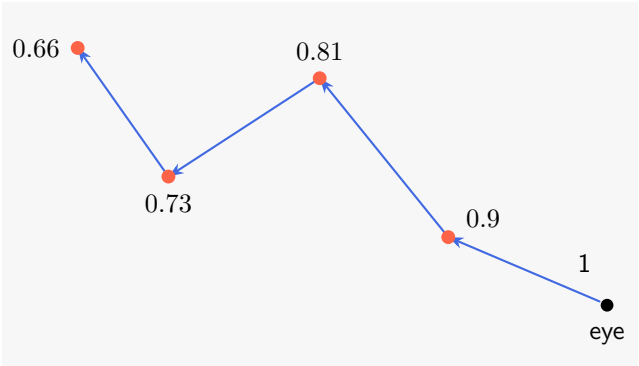
Absorbed or scattered: the albedo α



the path's weight, $\alpha = 0.9$

- $\alpha = \sigma_s/\sigma_t$: the share of hits that scatter.

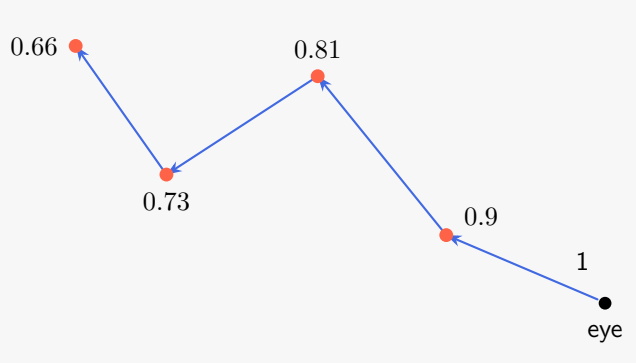
Absorbed or scattered: the albedo α



the path's weight, $\alpha = 0.9$

- $\alpha = \sigma_s/\sigma_t$: the share of hits that scatter.
- Each turn multiplies the path's weight by α.

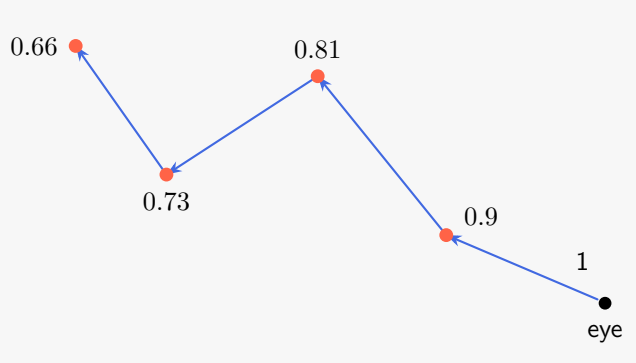
Absorbed or scattered: the albedo α



the path's weight, $\alpha = 0.9$

- $\alpha = \sigma_s/\sigma_t$: the share of hits that scatter.
- Each turn multiplies the path's weight by α.
- Or a coin: go on with chance α, weight unchanged.

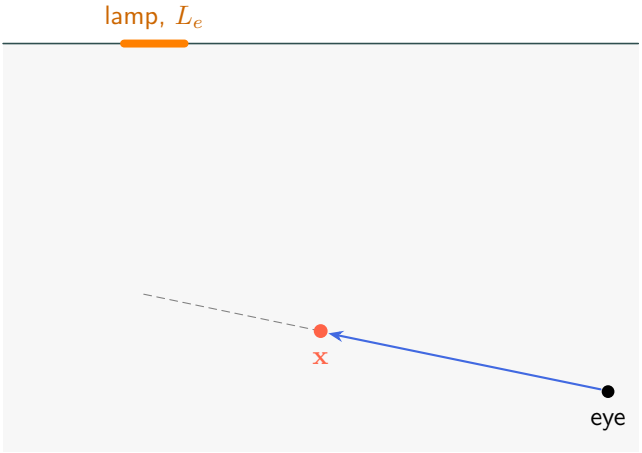
Absorbed or scattered: the albedo α



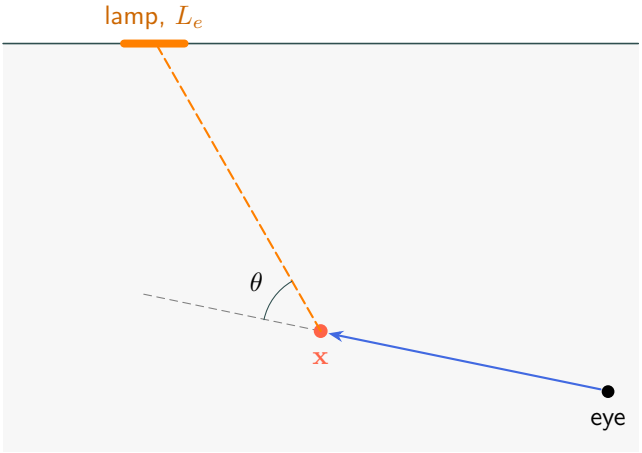
the path's weight, $\alpha = 0.9$

- $\alpha = \sigma_s/\sigma_t$: the share of hits that scatter.
- Each turn multiplies the path's weight by α.
- Or a coin: go on with chance α, weight unchanged.
- One α per colour gives the medium its colour.

Shadow rays in fog

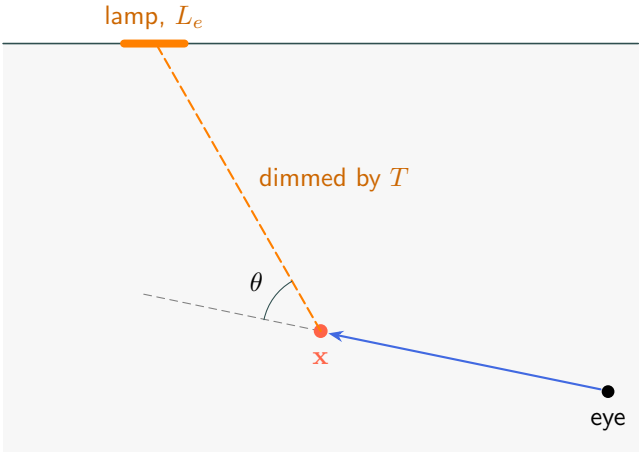


Shadow rays in fog



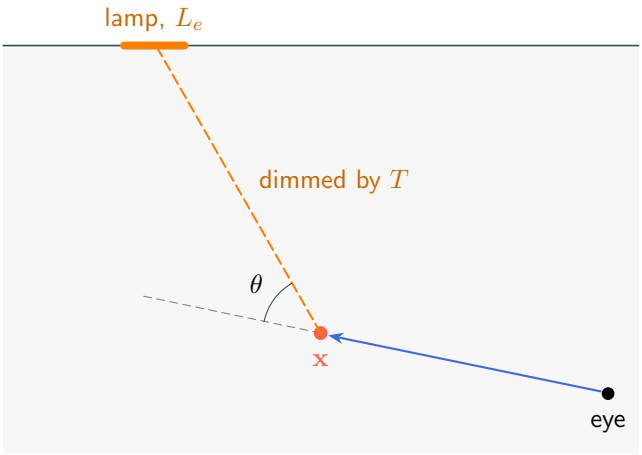
- At every turn, aim a shadow ray at the lamp (lecture 20, Extra 1).

