Extra 2: Materials

A Surface Made of Tiny Mirrors

CS 453 — Computer Graphics

Fakhir Shaheen

Forman Christian University

Today: from a guessed highlight to tiny mirrors

Extra 2: self-study, not on the exams

$$\rho = \frac{R}{\pi} + R\,(\mathbf{n} \cdot \mathbf{h})^s$$

lecture 17's Blinn–Phong:
a guessed highlight

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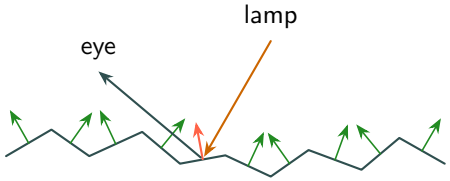

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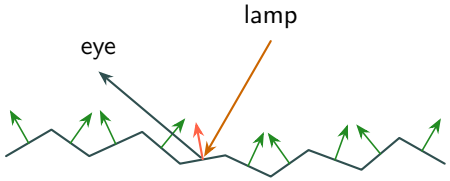

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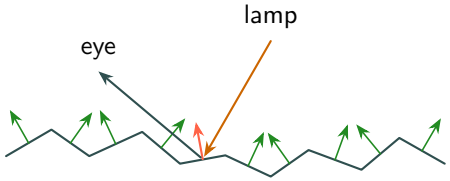


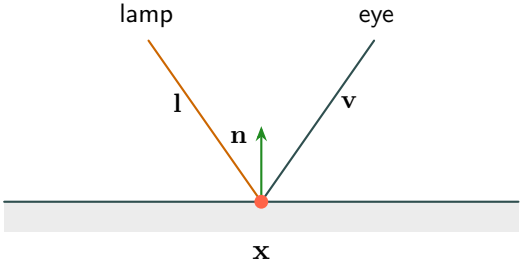
Extra 2: a surface of tiny mirrors

- Lecture 17's highlight sends back more light than arrives.
- Tiny mirrors give a BRDF that keeps the rules, and a way to pick by it.

Not a BRDF

Two tests, and a surface under a microscope

Two rules every BRDF obeys



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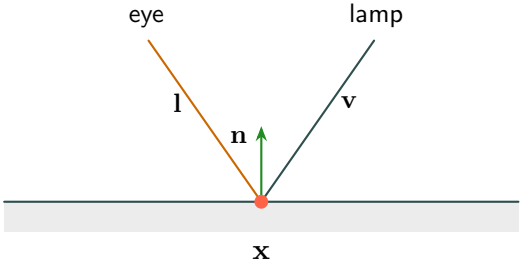
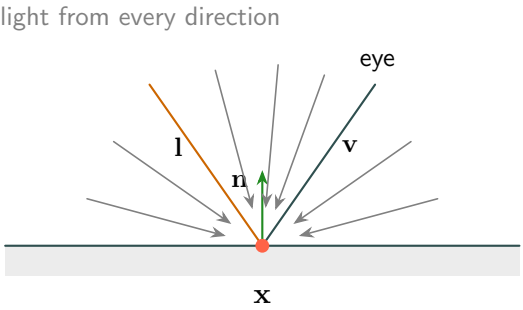


- Reciprocity: swap the lamp and the eye, and ρ stays the same.

$$\rho(\mathbf{l}, \mathbf{v}) = \rho(\mathbf{v}, \mathbf{l})$$

Two rules every BRDF obeys



- Reciprocity: swap the lamp and the eye, and ρ stays the same.

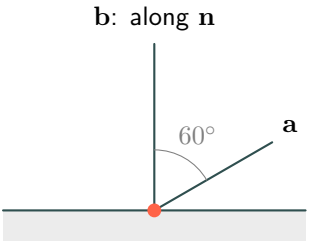
$$\rho(\mathbf{l}, \mathbf{v}) = \rho(\mathbf{v}, \mathbf{l})$$

- Energy: the surface sends back at most what arrives.

$$a(\mathbf{v}) = \int_{\text{dome}} \rho(\mathbf{l}, \mathbf{v}) \cos\theta_l \, d\sigma \;\leq\; 1$$

for every $\mathbf{v}$; chalk: $\dfrac{R}{\pi} \cdot \pi = R$ (lecture 19)

Test 1: swap the lamp and the eye



highlight term	lamp at $\mathbf{a}$, eye at $\mathbf{b}$	lamp at $\mathbf{b}$, eye at $\mathbf{a}$

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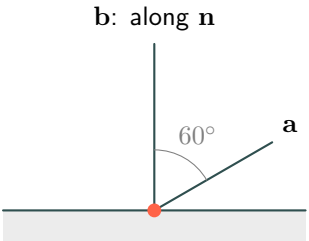

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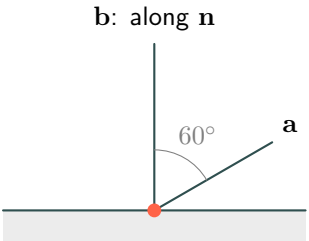
lecture 17: $R\,(\mathbf{n}\cdot\mathbf{h})^s$	$R\,(\mathbf{n}\cdot\mathbf{h})^s$	$R\,(\mathbf{n}\cdot\mathbf{h})^s$

- Lecture 17 passes: $\mathbf{h}$ is the same both ways, and E carries the cosine.

Test 1: swap the lamp and the eye

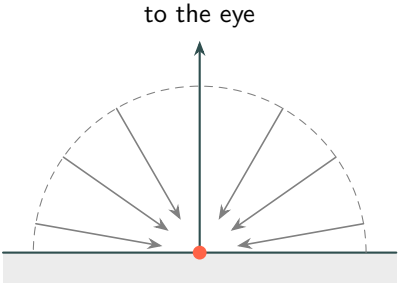


highlight term	lamp at **a**, eye at **b**	lamp at **b**, eye at **a**
lecture 17: $R\,(\mathbf{n}\cdot\mathbf{h})^s$	$R\,(\mathbf{n}\cdot\mathbf{h})^s$	$R\,(\mathbf{n}\cdot\mathbf{h})^s$
OpenGL: $\dfrac{k_s\,(\mathbf{n}\cdot\mathbf{h})^s}{\cos\theta_l}$	$\times 2$	$\times 1$

- Lecture 17 passes: $\mathbf{h}$ is the same both ways, and E carries the cosine.
- OpenGL's highlight leaves the cosine out: twice as bright one way.

OpenGL's and Phong's 1975 highlight: $k_s(\mathbf{n}\cdot\mathbf{h})^s I$, with no $\cos\theta_l$; as a BRDF, divide by $\cos\theta_l$

Worked: a white Blinn–Phong surface under a grey sky

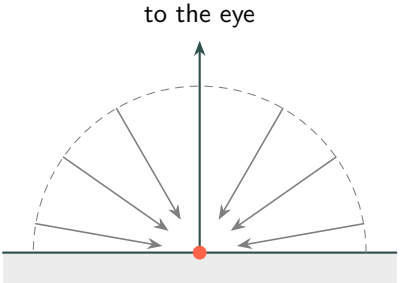


the same L from every direction; $R = 1$

Try first: what share comes back, head-on?

part of ρ	head-on	eye at $60°$
chalk, R/π		
highlight, $s = 100$		
highlight, $s = 10$		

Worked: a white Blinn–Phong surface under a grey sky



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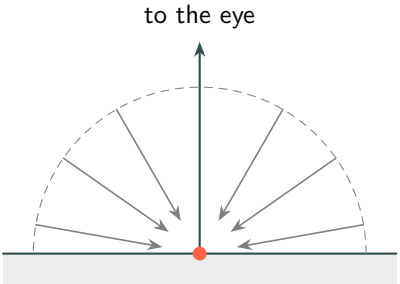
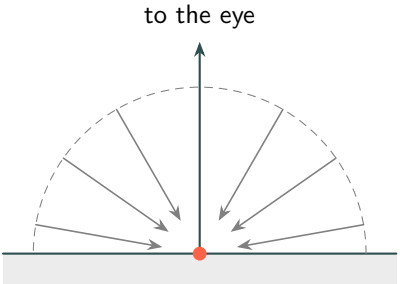


the same L from every direction; $R = 1$

Try first: what share comes back, head-on?

part of ρ	head-on	eye at $60°$
chalk, R/π	1	1
highlight, $s = 100$	0.24	0.06
highlight, $s = 10$	1.50	0.60

Worked: a white Blinn–Phong surface under a grey sky



the same L from every direction; $R = 1$

Try first: what share comes back, head-on?

part of ρ	head-on	eye at $60°$
chalk, R/π	1	1
highlight, $s = 100$	0.24	0.06
highlight, $s = 10$	1.50	0.60
total, $s = 100/10$	1.24 / 2.50	1.06 / 1.60

- More light leaves than arrives: not a BRDF.

  s: lecture 17's p; each share: $\int \rho \cos\theta_l \, d\sigma$, numerically

Rendered: the white furnace



chalk, $R = 1$ Blinn–Phong, $s = 100$ Blinn–Phong, $s = 10$

Rendered: the white furnace



chalk, $R = 1$ Blinn–Phong, $s = 100$ Blinn–Phong, $s = 10$

- Chalk vanishes.

Rendered: the white furnace

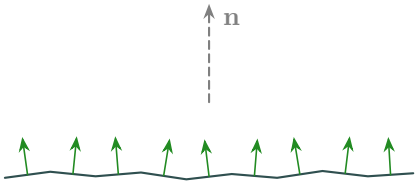


chalk, $R = 1$ Blinn–Phong, $s = 100$ Blinn–Phong, $s = 10$

- Chalk vanishes.
- The Blinn–Phong balls glow: they make light.

Up close, a surface is many tiny mirrors

seen through a microscope (not to scale)



polished:
the facet normals stay close to $\mathbf{n}$

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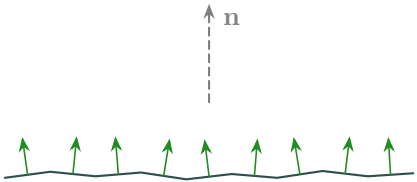

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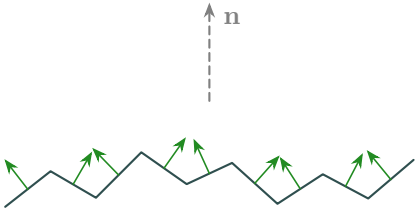

rough:
the facet normals spread

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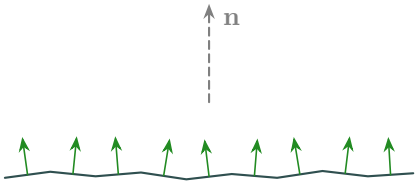

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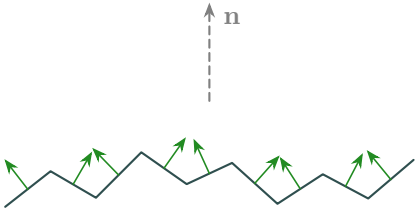
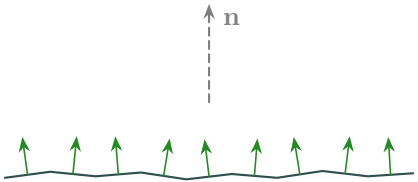


rough:
the facet normals spread

- Each facet is a perfect mirror (lecture 17), with Fresnel (lecture 18).

Up close, a surface is many tiny mirrors

seen through a microscope (not to scale)



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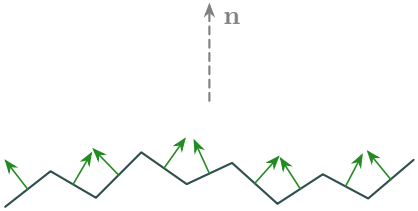
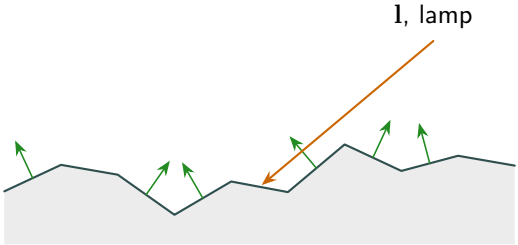


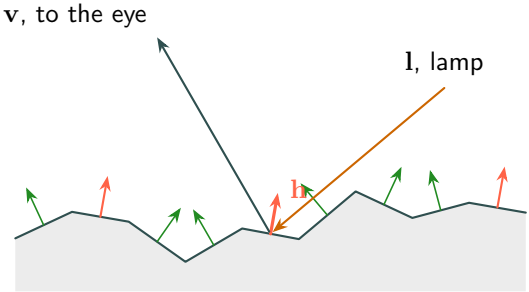
rough:
the facet normals spread

- Each facet is a perfect mirror (lecture 17), with Fresnel (lecture 18).
- Roughness α: how widely the facet normals spread. $\alpha = 0$: one mirror.