Shadow rays in fog



- At every turn, aim a shadow ray at the lamp (lecture 20, Extra 1).
- $p(\theta)$ in place of the BRDF and its cosine; the fog dims it by T.

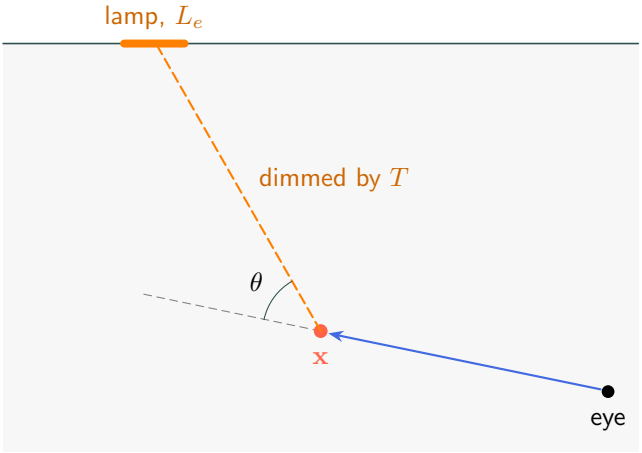
Shadow rays in fog



$$L_e \; p(\theta) \; T \; \frac{A \cos \theta_y}{r^2}$$

- At every turn, aim a shadow ray at the lamp (lecture 20, Extra 1).
- $p(\theta)$ in place of the BRDF and its cosine; the fog dims it by T.
- Here T is computed: a shadow ray must say how much gets through.

Shadow rays in fog

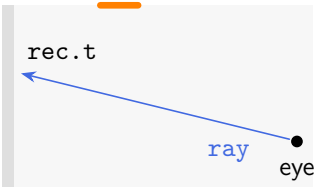


$$L_e \; p(\theta) \; T \; \frac{A \cos \theta_y}{r^2}$$

- At every turn, aim a shadow ray at the lamp (lecture 20, Extra 1).
- $p(\theta)$ in place of the BRDF and its cosine; the fog dims it by T.
- Here T is computed: a shadow ray must say how much gets through.
- Heckbert's letters gain V, a turn in a medium: a beam is L V E.

The medium step, in Python

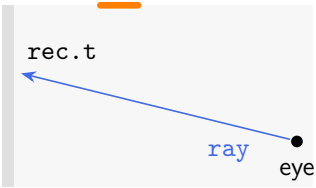
```
def trace(ray, depth):
    if depth == 0:  return Vec3(0, 0, 0)
    rec = scene.hit(ray)
```




```
    the surface branches of lecture 20, unchanged
```

The medium step, in Python

```
def trace(ray, depth):
    if depth == 0:  return Vec3(0, 0, 0)
    rec = scene.hit(ray)
    t = -math.log(1 - random.random()) / fog.sigma_t
```

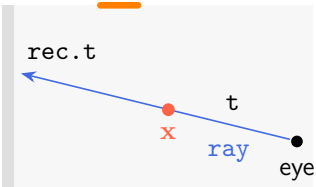



```
    the surface branches of lecture 20, unchanged
```

- `t`: a random distance (C20). It is compared with the nearest surface.

The medium step, in Python

```
def trace(ray, depth):
    if depth == 0:  return Vec3(0, 0, 0)
    rec = scene.hit(ray)
    t = -math.log(1 - random.random()) / fog.sigma_t
    if t < rec.t:
        x = ray.evaluate(t)
```



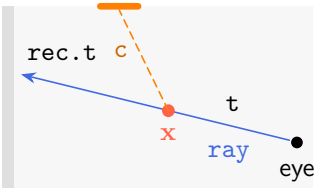

```
    the surface branches of lecture 20, unchanged
```

- `t`: a random distance (C20). It is compared with the nearest surface.

The medium step, in Python

```
def trace(ray, depth):
    if depth == 0:  return Vec3(0, 0, 0)
    rec = scene.hit(ray)
    t = -math.log(1 - random.random()) / fog.sigma_t
    if t < rec.t:
        x = ray.evaluate(t)
        c = shadow_ray(x, ray.direction)


    the surface branches of lecture 20, unchanged
```



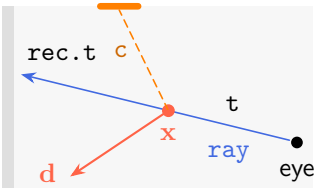
- `t`: a random distance (C20). It is compared with the nearest surface.
- `shadow_ray` returns C25's term: the lamp's light, times p and T.

The medium step, in Python

```
def trace(ray, depth):
    if depth == 0:  return Vec3(0, 0, 0)
    rec = scene.hit(ray)

    t = -math.log(1 - random.random()) / fog.sigma_t
    if t < rec.t:
        x = ray.evaluate(t)
        c = shadow_ray(x, ray.direction)
        d = sample_hg(ray.direction, fog.g)


    the surface branches of lecture 20, unchanged
```