Only the facets facing $\mathbf{h}$ send $\mathbf{l}$ to $\mathbf{v}$



Only the facets facing $\mathbf{h}$ send $\mathbf{l}$ to $\mathbf{v}$



- A mirror facet sends $\mathbf{l}$ to $\mathbf{v}$ only if its normal is $\mathbf{h}$, lecture 17's half vector.

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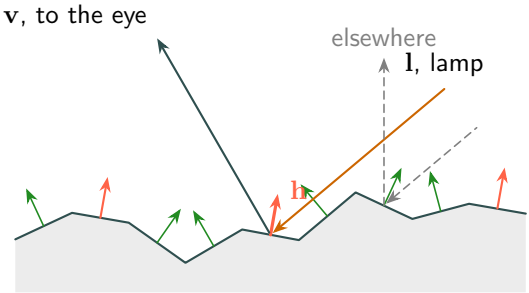
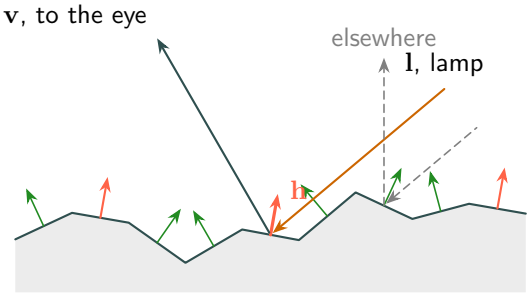


- A mirror facet sends $\mathbf{l}$ to $\mathbf{v}$ only if its normal is $\mathbf{h}$, lecture 17's half vector.
- Every other facet sends the lamp's light somewhere else.

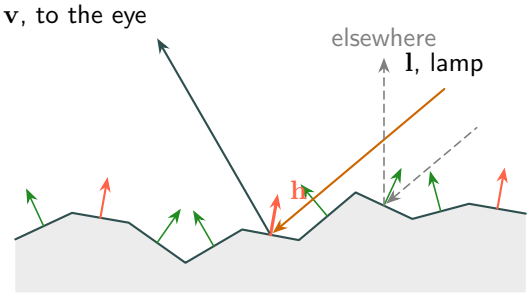
Only the facets facing $\mathbf{h}$ send $\mathbf{l}$ to $\mathbf{v}$



- A mirror facet sends $\mathbf{l}$ to $\mathbf{v}$ only if its normal is $\mathbf{h}$, lecture 17's half vector.
- Every other facet sends the lamp's light somewhere else.
- So $\rho(\mathbf{l}, \mathbf{v})$ counts the facets whose normal is $\mathbf{h}$.

$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{|\mathbf{l} + \mathbf{v}|}$$

Only the facets facing $\mathbf{h}$ send $\mathbf{l}$ to $\mathbf{v}$



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$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{|\mathbf{l} + \mathbf{v}|}$$

lecture 17's $(\mathbf{n} \cdot \mathbf{h})^s$ was a guess at that count

Recap: not a BRDF

a BRDF: the same both ways; never more out than in

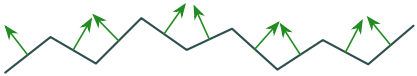
lecture 17's highlight sends back up to 2.5 times

tiny mirrors: only facets facing $\mathbf{h}$ send $\mathbf{l}$ to $\mathbf{v}$

The three factors

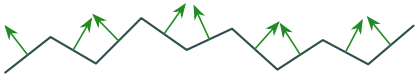
How many facets face $\mathbf{h}$, how many are seen, how much they mirror

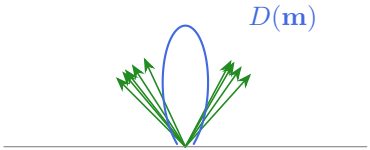
D: how crowded the facet normals are



- $\mathbf{m}$: a facet's normal; θ_m: from $\mathbf{n}$ to $\mathbf{m}$.

D: how crowded the facet normals are





the same normals, gathered at one point

- $\mathbf{m}$: a facet's normal; θ_m: from $\mathbf{n}$ to $\mathbf{m}$.
- $D(\mathbf{m})$: how much facet area faces $\mathbf{m}$, per unit of directions.

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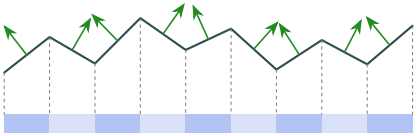
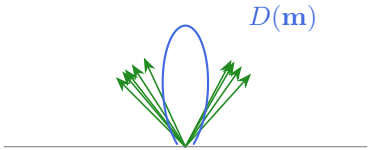




the same normals, gathered at one point

- $\mathbf{m}$: a facet's normal; θ_m: from $\mathbf{n}$ to $\mathbf{m}$.
- $D(\mathbf{m})$: how much facet area faces $\mathbf{m}$, per unit of directions.
- The facets' shadows cover the flat patch exactly once:

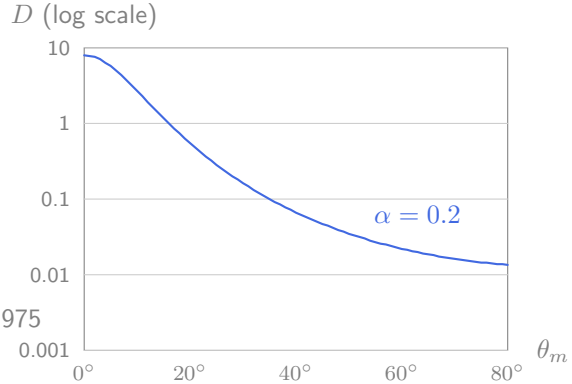
$$\int_{\text{dome}} D(\mathbf{m}) \cos\theta_m \, d\sigma_m = 1$$

per unit area of the flat patch

GGX: the distribution everyone uses

$$D(\mathbf{m}) = \frac{\alpha^2}{\pi \cos^4 \theta_m \, (\alpha^2 + \tan^2 \theta_m)^2}$$

- Head-on, $D = 1/(\pi\alpha^2)$: 7.96 for $\alpha = 0.2$.



GGX: Walter et al. 2007; Trowbridge, Reitz 1975

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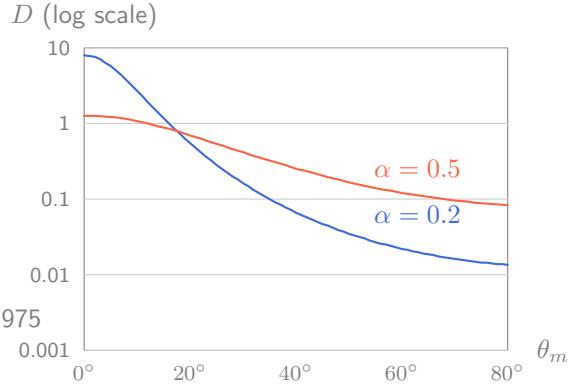

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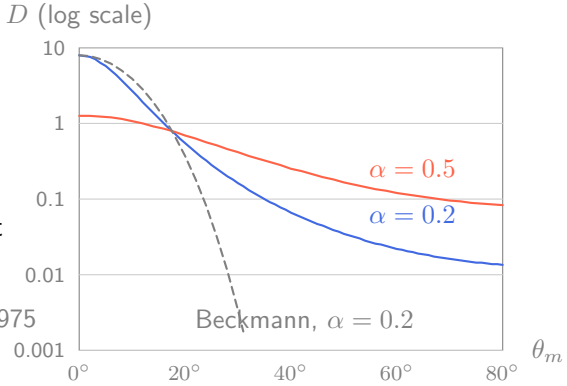
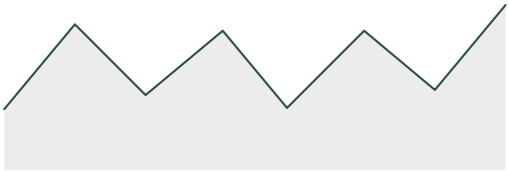
- Head-on, $D = 1/(\pi\alpha^2)$: 7.96 for $\alpha = 0.2$.
- Rougher: a lower peak, a wider spread. 1.27 for $\alpha = 0.5$.
- GGX's long tail gives every highlight a soft glow, as real metals have.

GGX: Walter et al. 2007; Trowbridge, Reitz 1975



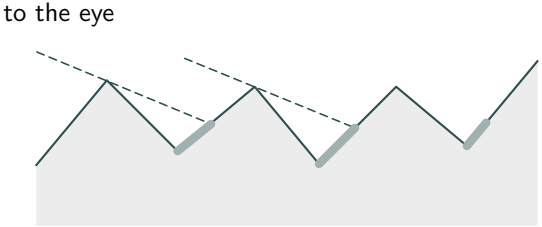
G: some facets hide behind others



a rough surface, close up

G: some facets hide behind others

- Masking: the eye cannot see some facets that face it.



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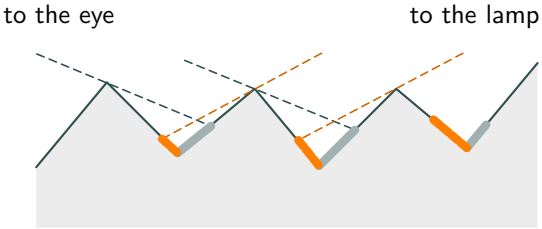
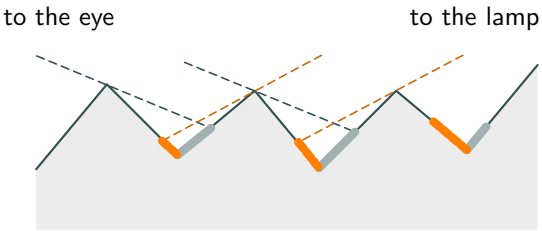


a rough surface, close up

- Masking: the eye cannot see some facets that face it.
- Shadowing: the lamp cannot reach some facets that face it.

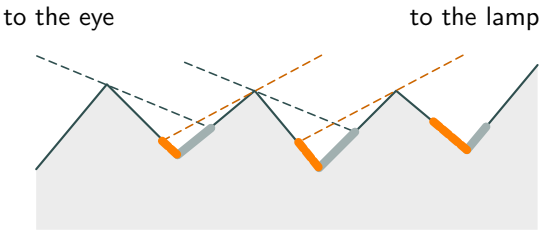
G: some facets hide behind others



a rough surface, close up

- Masking: the eye cannot see some facets that face it.
- Shadowing: the lamp cannot reach some facets that face it.
- $G(\mathbf{l}, \mathbf{v})$: the share of facets that are both seen and lit.

G: some facets hide behind others



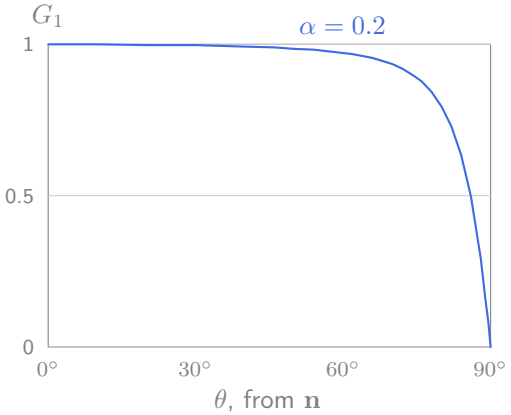
a rough surface, close up

- Masking: the eye cannot see some facets that face it.
- Shadowing: the lamp cannot reach some facets that face it.
- $G(\mathbf{l}, \mathbf{v})$: the share of facets that are both seen and lit.
- Grazing directions and rough surfaces lose the most.

Smith's G for GGX

$$G_1(\theta) = \frac{2}{1 + \sqrt{1 + \alpha^2 \tan^2 \theta}}$$

- $G_1(\theta)$: the share of facets seen from one direction, θ from $\mathbf{n}$.



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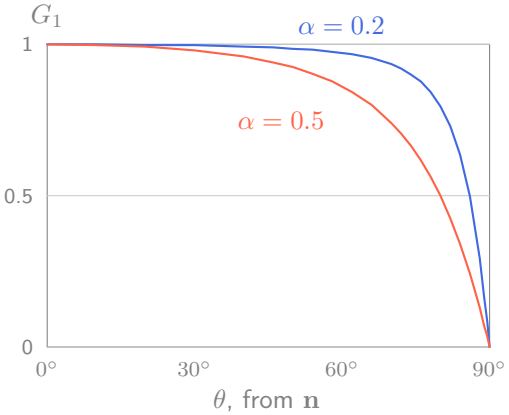
- $G_1(\theta)$: the share of facets seen from one direction, θ from $\mathbf{n}$.
- $\alpha = 0.5$: 1 head-on, 0.86 at $60°$, 0.50 at $80°$.



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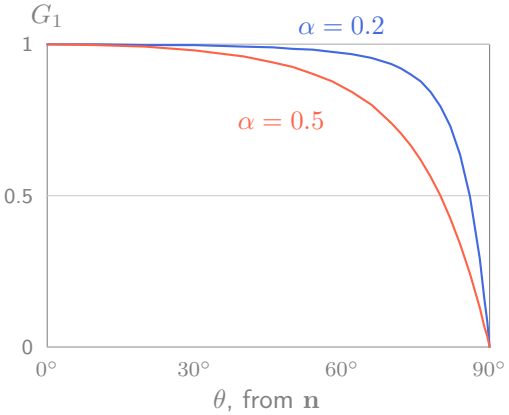
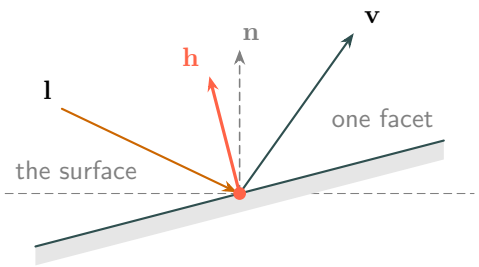
- $G_1(\theta)$: the share of facets seen from one direction, θ from $\mathbf{n}$.
- $\alpha = 0.5$: 1 head-on, 0.86 at $60°$, 0.50 at $80°$.
- Smith: the eye and the lamp hide facets on their own. Multiply.

$$G(\mathbf{l}, \mathbf{v}) = G_1(\theta_l)\, G_1(\theta_v)$$



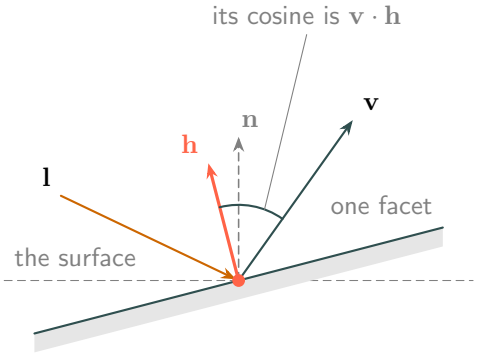
F: each facet is lecture 18's Fresnel mirror



$$F = F_0 + (1 - F_0)\,(1 - \mathbf{v} \cdot \mathbf{h})^5$$

lecture 18's Schlick; there R and R_0

F: each facet is lecture 18's Fresnel mirror



$$F = F_0 + (1 - F_0)\,(1 - \mathbf{v} \cdot \mathbf{h})^5$$

lecture 18's Schlick; there R and R_0

- Measure the angle from the facet's normal $\mathbf{h}$, not from $\mathbf{n}$.

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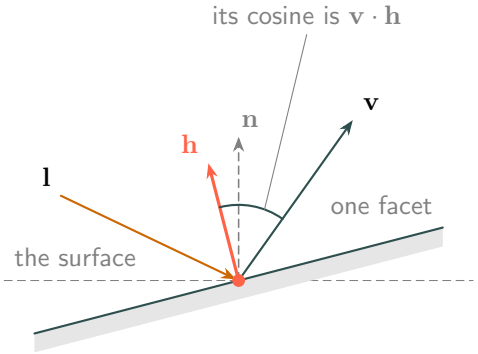

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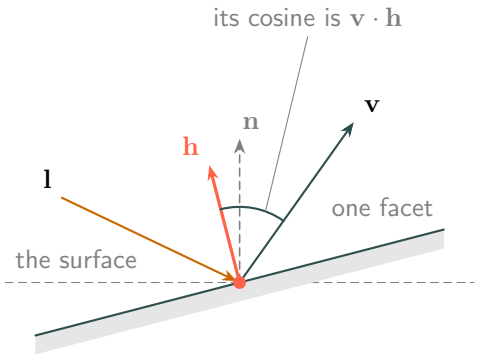
- Measure the angle from the facet's normal $\mathbf{h}$, not from $\mathbf{n}$.
- F_0: the share mirrored head-on. Glass 0.04, gold 1.00, 0.71, 0.29.

F: each facet is lecture 18's Fresnel mirror



$$F = F_0 + (1 - F_0)\,(1 - \mathbf{v} \cdot \mathbf{h})^5$$

lecture 18's Schlick; there R and R_0

- Measure the angle from the facet's normal $\mathbf{h}$, not from $\mathbf{n}$.
- F_0: the share mirrored head-on. Glass 0.04, gold 1.00, 0.71, 0.29.
- At a grazing angle F rises to 1: every surface becomes a mirror.

Cook–Torrance: the three factors together

$$\rho(\mathbf{l}, \mathbf{v}) = \frac{D(\mathbf{h})\ F(\mathbf{v} \cdot \mathbf{h})\ G(\mathbf{l}, \mathbf{v})}{4 \cos \theta_l \cos \theta_v}$$

Cook and Torrance, 1982 (with Beckmann's D); with GGX: Walter et al. 2007

Cook–Torrance: the three factors together

how many facets face $\mathbf{h}$

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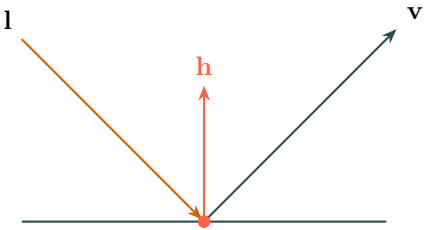
how many are seen and lit

from facets to directions, and to the flat surface

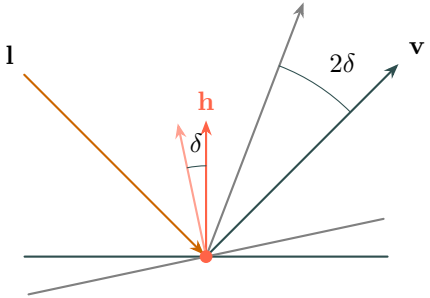
- The 4 and the cosines come from counting the light: the next two slides.

Cook and Torrance, 1982 (with Beckmann's D); with GGX: Walter et al. 2007

Turn a facet by δ, and its mirror ray turns by 2δ



Turn a facet by δ, and its mirror ray turns by 2δ



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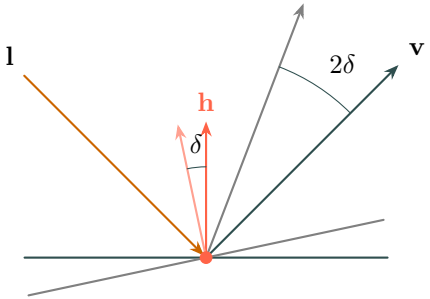

- Turn the facet by δ: its mirror ray turns by 2δ.
- In 3D a patch of facet normals lights a patch of directions:

$$d\sigma_v = 4\,(\mathbf{v}\cdot\mathbf{h})\,d\sigma_h$$

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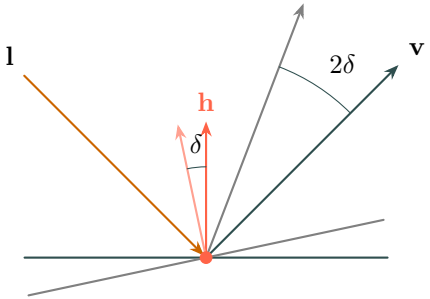


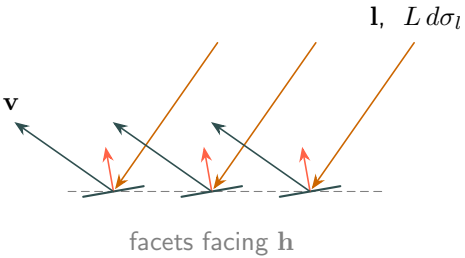
- Turn the facet by δ: its mirror ray turns by 2δ.
- In 3D a patch of facet normals lights a patch of directions:

$$d\sigma_v = 4\,(\mathbf{v}\cdot\mathbf{h})\,d\sigma_h$$

- Head-on: twice as wide each way, so 4. At an angle, the sideways turn shrinks by $\mathbf{v}\cdot\mathbf{h}$.

Count the light: where $4 \cos \theta_l \cos \theta_v$ comes from

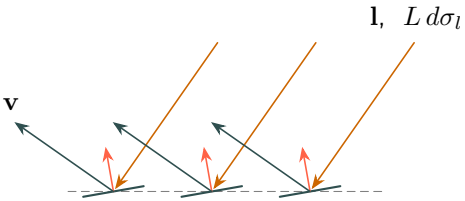
Per unit area of the flat surface:



Count the light: where $4 \cos \theta_l \cos \theta_v$ comes from

Per unit area of the flat surface:

1. On the facets facing $\mathbf{h}$: $L\, d\sigma_l\ (\mathbf{h} \cdot \mathbf{l})\ D(\mathbf{h})\, d\sigma_h$



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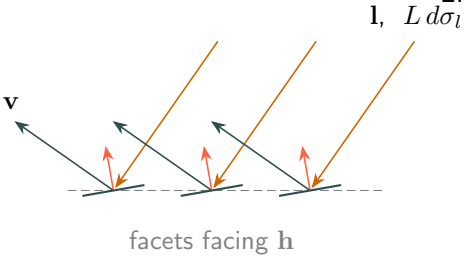

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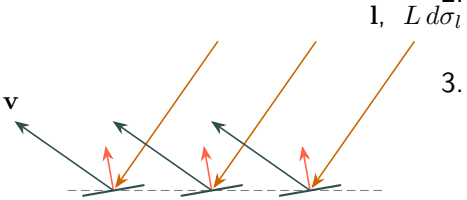
2. Mirrored, seen and lit: $\times\, F\, G$

Count the light: where $4 \cos\theta_l \cos\theta_v$ comes from



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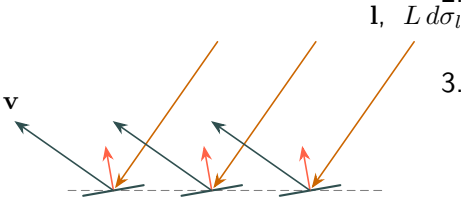
2. Mirrored, seen and lit: $\times\, F\,G$
3. Out, per $\cos\theta_v\,d\sigma_v$: $L_o = \dfrac{L\,d\sigma_l\,(\mathbf{h}\cdot\mathbf{l})\,D\,d\sigma_h\,F\,G}{\cos\theta_v \cdot 4(\mathbf{v}\cdot\mathbf{h})\,d\sigma_h}$

Count the light: where $4 \cos\theta_l \cos\theta_v$ comes from



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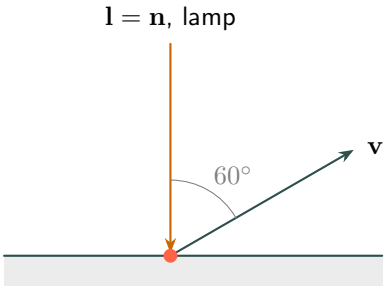
4. $\rho = \dfrac{L_o}{L\cos\theta_l\,d\sigma_l} = \dfrac{D\,F\,G}{4\cos\theta_l\cos\theta_v}$

$\mathbf{h}\cdot\mathbf{l} = \mathbf{v}\cdot\mathbf{h}$ and $d\sigma_h$ cancel

Worked: a grey metal, lamp overhead, eye at $60°$

Try first: $\mathbf{h}$, then D, G, F, and ρ.



$F_0 = 0.9, \;\; \alpha = 0.5$

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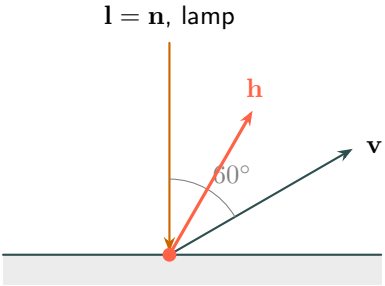
Try first: $\mathbf{h}$, then D, G, F, and ρ.