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        c = shadow_ray(x, ray.direction)
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        return fog.albedo * (c + trace(Ray(x, d), depth - 1))
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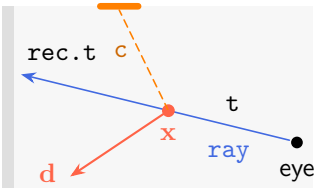
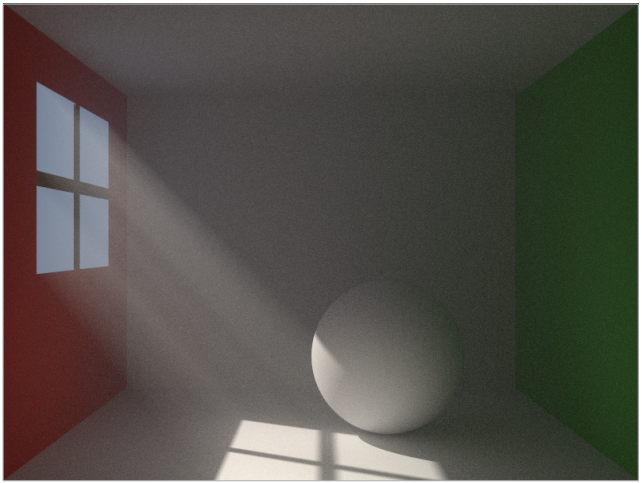
```



- `t`: a random distance (C20). It is compared with the nearest surface.
- `shadow_ray` returns C25's term: the lamp's light, times p and T.
- One recursive call, times α: the path goes on from the turn.

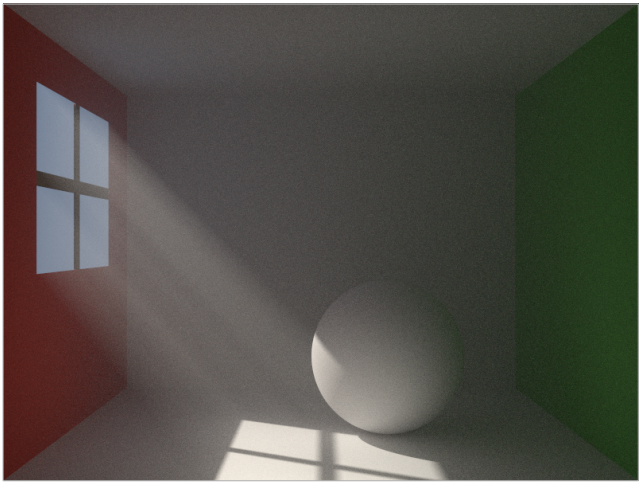
Rendered: sun through a window in fog



$\sigma_t = 0.22$, $\alpha = 0.95$, $g = 0.6$; 512 paths per pixel

- The room in thin fog, a window in the red wall, the sun behind it.

Rendered: sun through a window in fog



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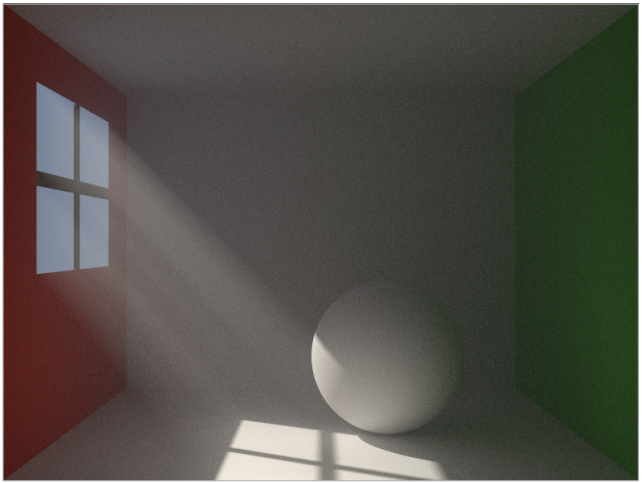
Rendered: sun through a window in fog



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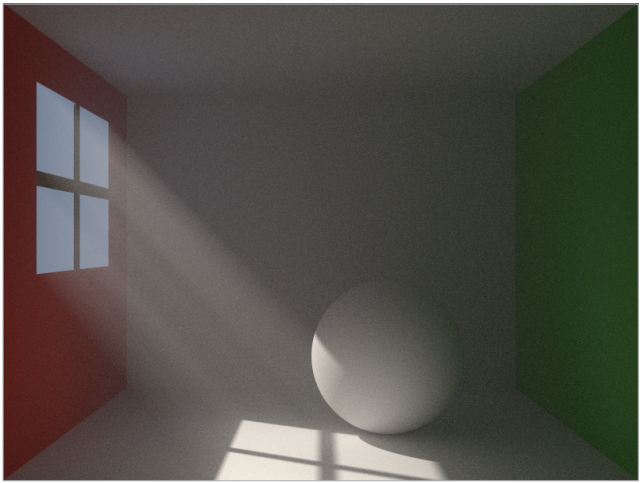

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- The room in thin fog, a window in the red wall, the sun behind it.
- The shafts: turns whose shadow rays reach the sun through the window.
- The bars' shadows run through the air down to the floor.
- Everything else: light that turned or bounced more than once.

Quick check 2: free flight

$t = -\ln(1-\xi)/\sigma_t$; **past the surface: the surface; a turn:** $\times\,\alpha$

1. $\sigma_t = 2$ per metre, a wall 1 m ahead, $\xi = 0.5$. Turn or wall?
2. The same: what share of all draws reach the wall?
3. $\alpha = 0.9$. A path's weight after three turns?
4. Why does `trace` never multiply by T?

- One draw decides where the path turns; Beer's law is in the chances.

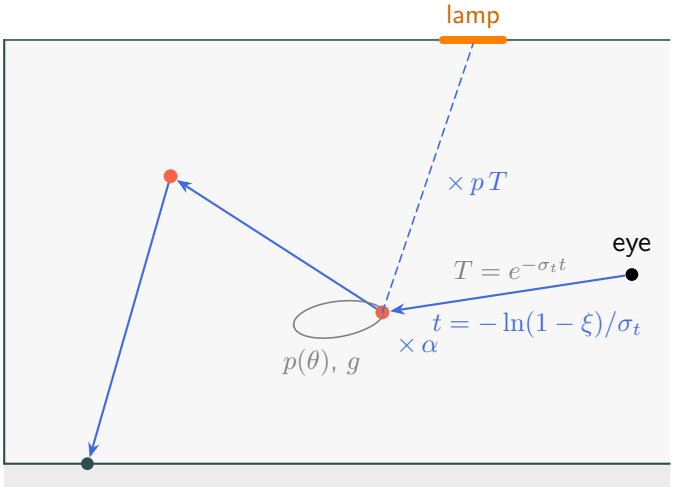
Quick check 2: free flight

$t = -\ln(1-\xi)/\sigma_t$; **past the surface: the surface; a turn:** $\times\,\alpha$

1.	$\sigma_t = 2$ per metre, a wall 1 m ahead, $\xi = 0.5$. Turn or wall?	a turn, at 0.35 m
2.	The same: what share of all draws reach the wall?	$e^{-2} = 0.14$
3.	$\alpha = 0.9$. A path's weight after three turns?	$0.9^3 = 0.73$