$\theta_h = 30^\circ, \quad \mathbf{v} \cdot \mathbf{h} = \cos 30^\circ = 0.866$



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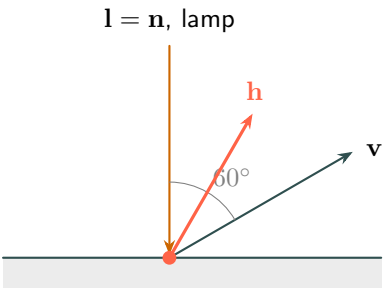


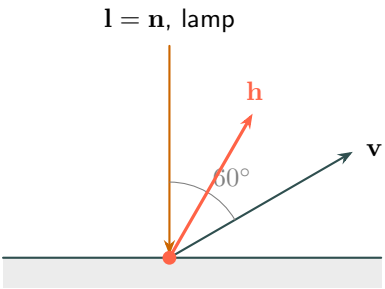
$F_0 = 0.9, \quad \alpha = 0.5$

Try first: $\mathbf{h}$, then D, G, F, and ρ.

$$\theta_h = 30^\circ, \qquad \mathbf{v} \cdot \mathbf{h} = \cos 30^\circ = 0.866$$

$$D = \frac{0.25}{\pi \times 0.5625 \times (0.25 + 0.333)^2} = 0.416$$

Worked: a grey metal, lamp overhead, eye at 60°



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$$G = G_1(0^\circ)\, G_1(60^\circ) = 1 \times \frac{2}{1 + \sqrt{1.75}} = 0.861$$

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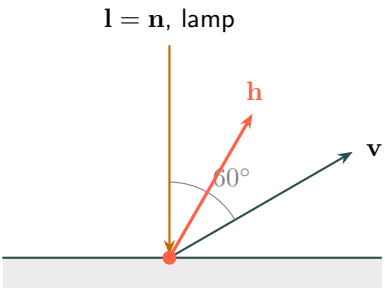

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$$F = 0.9 + 0.1 \times 0.134^5 = 0.900$$

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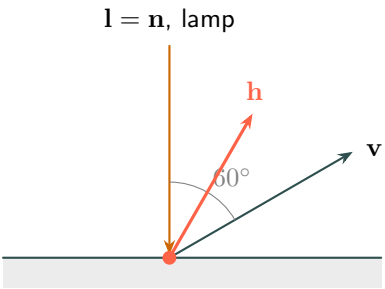

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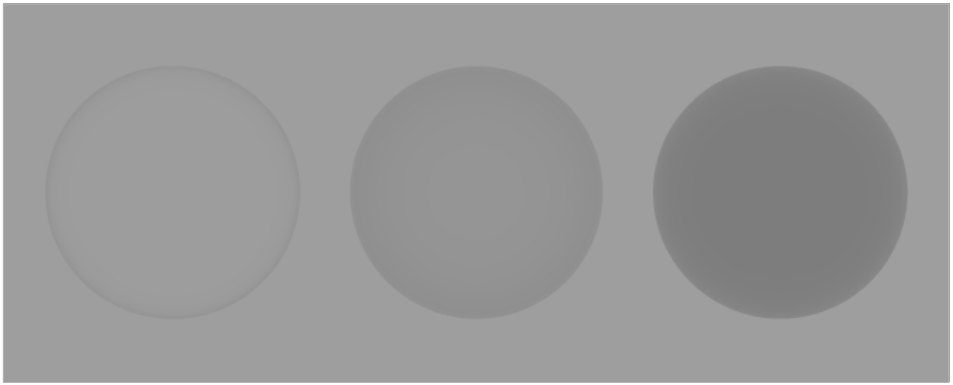
$$F = 0.9 + 0.1 \times 0.134^5 = 0.900$$

$$\rho = \frac{0.416 \times 0.900 \times 0.861}{4 \times 1 \times 0.5} = 0.161$$

for comparison, chalk with $R = 0.75$: $R/\pi = 0.239$

Rendered: Cook–Torrance passes both tests



$\alpha = 0.1$: 0.99 $\alpha = 0.3$: 0.88 $\alpha = 0.6$: 0.59

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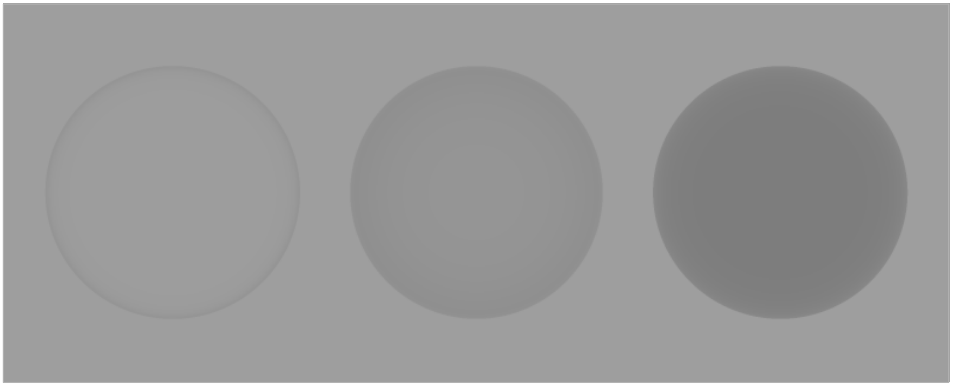

$\alpha = 0.1$: 0.99 $\alpha = 0.3$: 0.88 $\alpha = 0.6$: 0.59

- Swap $\mathbf{l}$ and $\mathbf{v}$: no factor changes. None sends back more than arrives.

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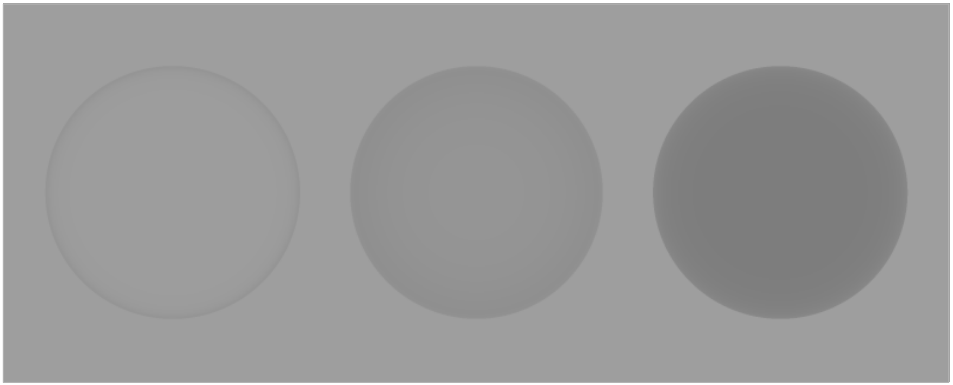

$\alpha = 0.1$: 0.99 $\alpha = 0.3$: 0.88 $\alpha = 0.6$: 0.59

- Swap $\mathbf{l}$ and $\mathbf{v}$: no factor changes. None sends back more than arrives.
- Rough balls lose light that bounces between facets. Kulla and Conty 2017 add it back

The three factors, in Python

```
def ggx_d(cos_m, alpha):
    k = cos_m**2 * (alpha**2 - 1) + 1
    return alpha**2 / (math.pi * k**2)
```

$$\frac{\alpha^2}{\pi \cos^4 \theta_m (\alpha^2 + \tan^2 \theta_m)^2}$$

k^2: the same denominator

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k^2: the same denominator

```
def smith_g1(cos_v, alpha):
    tan2 = (1 - cos_v**2) / cos_v**2
    return 2 / (1 + math.sqrt(1 + alpha**2 * tan2))
```

$$\frac{2}{1 + \sqrt{1 + \alpha^2 \tan^2 \theta}}$$

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$$\frac{2}{1 + \sqrt{1 + \alpha^2 \tan^2 \theta}}$$

```
def fresnel(c, f0):
    return f0 + (Vec3(1,1,1) - f0) * (1 - c)**5
```

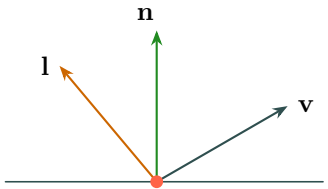
$$F_0 + (1 - F_0)(1 - c)^5$$

- Each takes a cosine. `f0` is a colour, so `fresnel` returns a colour.

A rough mirror material, in Python

```
class RoughMirror(Material):
    def __init__(self, f0, alpha):
        self.f0, self.alpha = f0, alpha

    def evaluate(self, l, v, n):
```



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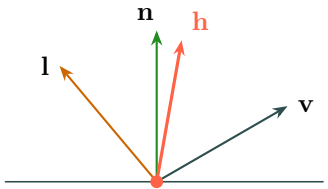
    def evaluate(self, l, v, n):
        h = (l + v) / (l + v).length()
        a = self.alpha
```



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    def evaluate(self, l, v, n):
        h = (l + v) / (l + v).length()
        a = self.alpha
        D = ggx_d(n.dot(h), a)
        G = smith_g1(n.dot(l), a) * smith_g1(n.dot(v), a)
        F = fresnel(v.dot(h), self.f0)
```

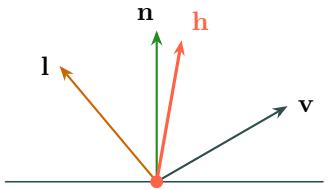

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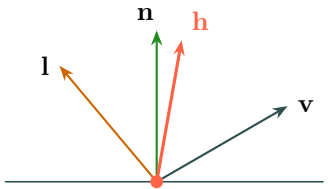
        a = self.alpha
        D = ggx_d(n.dot(h), a)
        G = smith_g1(n.dot(l), a) * smith_g1(n.dot(v), a)
        F = fresnel(v.dot(h), self.f0)
        return D * F * G / (4 * n.dot(l) * n.dot(v))
```

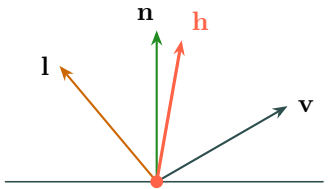


- `evaluate` returns ρ, as lecture 17's `BlinnPhong` did.

A rough mirror material, in Python

```
class RoughMirror(Material):
    def __init__(self, f0, alpha):
        self.f0, self.alpha = f0, alpha

    def evaluate(self, l, v, n):
        h = (l + v) / (l + v).length()
        a = self.alpha
        D = ggx_d(n.dot(h), a)
        G = smith_g1(n.dot(l), a) * smith_g1(n.dot(v), a)
        F = fresnel(v.dot(h), self.f0)
        return D * F * G / (4 * n.dot(l) * n.dot(v))
```



- `evaluate` returns ρ, as lecture 17's `BlinnPhong` did.
- Extra 1's `lamp_share` calls `evaluate` in place of `albedo / math.pi`.

Quick check 1: the three factors

GGX: $D(\mathbf{n}) = 1/(\pi\alpha^2)$; **Smith:** $G_1 = 2/(1 + \sqrt{1 + \alpha^2 \tan^2\theta})$; **Schlick's** F.

1. $\alpha = 0.25$. What is D for facets facing along $\mathbf{n}$?
2. Seen from straight above, what is G_1, for any α?
3. Gold, $F_0 = (1.00, 0.71, 0.29)$: what is F when $\mathbf{v} \cdot \mathbf{h} = 1$?
4. Which factor turns every surface into a mirror at grazing angles?

- $\rho = D\,F\,G/(4\cos\theta_l \cos\theta_v)$: count, mirror, hide.

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1. $\alpha = 0.25$. What is D for facets facing along $\mathbf{n}$? 5.09
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4. Which factor turns every surface into a mirror at grazing angles? F

- $\rho = D\,F\,G/(4\cos\theta_l \cos\theta_v)$: count, mirror, hide.

Recap: the three factors

$$\rho = \frac{D(\mathbf{h})\, F(\mathbf{v} \cdot \mathbf{h})\, G(\mathbf{l}, \mathbf{v})}{4 \cos \theta_l \cos \theta_v}$$

D: GGX; G: Smith; F: Schlick at the facet

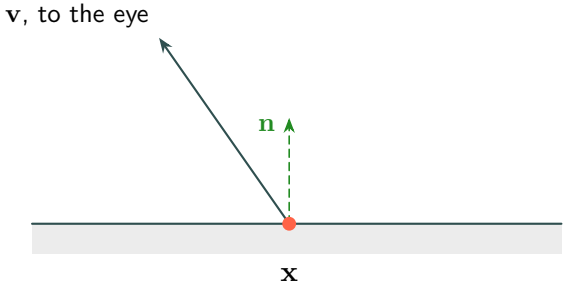
passes both tests; rough ones lose a little light

Picking a facet

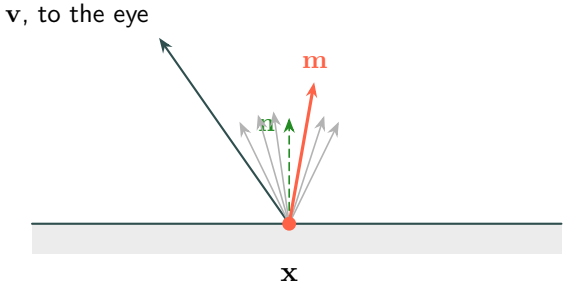
Extra 1's lobe pick, for tiny mirrors

Pick a facet, then reflect

- No neat formula picks $\mathbf{l}$ by ρ itself.



Pick a facet, then reflect



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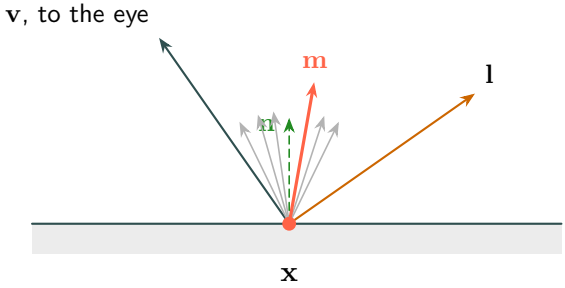
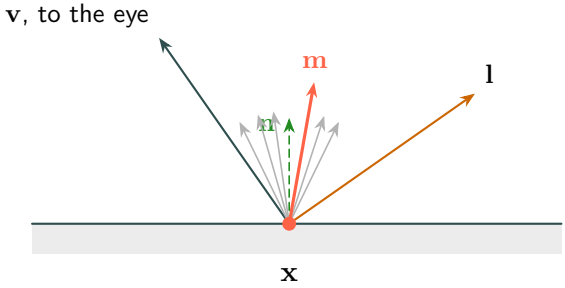
Pick a facet, then reflect



- No neat formula picks $\mathbf{l}$ by ρ itself.
- Pick a facet's normal $\mathbf{m}$ by D.
- Reflect $\mathbf{v}$ about it: lecture 17's mirror, with $\mathbf{m}$ for $\mathbf{n}$.

$$\mathbf{l} = 2(\mathbf{v} \cdot \mathbf{m})\,\mathbf{m} - \mathbf{v}$$

Pick a facet, then reflect



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$$\mathbf{l} = 2(\mathbf{v} \cdot \mathbf{m})\,\mathbf{m} - \mathbf{v}$$

$\mathbf{m}$ is then the half vector of $\mathbf{l}$ and $\mathbf{v}$

GGX's pick for the facet normal

$$\tan^2 \theta_m = \alpha^2 \frac{\xi_1}{1 - \xi_1}, \quad \text{turn} = 2\pi \, \xi_2$$

12 picks of $\mathbf{m}$, seen from the side



$\alpha = 0.2$

GGX's pick for the facet normal

$$\tan^2 \theta_m = \alpha^2 \frac{\xi_1}{1 - \xi_1}, \quad \text{turn} = 2\pi\, \xi_2$$

- Half the picks fall within $\tan^{-1} \alpha$ of $\mathbf{n}$: $11°$ for 0.2, $27°$ for 0.5.

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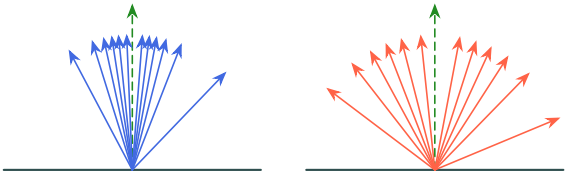

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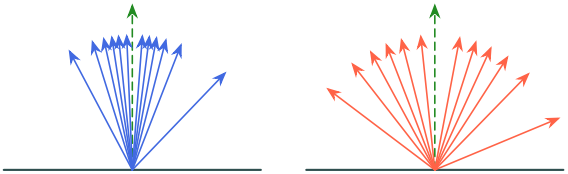

$\alpha = 0.2$ $\alpha = 0.5$

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- Half the picks fall within $\tan^{-1}\alpha$ of $\mathbf{n}$: $11°$ for 0.2, $27°$ for 0.5.
- Their density is the shadows rule's $D\cos\theta_m$:

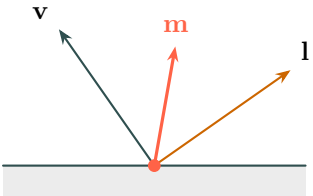
$$p_m(\mathbf{m}) = D(\mathbf{m})\cos\theta_m$$

it adds up to 1: the shadows rule

The density of l, and what one pick is worth

$$p(\mathbf{l}) = \frac{p_m(\mathbf{m})}{4\,(\mathbf{v}\cdot\mathbf{m})} = \frac{D(\mathbf{m})\cos\theta_m}{4\,(\mathbf{v}\cdot\mathbf{m})}$$

the 4 of the turning facet: $d\sigma_l = 4(\mathbf{v}\cdot\mathbf{m})\,d\sigma_m$



The density of l, and what one pick is worth

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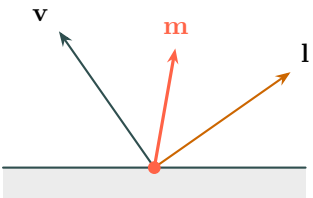

$$\frac{\rho\,\cos\theta_l}{p(\mathbf{l})} = \frac{DFG}{4\cos\theta_l\cos\theta_v}\,\cos\theta_l\,\frac{4\,(\mathbf{v}\cdot\mathbf{m})}{D\cos\theta_m}$$

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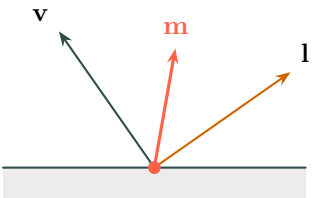

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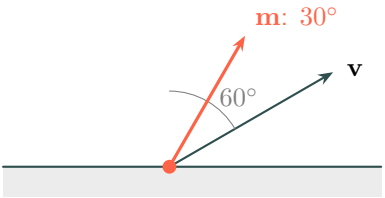
$$= \frac{F\,G\,(\mathbf{v}\cdot\mathbf{m})}{\cos\theta_v\cos\theta_m}$$

- D cancels: no pick is huge, unless $\cos\theta_v$ is tiny.

Worked: one pick on the grey metal

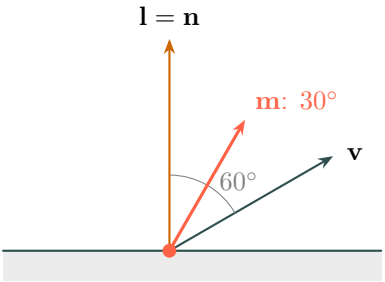
Try first: the pick's $\mathbf{l}$, $p(\mathbf{l})$, and its worth.

$\xi_1 = 0.571$: $\tan^2\theta_m = 0.25 \times 0.571/0.429 = 1/3$, toward $\mathbf{v}$



$F_0 = 0.9,\ \ \alpha = 0.5$

Worked: one pick on the grey metal



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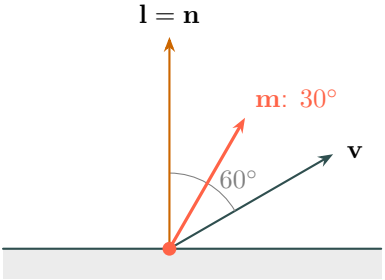

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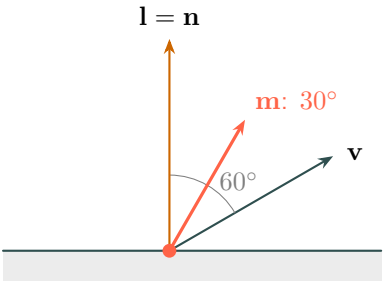
$\mathbf{v}$ mirrored about $\mathbf{m}$: $\mathbf{l} = \mathbf{n}$, straight up

$$p(\mathbf{l}) = \frac{0.416 \times 0.866}{4 \times 0.866} = 0.104$$

Worked: one pick on the grey metal



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$$p(\mathbf{l}) = \frac{0.416 \times 0.866}{4 \times 0.866} = 0.104$$

long way: $$\frac{\rho \cos \theta_l}{p} = \frac{0.161 \times 1}{0.104} = 1.55$$

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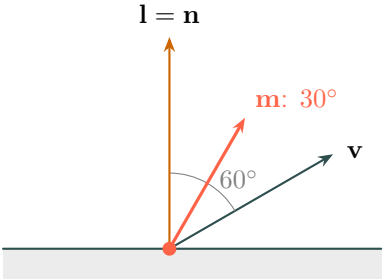

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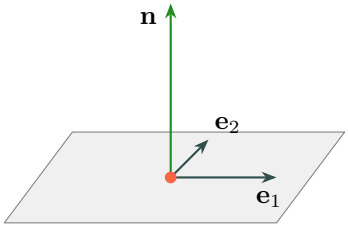
long way: $$\frac{\rho \cos \theta_l}{p} = \frac{0.161 \times 1}{0.104} = 1.55$$

short way: $$\frac{0.9 \times 0.861 \times 0.866}{0.5 \times 0.866} = 1.55$$

$\rho = 0.161$, $D = 0.416$, $G = 0.861$: the worked metal slide

Picking a facet, in Python

```
def basis(n):
    a = Vec3(0,1,0) if abs(n.x) > 0.9 else Vec3(1,0,0)
    e1 = a.cross(n).normalized()
    return e1, n.cross(e1)
```



- `basis` gives two unit arrows on the surface, perpendicular to $\mathbf{n}$ and to each other.

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def ggx_normal(n, alpha):
    xi1, xi2 = random.random(), random.random()
    tan2 = alpha**2 * xi1 / (1 - xi1)
```

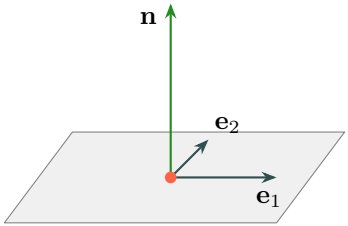

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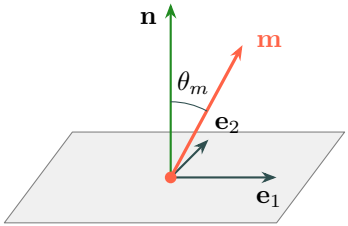
    sin_m = math.sqrt(1 - cos_m**2)
```



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    turn = 2 * math.pi * xi2
    e1, e2 = basis(n)
```

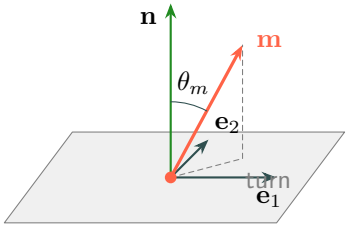

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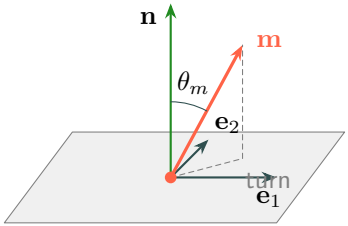
    turn = 2 * math.pi * xi2
    e1, e2 = basis(n)
    return (sin_m * math.cos(turn) * e1 + sin_m * math.sin(turn) * e2
            + cos_m * n)
```



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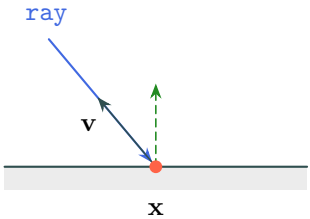
A rough mirror, in Python

```
elif isinstance(material, RoughMirror):
    if depth == 0:  return Vec3(0,0,0)
    x = ray.evaluate(rec.t)
    n, v = rec.normal, -ray.d
```



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    m = ggx_normal(n, material.alpha)
    l = 2 * v.dot(m) * m - v
```

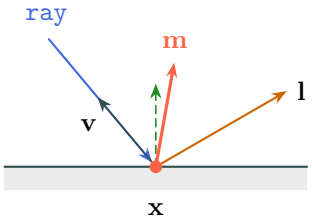

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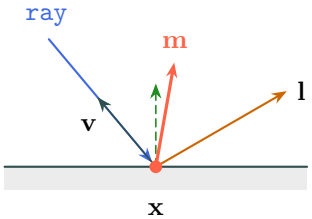
    l = 2 * v.dot(m) * m - v
    if v.dot(m) <= 0 or n.dot(l) <= 0:
        return Vec3(0,0,0)
```



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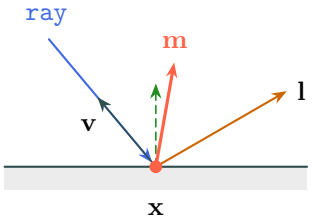

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    l = 2 * v.dot(m) * m - v
    if v.dot(m) <= 0 or n.dot(l) <= 0:
        return Vec3(0,0,0)
    a = material.alpha
    G = smith_g1(n.dot(l), a) * smith_g1(n.dot(v), a)
    F = fresnel(v.dot(m), material.f0)
    worth = F * G * v.dot(m) / (n.dot(v) * n.dot(m))
    return worth * trace(Ray(x, l), depth - 1)
```
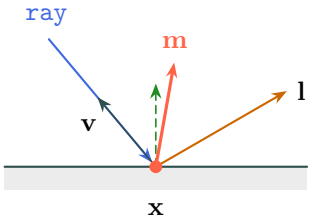
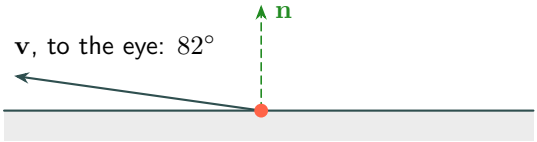



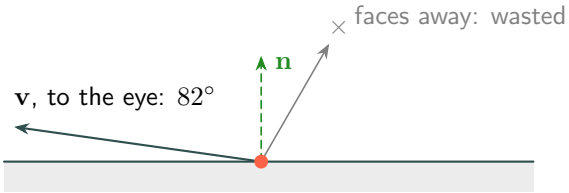
- A facet that faces away, or an $\mathbf{l}$ under the surface: no light.
- Grey: lecture 17's mirror branch. With Extra 1's MIS, pass $p(\mathbf{l})$ on as `p_brdf`.