4.	Why does `trace` never multiply by T?	the draws dim

- One draw decides where the path turns; Beer's law is in the chances.

Recap: one path through fog

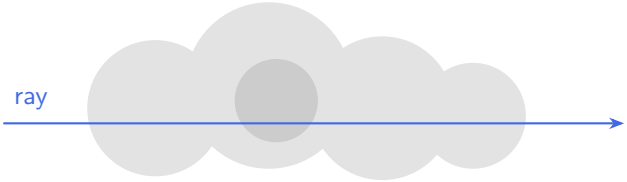


1. $L = T(d)\, L_{\text{wall}} + \int T\, \sigma_s L_s\, dt$
2. $p(\theta)$: Henyey–Greenstein, g
3. draw t; $\times\, \alpha$; shadow ray $\times\, p\,T$

Uneven media

Clouds, smoke and skin

A cloud: σ_t changes from point to point



- A cloud: $\sigma_t = \sigma$ times a density, kept in a 3D grid of numbers.

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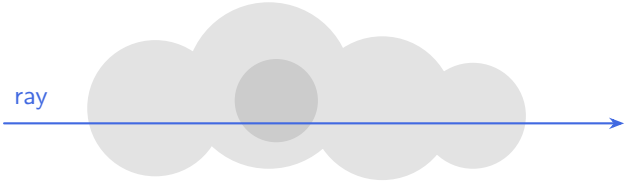

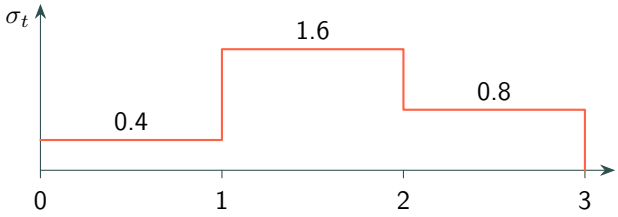

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- Along a ray, σ_t goes up and down.

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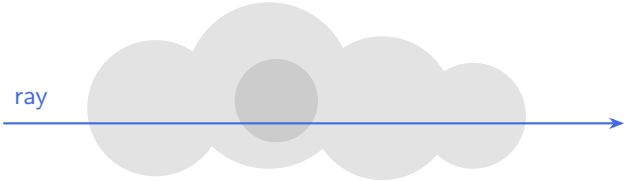

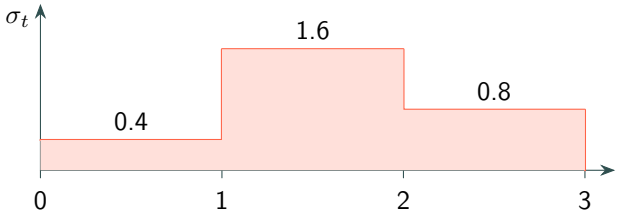

- A cloud: $\sigma_t = \sigma$ times a density, kept in a 3D grid of numbers.
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- $T = e^{-\tau}$; τ, the optical depth, is the area under σ_t.

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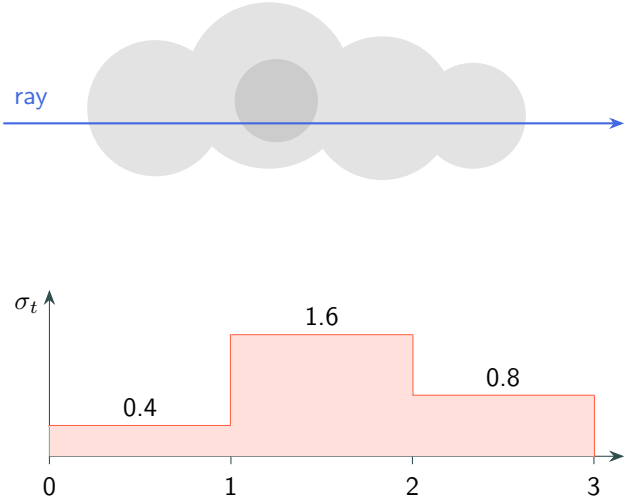
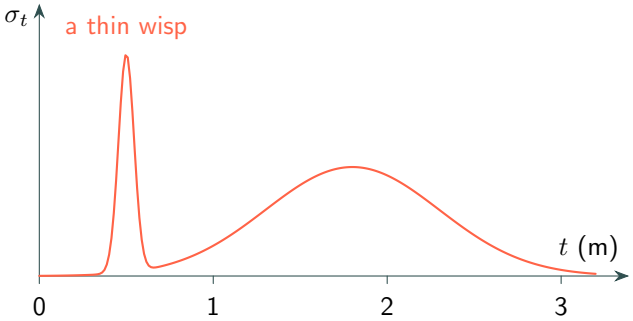


- A cloud: $\sigma_t = \sigma$ times a density, kept in a 3D grid of numbers.
- Along a ray, σ_t goes up and down.
- $T = e^{-\tau}$; τ, the optical depth, is the area under σ_t.
- C20's draw needs τ as a formula. A grid has none.

Ray marching: add up σ_t in steps



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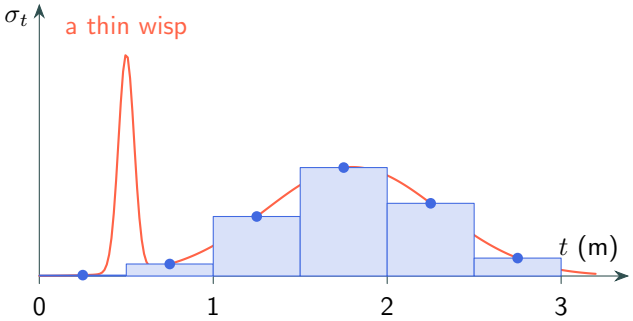

- Step along the ray; add $\sigma_t(\mathbf{x})\,\Delta s$ at each step: τ.

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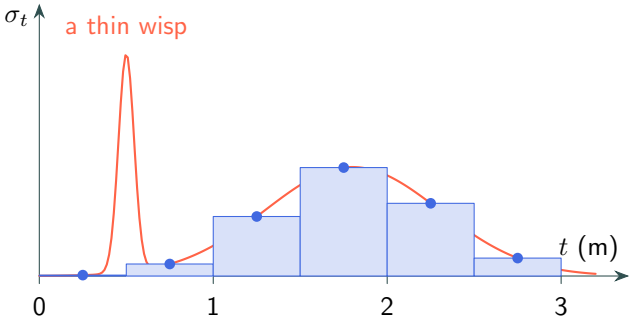

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- Fast, and easy on a GPU: games do this (Extra 6), and NeRF (Extra 7).

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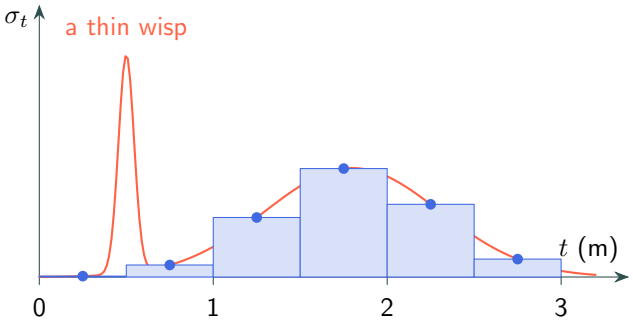

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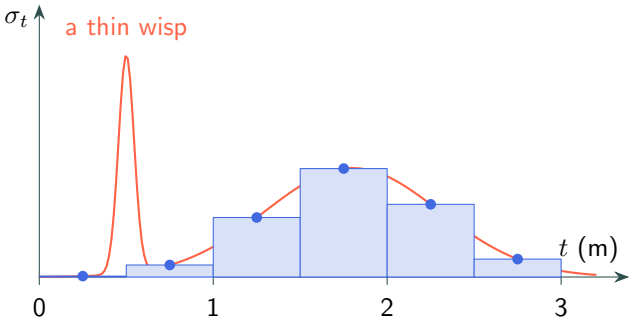

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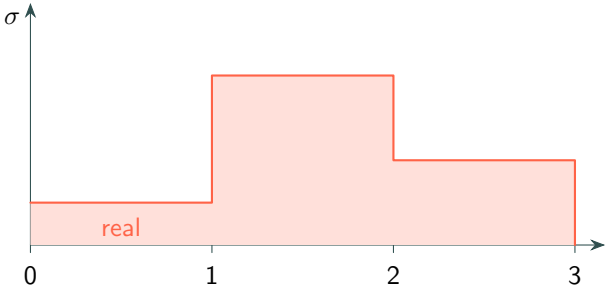
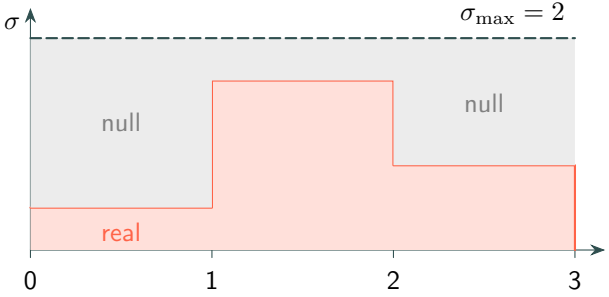
- Fast, and easy on a GPU: games do this (Extra 6), and NeRF (Extra 7).
- But a wisp between two steps is missed: the answer is wrong on average.
- Smaller steps cost more, and are still not exact.

Delta tracking: fill up with null particles



Delta tracking: fill up with null particles

- Add null particles until $\sigma = \sigma_{\max}$ everywhere: even fog.



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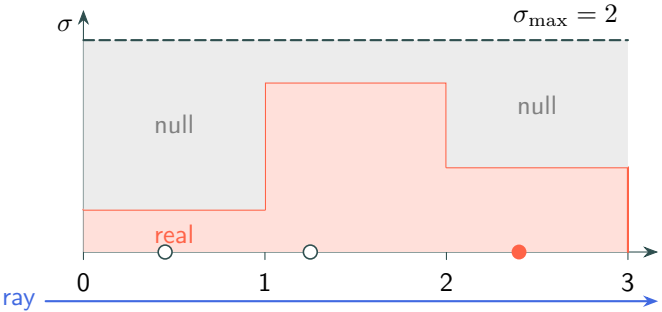

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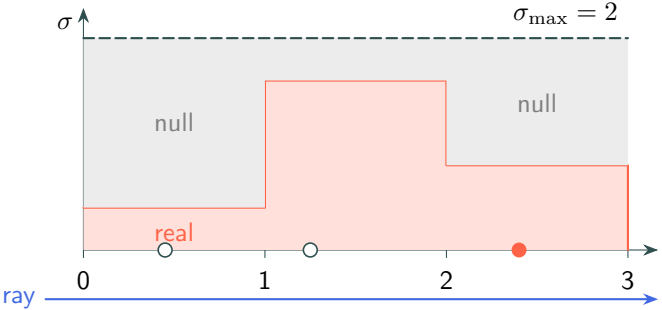

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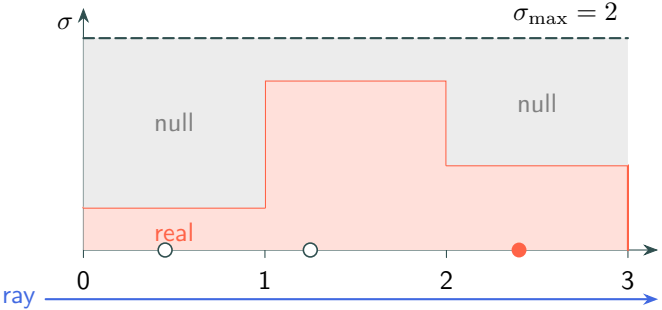

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- Draw hits with $\sigma_{\max}$, C20's formula.
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- A null particle changes nothing, so the answer is exact.

Woodcock 1965: neutrons in reactors

Worked: one ray through the cloud's edge

σ: 0.4 on $[0,1)$, 1.6 on $[1,2)$, 0.8 on $[2,3)$; $\sigma_{\max} = 2$

steps drawn with $\sigma_{\max}$: 0.45, 0.80, 1.15; coins: 0.55, 0.90, 0.30

- Try first: where does the path turn?



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- Thin parts are mostly null hits: rays pass through them.

Ratio tracking: a fraction, not 0 or 1

A shadow ray across the same cloud: how much light gets through? True: $e^{-2.8} = 0.061$

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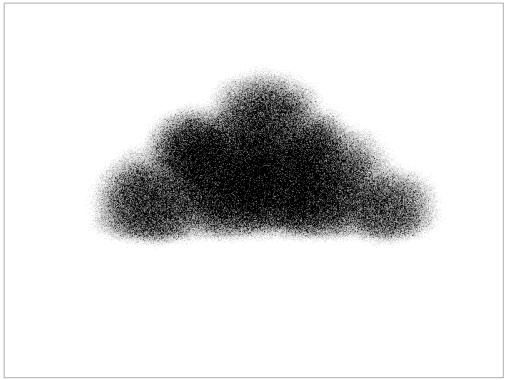
$(1 - 0.2) \times (1 - 0.8) \times (1 - 0.4) = 0.8 \times 0.2 \times 0.6 = 0.096$

- On average also 0.061, but every answer is a fraction near it.
- Same tentative hits, no coins: far less noise in shadows.

  Novák, Selle and Jarosz 2014; the next draw (past 3 m) ends the ray

Rendered: what one shadow ray reports

each pixel: T through a thin cloud, against white; 4 rays per pixel



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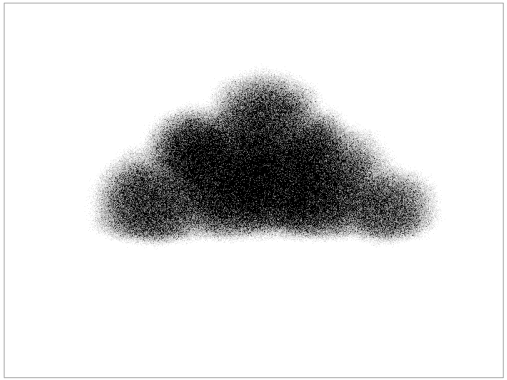

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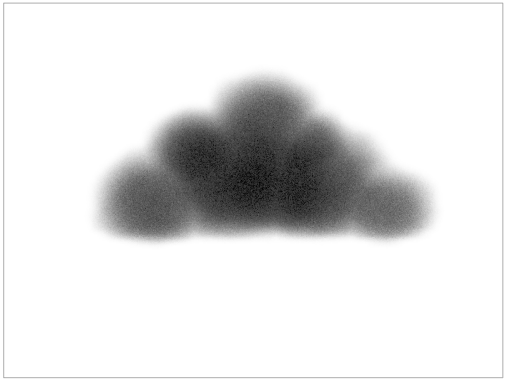

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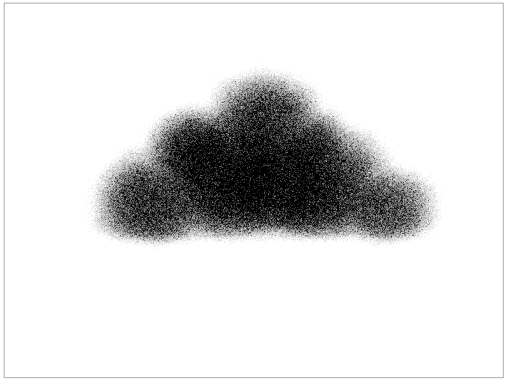