At a grazing view, a few picks are huge



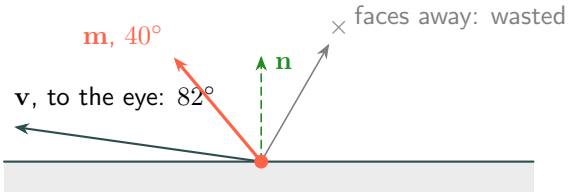
- The pick by $D \cos \theta_m$ does not look at where the eye is.

At a grazing view, a few picks are huge



- The pick by $D \cos \theta_m$ does not look at where the eye is.
- Facets that face away from the eye are picked and wasted.

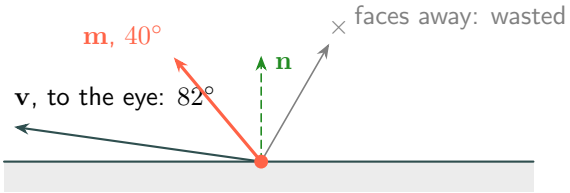
At a grazing view, a few picks are huge



$$\frac{\mathbf{v} \cdot \mathbf{m}}{\cos\theta_v \cos\theta_m} = \frac{0.74}{0.14 \times 0.77} = 7.0$$

- The pick by $D \cos\theta_m$ does not look at where the eye is.
- Facets that face away from the eye are picked and wasted.
- Steep facets facing the eye bring back huge values.

At a grazing view, a few picks are huge



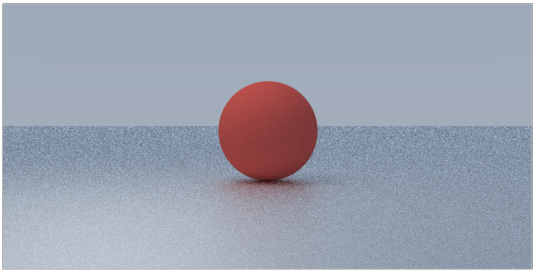
$$\frac{\mathbf{v}\cdot\mathbf{m}}{\cos\theta_v\cos\theta_m} = \frac{0.74}{0.14\times 0.77} = 7.0$$

- The pick by $D\cos\theta_m$ does not look at where the eye is.
- Facets that face away from the eye are picked and wasted.
- Steep facets facing the eye bring back huge values.
- Heitz 2018: pick only facets the eye can see. One pick is then worth $F\,G_1(\theta_l)$.

a grazing floor: spread 0.86, against 0.09

Rendered: a grazing floor, two picks

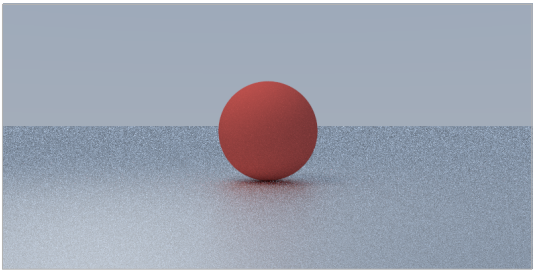
a floor of rough metal, $\alpha = 0.3$, seen from low; 64 paths per pixel each



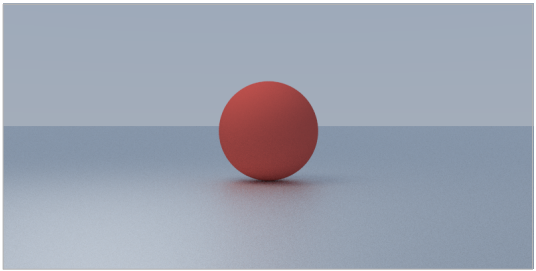
picking facets by $D \cos \theta_m$

Rendered: a grazing floor, two picks

a floor of rough metal, $\alpha = 0.3$, seen from low; 64 paths per pixel each



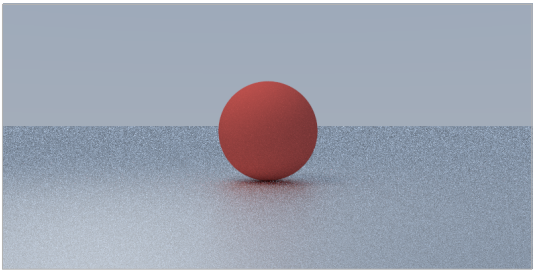
picking facets by $D \cos \theta_m$



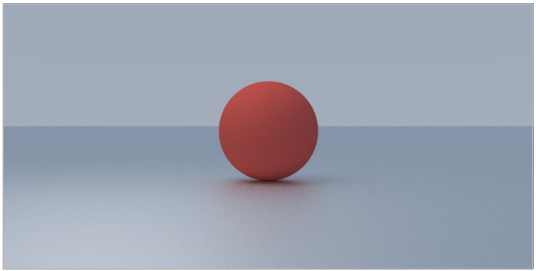
picking facets the eye sees (Heitz)

Rendered: a grazing floor, two picks

a floor of rough metal, $\alpha = 0.3$, seen from low; 64 paths per pixel each



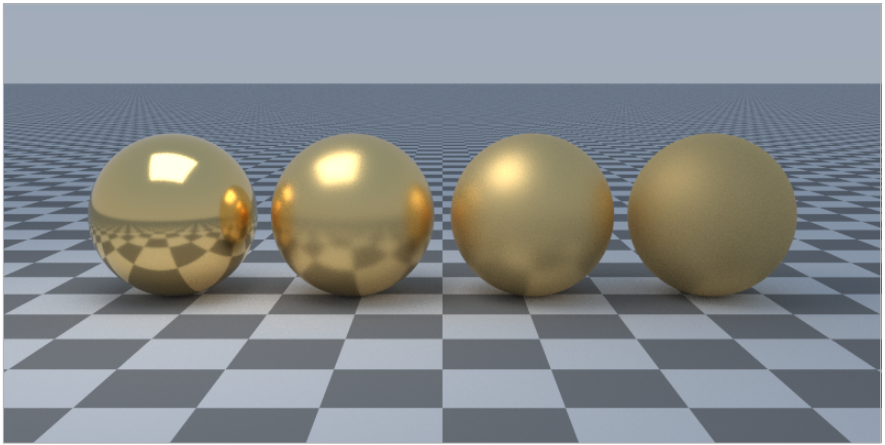
picking facets by $D \cos \theta_m$



picking facets the eye sees (Heitz)

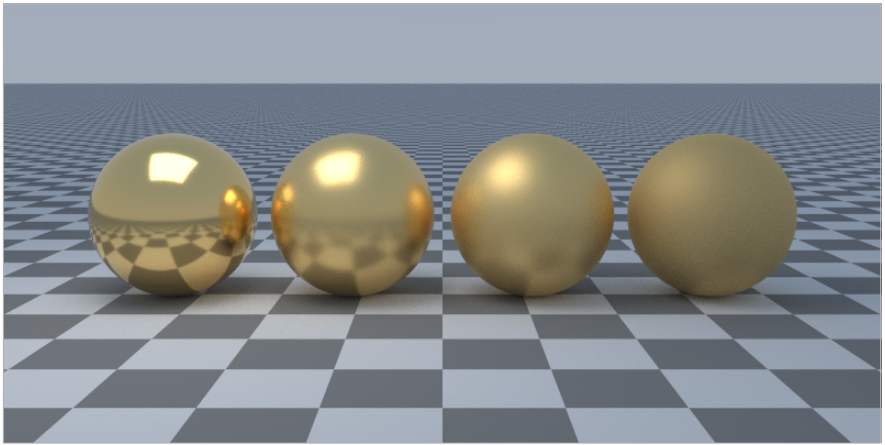
- Same average, far less noise where the floor is seen at a grazing angle.
- The pictures of this part from now on use Heitz's pick.

Rendered: gold, from polished to rough



$\alpha = 0.02$ $\alpha = 0.1$ $\alpha = 0.25$ $\alpha = 0.5$

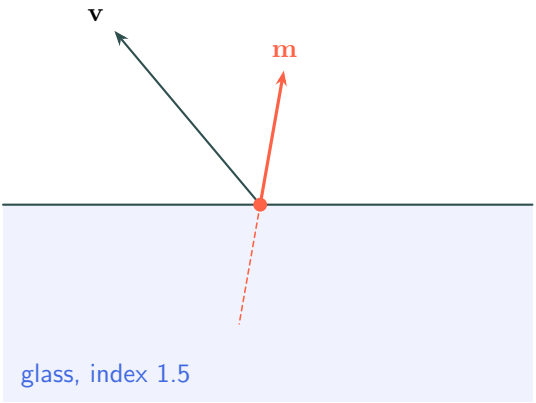
Rendered: gold, from polished to rough



$\alpha = 0.02$ $\alpha = 0.1$ $\alpha = 0.25$ $\alpha = 0.5$

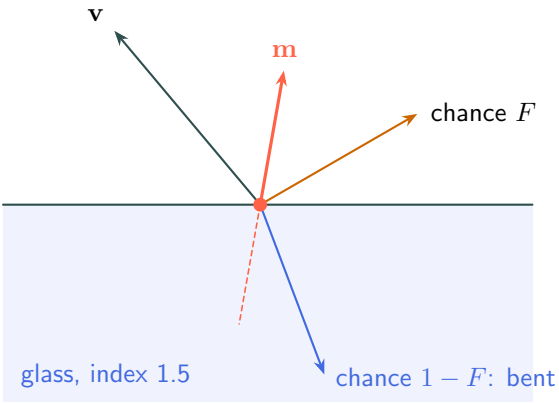
- The lamp's reflection grows and dims; the floor's squares blur, then vanish.

Rough glass: every facet is lecture 18's glass



- Pick a facet $\mathbf{m}$ as for the rough mirror.

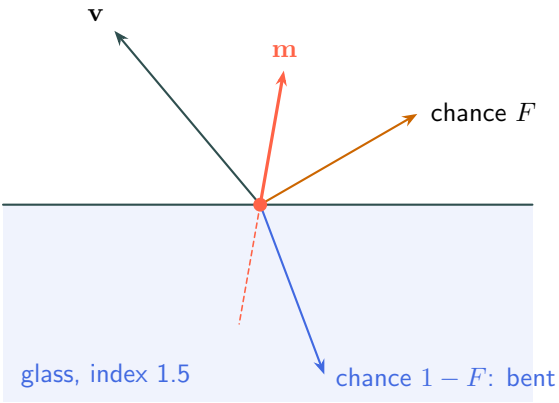
Rough glass: every facet is lecture 18's glass



- Pick a facet $\mathbf{m}$ as for the rough mirror.
- With chance $F(\mathbf{v} \cdot \mathbf{m})$ reflect about $\mathbf{m}$; otherwise refract through it.

lecture 18's `refract`, `schlick`; $\mathbf{m}$ for $\mathbf{n}$

Rough glass: every facet is lecture 18's glass



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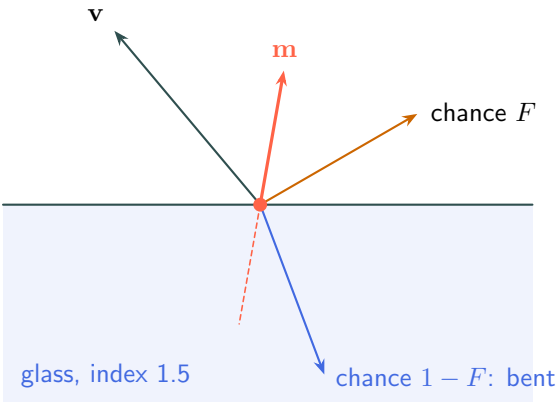
  lecture 18's `refract`, `schlick`; $\mathbf{m}$ for $\mathbf{n}$
- Lecture 18 followed both rays; a path follows one (lecture 20).