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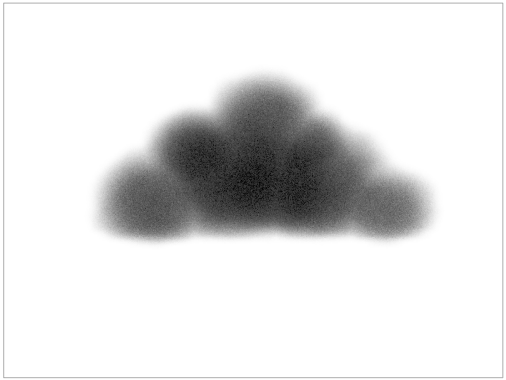
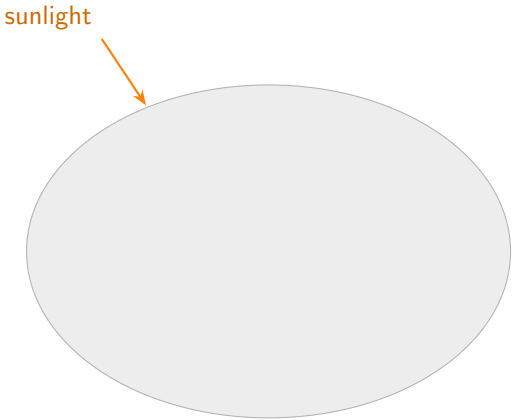


ratio tracking: a fraction

- The same average; ratio tracking gives a smooth grey.

Thick media: light turns many times



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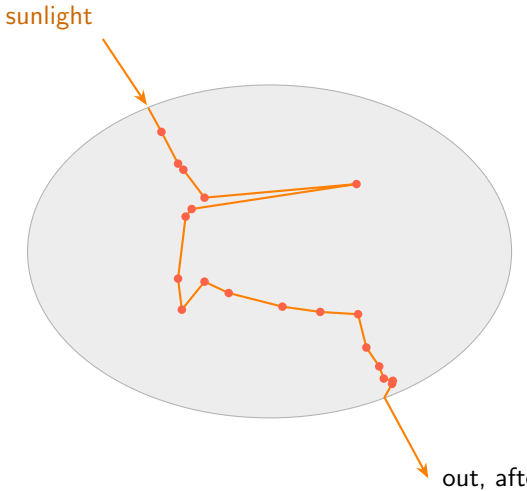

- In a cloud a path turns tens or hundreds of times before it leaves.

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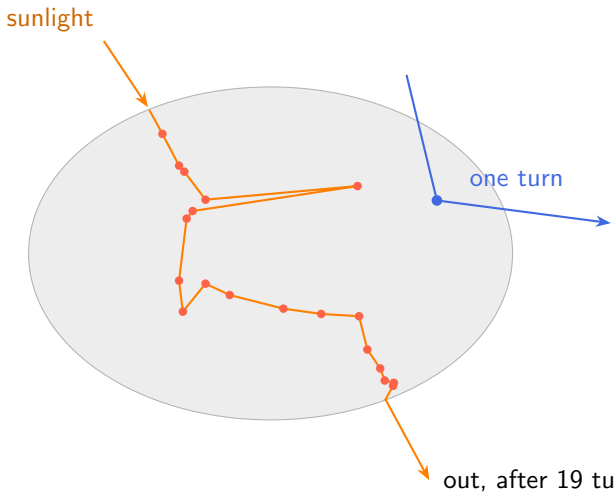

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- Single scattering: only paths that turned once.

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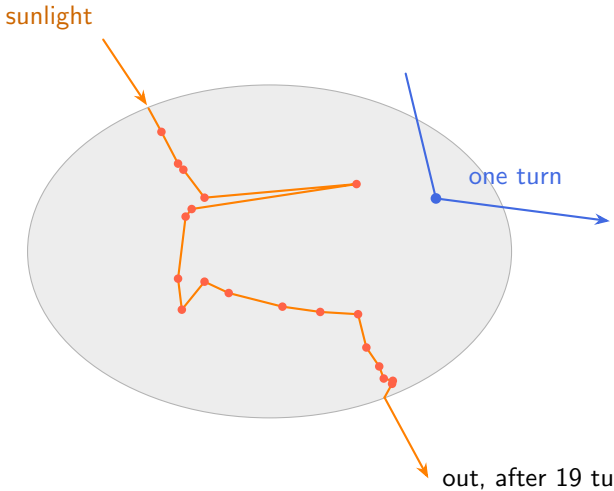
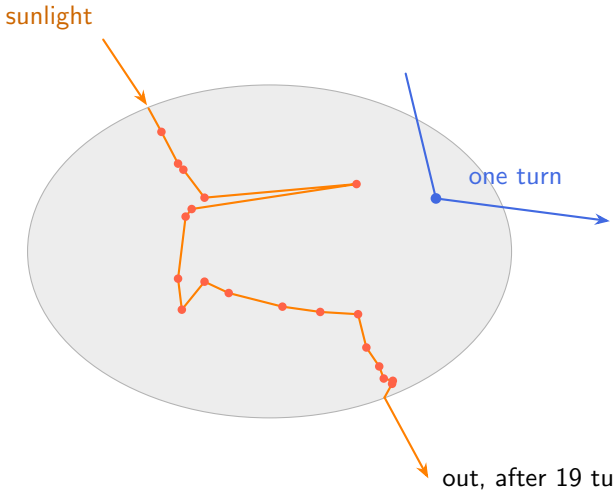


- In a cloud a path turns tens or hundreds of times before it leaves.
- Single scattering: only paths that turned once.
- Thin media: nearly enough. White, thick media: far from it.

Thick media: light turns many times



- In a cloud a path turns tens or hundreds of times before it leaves.
- Single scattering: only paths that turned once.
- Thin media: nearly enough. White, thick media: far from it.
- α near 1 matters: $0.995^{100} = 0.61$, while $0.9^{100} = 0.00003$.

Rendered: a cloud, one turn or all of them

$\sigma = 10$ where the density is 1, $\alpha = 0.995$, $g = 0.8$; the sun from the left



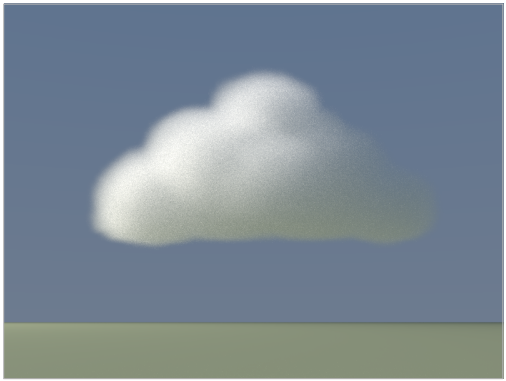
light that turned once

Rendered: a cloud, one turn or all of them

$\sigma = 10$ where the density is 1, $\alpha = 0.995$, $g = 0.8$; the sun from the left



light that turned once



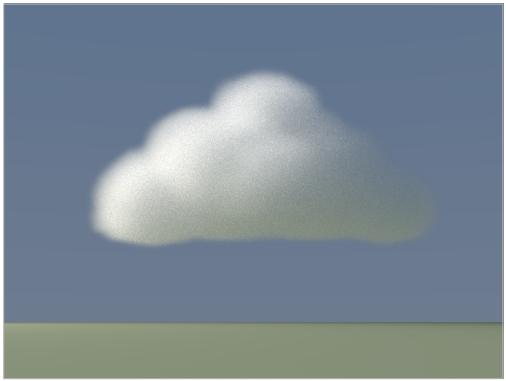
every turn

Rendered: a cloud, one turn or all of them

$\sigma = 10$ where the density is 1, $\alpha = 0.995$, $g = 0.8$; the sun from the left



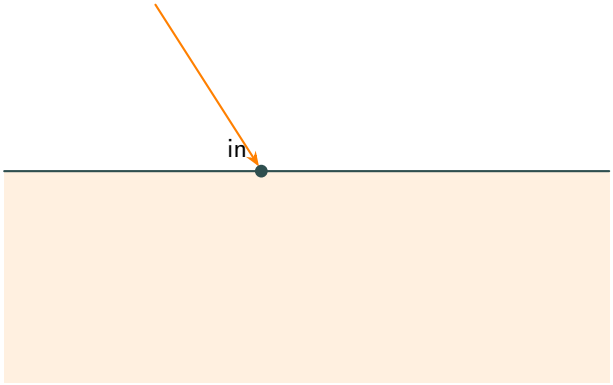
light that turned once



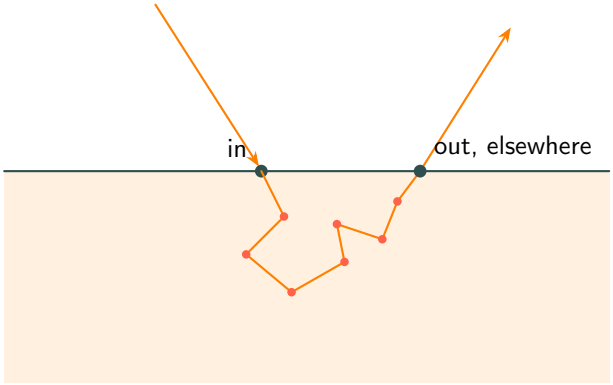
every turn

- The white of a cloud is light that turned many times.

Skin, wax, milk, marble: a medium under a surface

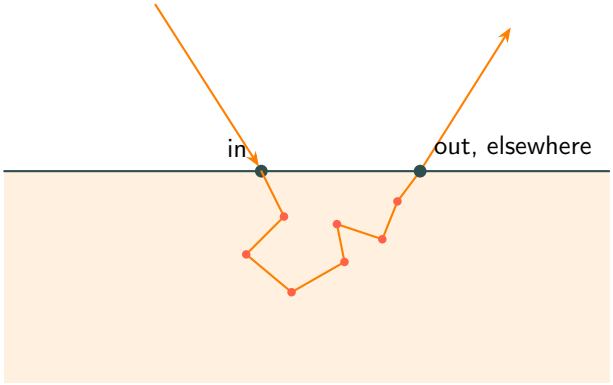


Skin, wax, milk, marble: a medium under a surface



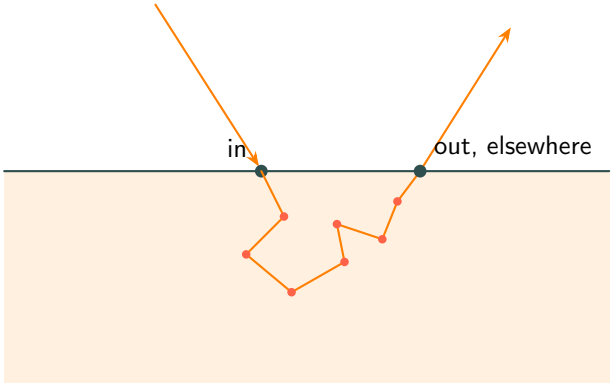
- Light goes in, wanders, and comes out somewhere else.