Rough glass: every facet is lecture 18's glass



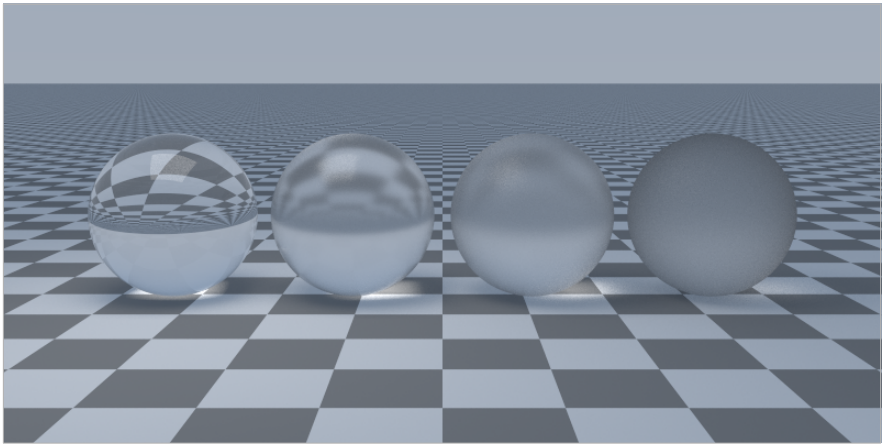
- Pick a facet $\mathbf{m}$ as for the rough mirror.
- With chance $F(\mathbf{v} \cdot \mathbf{m})$ reflect about $\mathbf{m}$; otherwise refract through it.

  lecture 18's `refract`, `schlick`; $\mathbf{m}$ for $\mathbf{n}$
- Lecture 18 followed both rays; a path follows one (lecture 20).
- Either way, one pick is worth:

$$\frac{G\,(\mathbf{v} \cdot \mathbf{m})}{\cos\theta_v \cos\theta_m}$$

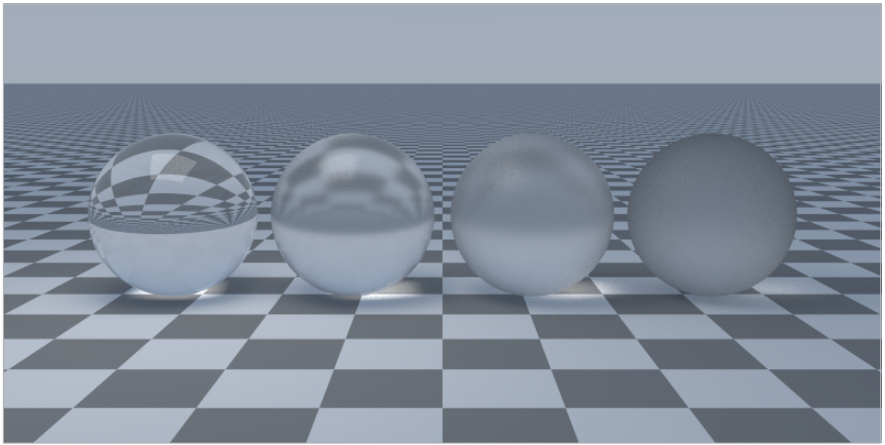
Heitz: $G_1(\theta_l)$

Rendered: glass, from clear to frosted



$\alpha = 0$ $\alpha = 0.05$ $\alpha = 0.15$ $\alpha = 0.4$

Rendered: glass, from clear to frosted



$\alpha = 0$ $\alpha = 0.05$ $\alpha = 0.15$ $\alpha = 0.4$

- The floor seen through the glass blurs, until the ball looks frosted.

Quick check 2: picking facets

$\tan^2\theta_m = \alpha^2\xi_1/(1-\xi_1)$; **one pick:** $F\,G\,(\mathbf{v}\cdot\mathbf{m})/(\cos\theta_v\cos\theta_m)$.

1. $\alpha = 0.5$. What share of the picks tilt less than $26.6°$?
2. The eye straight above, $\mathbf{v} = \mathbf{n}$: what is one pick worth?
3. A pick has $\mathbf{v}\cdot\mathbf{m} < 0$. What does it bring back?
4. Rough glass, $F = 0.1$. Of 100 paths, how many reflect?

- Pick a facet by D, reflect or bend through it; D cancels from the worth.

Quick check 2: picking facets

$\tan^2\theta_m = \alpha^2\xi_1/(1-\xi_1)$; **one pick:** $F\,G\,(\mathbf{v}\cdot\mathbf{m})/(\cos\theta_v\cos\theta_m)$.

1. $\alpha = 0.5$. What share of the picks tilt less than $26.6°$?	a half
2. The eye straight above, $\mathbf{v} = \mathbf{n}$: what is one pick worth?	$F\,G$
3. A pick has $\mathbf{v}\cdot\mathbf{m} < 0$. What does it bring back?	nothing
4. Rough glass, $F = 0.1$. Of 100 paths, how many reflect?	about 10

- Pick a facet by D, reflect or bend through it; D cancels from the worth.

Recap: picking a facet

pick $\mathbf{m}$ with density $D \cos\theta_m$; $\mathbf{l} = 2(\mathbf{v} \cdot \mathbf{m})\mathbf{m} - \mathbf{v}$

one pick: $\dfrac{F\,G\,(\mathbf{v} \cdot \mathbf{m})}{\cos\theta_v \cos\theta_m}$

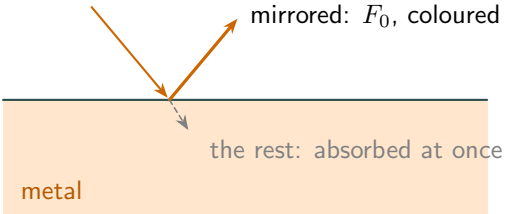
rough glass: a coin with chance F; Heitz: $F\,G_1(\theta_l)$

Real materials

Metals, plastics, knobs and textures

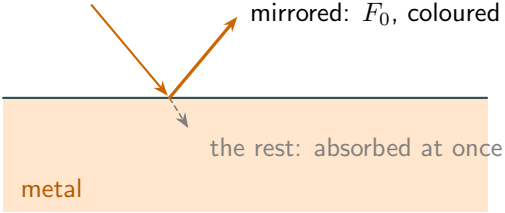
Metals and dielectrics

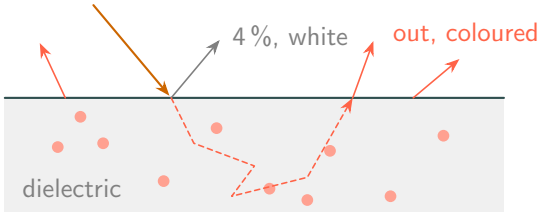
- A metal: F_0 is large and coloured; no light comes back from inside.



Metals and dielectrics

- A metal: F_0 is large and coloured; no light comes back from inside.
- A dielectric: F_0 is small and grey; what enters comes back out coloured.





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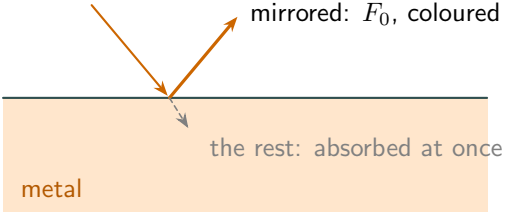
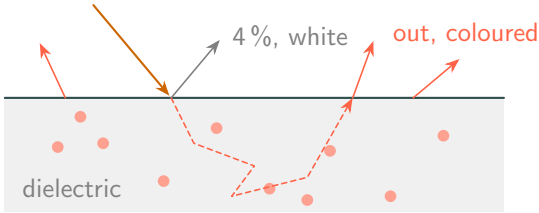


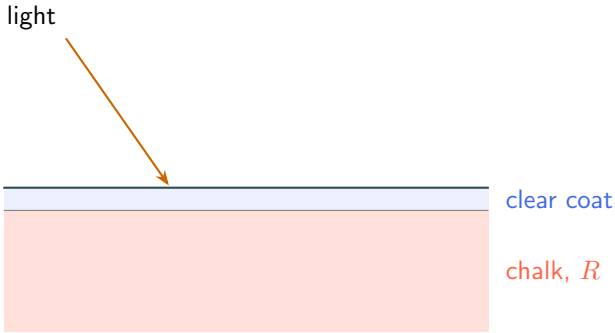


F_0: gold 1.00, 0.71, 0.29 | copper 0.95, 0.64, 0.54 | silver 0.95, 0.93, 0.88 | aluminium 0.91, 0.92, 0.92

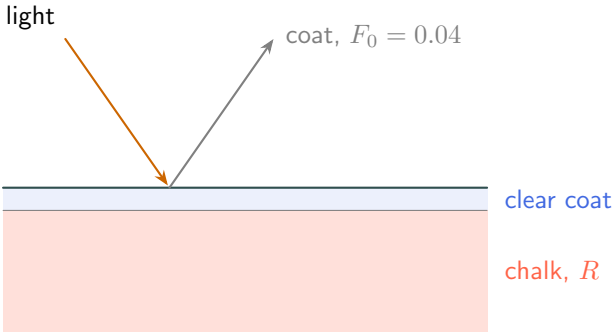
water 0.02
glass, plastic 0.04
diamond 0.17

metals: published measurements, linear RGB; dielectrics: lecture 18's R_0

A plastic: chalk under a clear coat

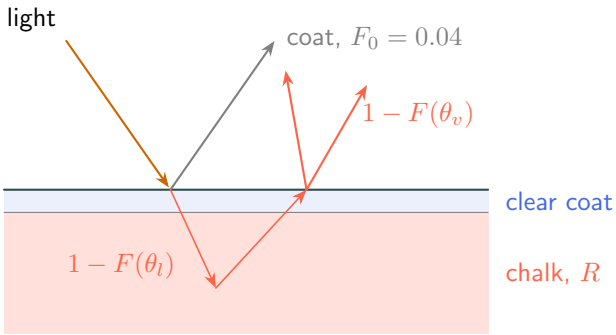


A plastic: chalk under a clear coat



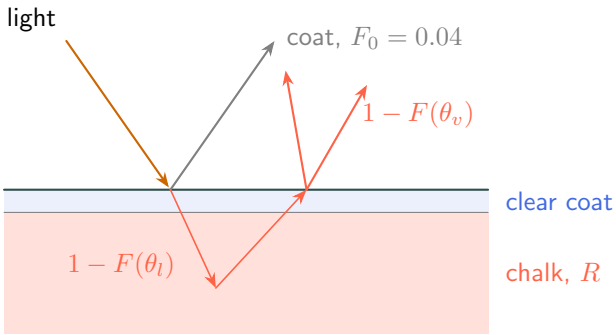
- The coat mirrors a little, white, as a rough mirror.

A plastic: chalk under a clear coat



- The coat mirrors a little, white, as a rough mirror.
- The chalk gets only what the coat lets in, and sends out only what it lets out.

A plastic: chalk under a clear coat

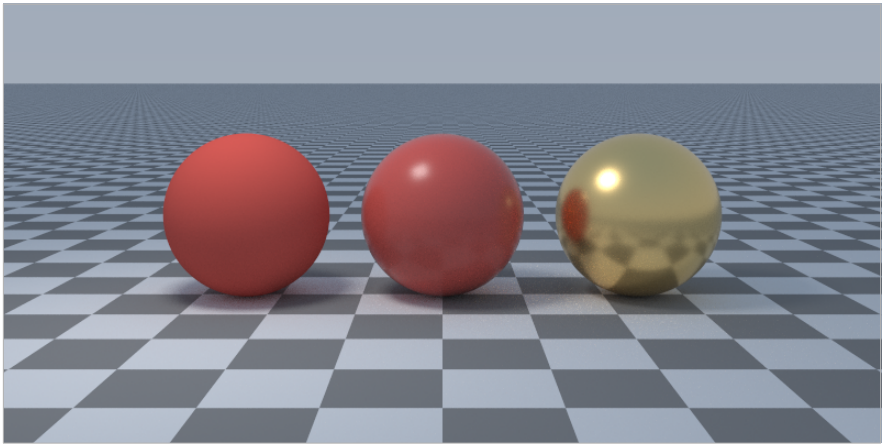


$$\rho = \rho_{\text{coat}} + \big(1 - F(\theta_l)\big)\big(1 - F(\theta_v)\big)\,\frac{R}{\pi}$$

- The coat mirrors a little, white, as a rough mirror.
- The chalk gets only what the coat lets in, and sends out only what it lets out.

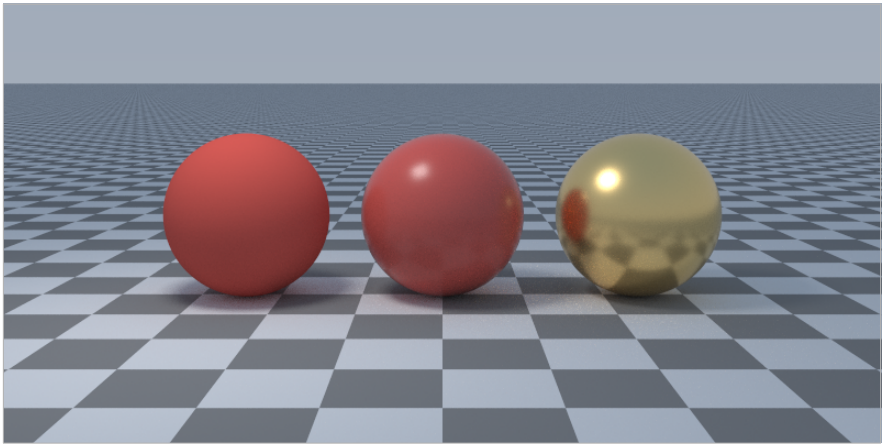
a simple model; PBRT's is exact

Rendered: chalk, plastic and metal



red chalk red plastic gold metal

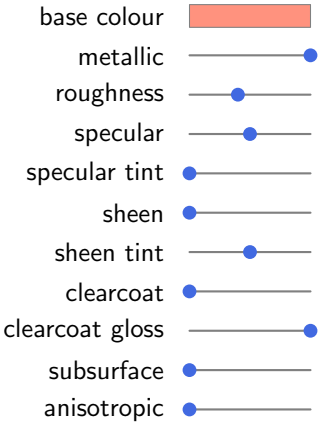
Rendered: chalk, plastic and metal



red chalk red plastic gold metal

- Plastic: a white highlight on red. Metal: a gold highlight, and no chalk colour.

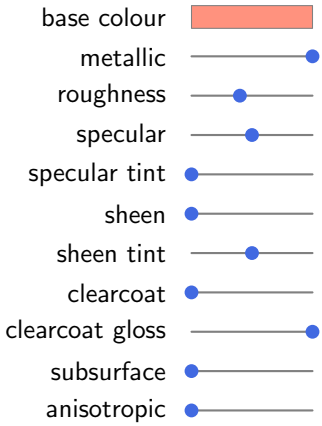
Disney's principled BRDF: knobs from 0 to 1



Burley 2012

- An artist's knobs, each from 0 to 1, in place of physical numbers.

Disney's principled BRDF: knobs from 0 to 1



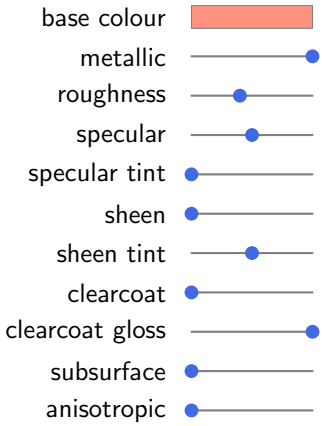
Burley 2012

- An artist's knobs, each from 0 to 1, in place of physical numbers.
- *metallic* chooses between the two cases of the metals slide:

$$F_0 = (1 - \text{metallic})\, 0.04 + \text{metallic} \cdot \text{base}$$

$$\text{chalk colour} = (1 - \text{metallic}) \cdot \text{base}$$

Disney's principled BRDF: knobs from 0 to 1



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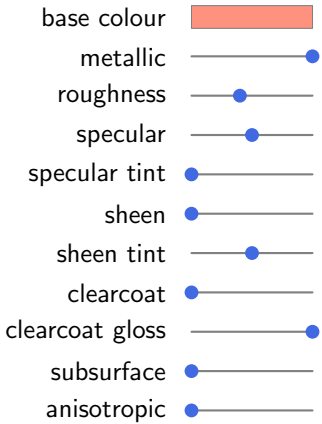
- $\alpha = \text{roughness}^2$, so that even steps of the knob look even.

Disney's principled BRDF: knobs from 0 to 1



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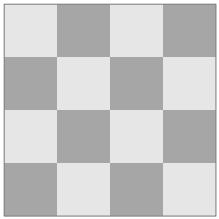
$$F_0 = (1 - \text{metallic})\, 0.04 + \text{metallic} \cdot \text{base}$$

$$\text{chalk colour} = (1 - \text{metallic}) \cdot \text{base}$$

- $\alpha = \text{roughness}^2$, so that even steps of the knob look even.
- Blender, Unreal, Unity and glTF files use the same few knobs.

A texture as a BRDF parameter

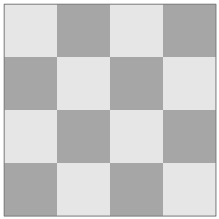


a roughness texture

dark: $\alpha = 0.03$; light: $\alpha = 0.3$

- Any knob can vary over a surface: α, base colour, metallic.

A texture as a BRDF parameter



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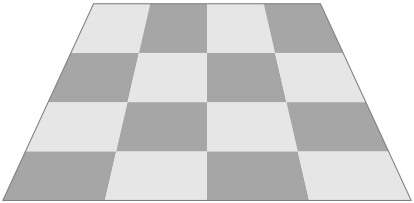
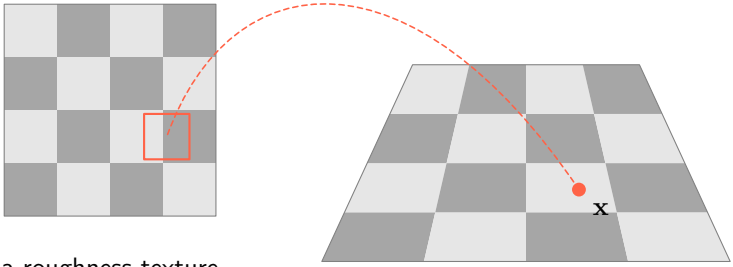


the floor

- Any knob can vary over a surface: α, base colour, metallic.

A texture as a BRDF parameter



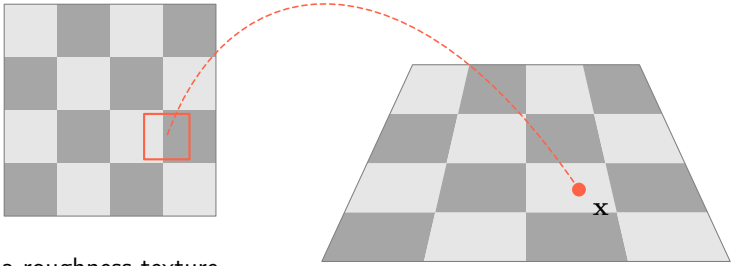
a roughness texture

dark: $\alpha = 0.03$; light: $\alpha = 0.3$

the floor

- Any knob can vary over a surface: α, base colour, metallic.
- At a hit, the material looks up the texture's pixel for that point.

A texture as a BRDF parameter



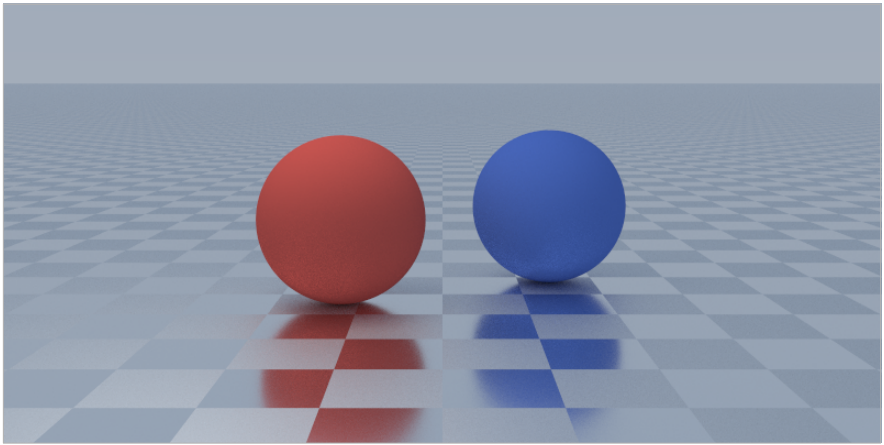
a roughness texture

dark: $\alpha = 0.03$; light: $\alpha = 0.3$

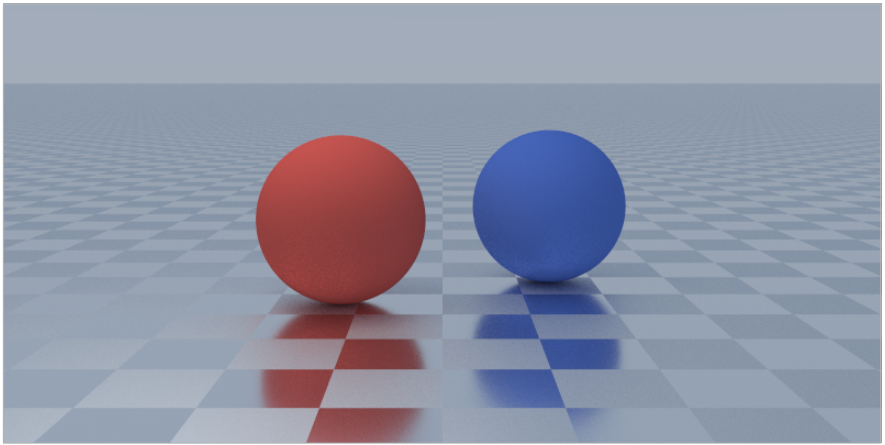
the floor

- Any knob can vary over a surface: α, base colour, metallic.
- At a hit, the material looks up the texture's pixel for that point.
- Our floor: a metal whose α is 0.03 or 0.3, square by square.

Rendered: a floor whose roughness is a texture



Rendered: a floor whose roughness is a texture



- One metal, one texture: sharp reflections in some squares, blurred in the rest.

Recap: real materials

metal: coloured F_0, no chalk; dielectric: $F_0 = 0.04$ over chalk

Disney: base colour, metallic, roughness, ...

a texture sets a knob point by point

Summary

1 Not a BRDF

the same both ways;

never more out than in

Blinn–Phong: up to 2.5 times

2 The three factors

$$\rho = \frac{D\,F\,G}{4\cos\theta_l\cos\theta_v}$$

GGX, Schlick at the facet, Smith

3 Picking a facet

one pick: $\dfrac{F\,G\,(\mathbf{v}\cdot\mathbf{m})}{\cos\theta_v\cos\theta_m}$

rough glass: a coin with chance F

4 Real materials

metal: coloured F_0, no chalk

plastic: chalk under a coat

Disney's knobs; textures

Reading

Pharr, Jakob and Humphreys, *Physically Based Rendering*, 4th edition (2023; free at pbr-book.org)

chapter 9: microfacets, rough conductors and rough dielectrics

Walter, Marschner, Li and Torrance, "Microfacet models for refraction through rough surfaces", EGSR 2007

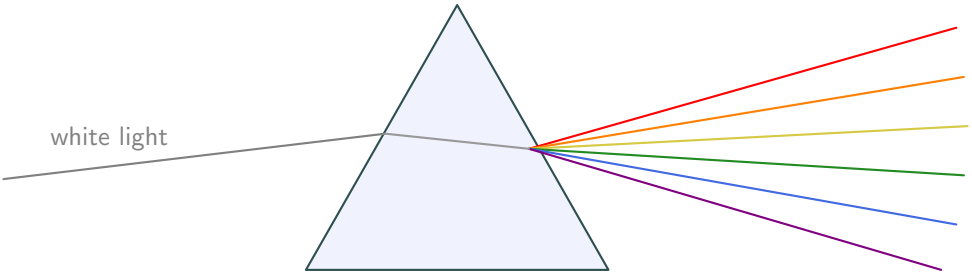
GGX, its Smith G_1, the facet pick, rough glass

Heitz, "Sampling the GGX distribution of visible normals", JCGT 2018; and "Understanding the masking-shadowing function", JCGT 2014

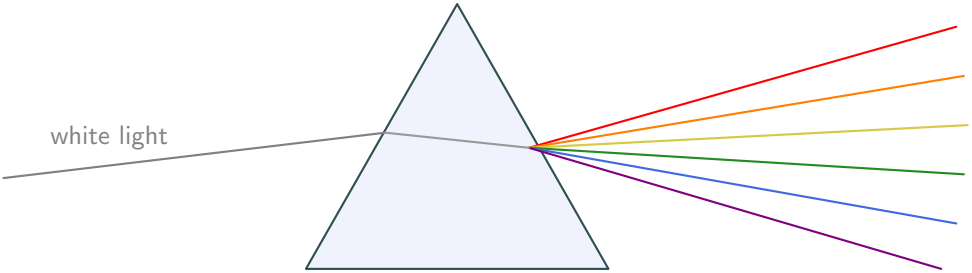
Burley, "Physically-based shading at Disney", SIGGRAPH 2012 course notes

the principled BRDF and its knobs

Next: Extra 3, colour and wavelength

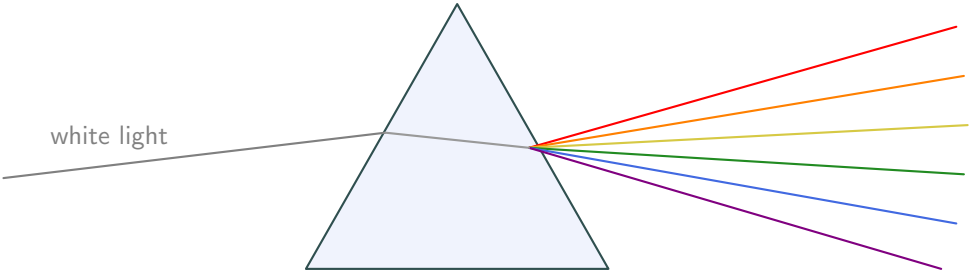


Next: Extra 3, colour and wavelength



- F_0 and the refractive index depend on the wavelength.

Next: Extra 3, colour and wavelength



- F_0 and the refractive index depend on the wavelength.
- Extra 3: spectra, prisms, rainbows, soap bubbles.