Skin, wax, milk, marble: a medium under a surface



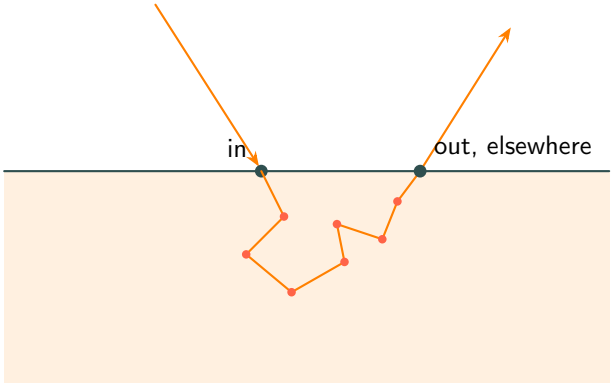
- Light goes in, wanders, and comes out somewhere else.
- Thin parts glow, shadow edges soften, and the colour deepens.

Skin, wax, milk, marble: a medium under a surface



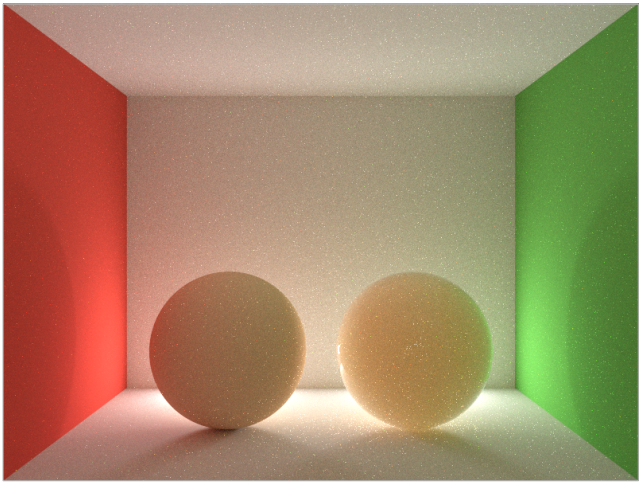
- Light goes in, wanders, and comes out somewhere else.
- Thin parts glow, shadow edges soften, and the colour deepens.
- Subsurface scattering: part three's tracer, inside a surface.

Skin, wax, milk, marble: a medium under a surface



- Light goes in, wanders, and comes out somewhere else.
- Thin parts glow, shadow edges soften, and the colour deepens.
- Subsurface scattering: part three's tracer, inside a surface.
- Films' renderers walk like this; the 2001 shortcut was a formula.

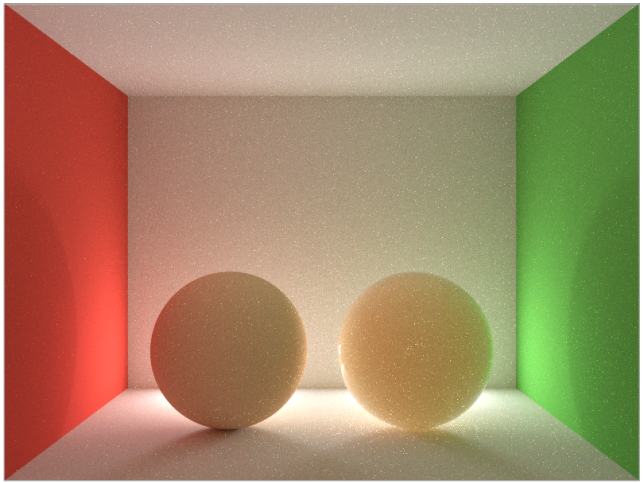
Rendered: chalk and wax, lit from behind



wax: $\sigma_t = 12$, $\alpha = (0.998, 0.95, 0.80)$; 512 paths per pixel

- Two balls; a small lamp on the back wall, hidden behind each.

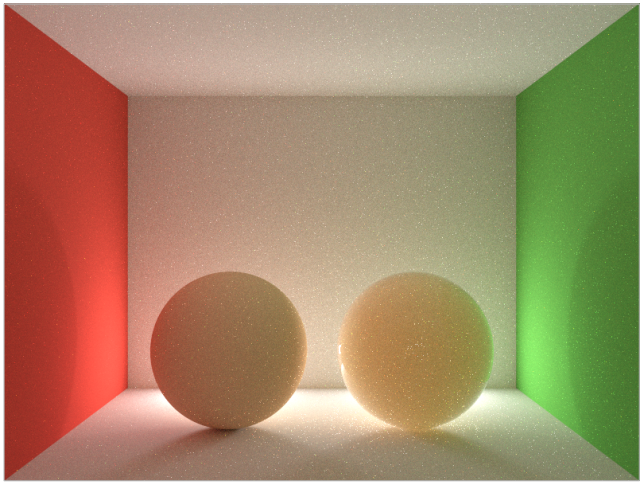
Rendered: chalk and wax, lit from behind



wax: $\sigma_t = 12$, $\alpha = (0.998, 0.95, 0.80)$; 512 paths per pixel

- Two balls; a small lamp on the back wall, hidden behind each.
- Chalk, left: its front gets only light bounced off the room.

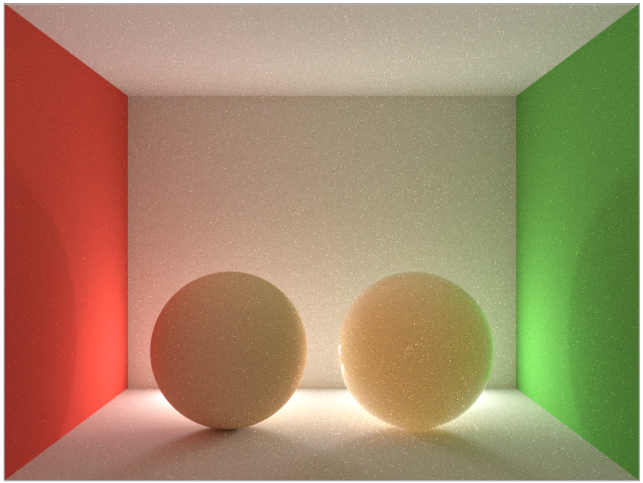
Rendered: chalk and wax, lit from behind



wax: $\sigma_t = 12$, $\alpha = (0.998, 0.95, 0.80)$; 512 paths per pixel

- Two balls; a small lamp on the back wall, hidden behind each.
- Chalk, left: its front gets only light bounced off the room.
- Wax, right: the lamp's light comes through, and the ball glows.

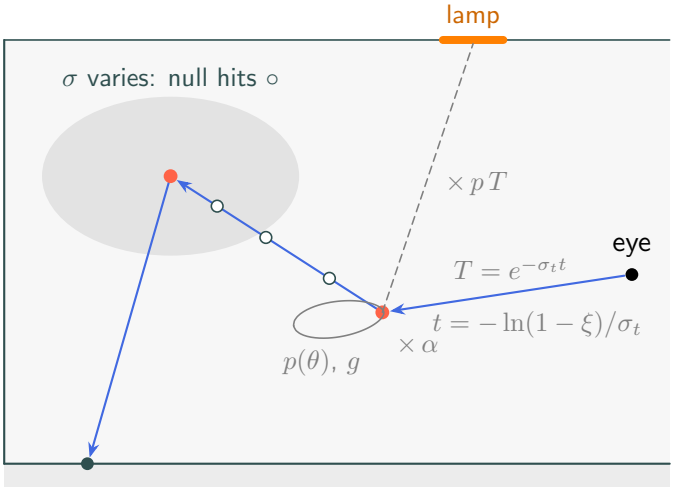
Rendered: chalk and wax, lit from behind



wax: $\sigma_t = 12$, $\alpha = (0.998, 0.95, 0.80)$; 512 paths per pixel

- Two balls; a small lamp on the back wall, hidden behind each.
- Chalk, left: its front gets only light bounced off the room.
- Wax, right: the lamp's light comes through, and the ball glows.
- Brightest near its rim, where the way through is short.

Recap: one path through fog



1. $L = T(d)\, L_{\text{wall}} + \int T\, \sigma_s L_s\, dt$
2. $p(\theta)$: Henyey–Greenstein, g
3. draw t; $\times\, \alpha$; shadow ray $\times\, p\,T$
4. $\sigma_{\max}$; real with chance $\sigma/\sigma_{\max}$

Summary

1 A ray through fog

$$L = T(d)\, L_{\text{wall}} + \int_0^d T\, \sigma_s L_s\, dt$$

$T = e^{-\sigma_t d}$; $\sigma_t = \sigma_a + \sigma_s$

2 Which way it turns

$p(\theta)$, Henyey–Greenstein g

sums to 1 over the sphere; air: $\sigma_s \propto 1/\lambda^4$

3 Free flight

$$t = -\frac{\ln(1-\xi)}{\sigma_t}$$

T cancels; $\times\, \alpha$; shadow rays $\times\, pT$

4 Uneven media

real with chance $\dfrac{\sigma(\mathbf{x})}{\sigma_{\max}}$

ratio tracking for shadows; thick: many turns

Reading

Pharr, Jakob and Humphreys, *Physically Based Rendering*, 4th edition (2023)

chapter 11 (volume scattering), chapter 14 (volume rendering); free at pbr-book.org

Novák, Georgiev, Hanika and Jarosz, "Monte Carlo methods for volumetric light transport simulation", Eurographics 2018

Fong, Wrenninge, Kulla and Habel, "Production volume rendering", SIGGRAPH 2017 course

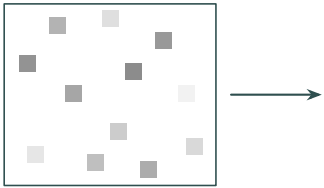
Novák, Selle and Jarosz, "Residual ratio tracking for estimating attenuation in participating media", 2014

Jensen, Marschner, Levoy and Hanrahan, "A practical model for subsurface light transport", SIGGRAPH 2001

Nishita, Sirai, Tadamura and Nakamae, "Display of the Earth taking into account atmospheric scattering", SIGGRAPH 1993

Next: Extra 6, real time

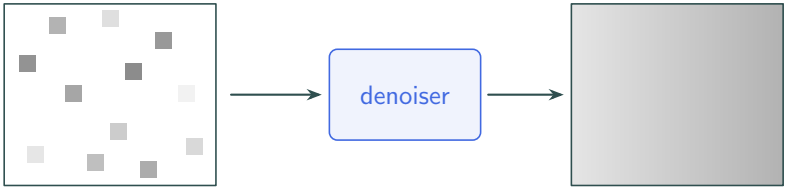
60 frames a second: about 16 ms for everything



1 path per pixel

Next: Extra 6, real time

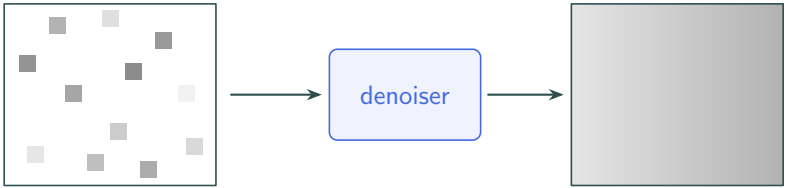
60 frames a second: about 16 ms for everything



- A game has one path per pixel, or less.

Next: Extra 6, real time

60 frames a second: about 16 ms for everything



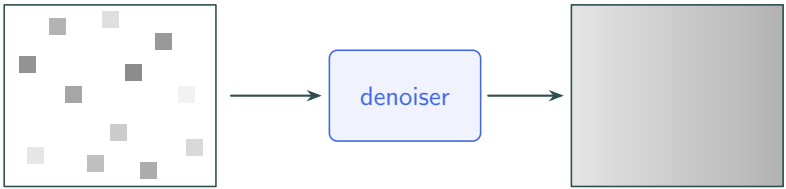
1 path per pixel

the frame

- A game has one path per pixel, or less.
- Denoisers, reused samples, ray tracing in hardware.

Next: Extra 6, real time

60 frames a second: about 16 ms for everything



1 path per pixel

the frame

- A game has one path per pixel, or less.
- Denoisers, reused samples, ray tracing in hardware.
- Extra 6: path tracing in real time.