Computer Graphics

Midterm Exam — **Fall 2026** (CS 453) — **ANSWER KEY**
Date: Wednesday, 21st October 2026

Time Limit: 50 minutes **Total: 50 marks**

This exam contains 8 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information at the top of this page, and put your roll no. on the top of every page, in case the pages become separated.

Following rules apply:

- **DO NOT** open this booklet unless you are explicitly instructed to do so.
- You **must not** use books, notes, mobile phone, smart watch, laptop, or any calculator on this exam.
- Write each answer in the blank, the box or the picture provided for it. Only what is written there is marked.
- Write in clear, intelligible handwriting. An answer that cannot be read, or has been overwritten, will receive **zero** credit.
- One extra sheet for rough work is attached at the end of this booklet.

Problem	Marks	Score
1	7	
2	8	
3	8	
4	8	
5	8	
6	11	
Total:	50	

Do not write in the table to the right.

Please fill out the following:

Name: _____

Roll no: _____

Section: _____

I _____ *(sign)* hereby pledge that the work done during this exam is my own.

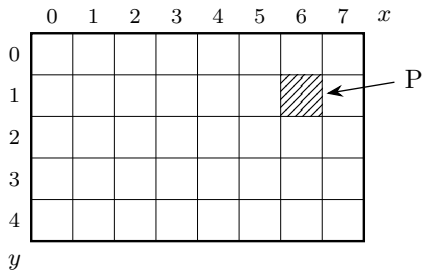
1. (7 marks) **Pixels** *(lectures 1 to 3)*

In every question: one square of a grid is one unit, and every picture is drawn to scale.

(a) (2 marks) The picture shows an image `img`, 8 pixels wide and 5 pixels high. Its columns x and its rows y are numbered.

(i) One line of code painted the hatched pixel P. Complete it:

```
img[ 1 ][ 6 ] = (255, 0, 0)
```

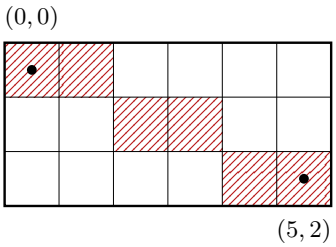


(ii) A gradient runs from the colour $A = (20, 30, 120)$ to the colour $B = (240, 150, 60)$. The lerp of A and B at $t = 0.5$ is

(130 , 90 , 90) .

(b) (3 marks) This loop draws the line from pixel $(0, 0)$ to pixel $(5, 2)$: $m = dy/dx = 2/5 = 0.4$.

```
err, y = 0.0, 0
for x in range(0, 6):
    set_pixel(img, x, y, color)
    err += 0.4
    if err > 0.5:
        y += 1
        err -= 1.0
```



(i) Shade the six pixels that the loop paints. *(2 marks)*

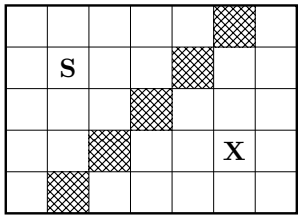
(ii) Another shallow line has $dx = 10$ and $dy = 3$. In its loop the three lines read

`err +=` 0.3 `if err >` 0.5 `:` `err -=` 1.0 *(1 mark)*

(c) (2 marks) Blending and filling.

(i) `blend_pixel` paints a red channel of 240 over a background of 80, with $\alpha = 0.25$. The pixel now stores 120 .

(ii) The cross-hatched pixels are a Bresenham line. `flood_fill` starts at S. Does the paint reach X?



with 4 neighbours: ☐ yes ☑ no

with 8 neighbours: ☑ yes ☐ no

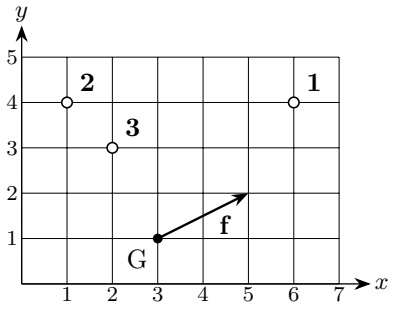
2. (8 marks) **Arrows and Curves** *(lectures 4 to 6)*

(a) (3 marks) A guard stands at G and faces along $\mathbf{f} = (2, 1)$. For each player the guard runs

```
rel = player_pos - guard_pos      facing = f.dot(rel) > 0
```

The players stand at the points 1, 2 and 3. Fill in the table. *(1 mark a row)*

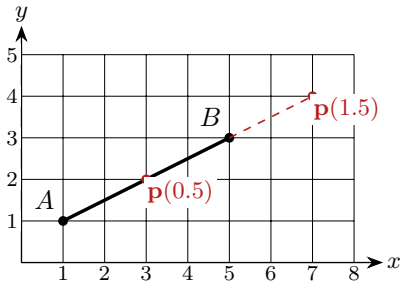
player	$\mathbf{f} \cdot$ `rel`	`facing` *(True or False)*
1	9	True
2	−1	False
3	0	False



(b) (2 marks) $\mathbf{p}(t) = A + t\,(B - A)$, with A and B as in the picture.

(i) On the picture, mark $\mathbf{p}(0.5)$ and $\mathbf{p}(1.5)$ with dots and label them.

(ii) `sample_parametric(f, 0, 1, steps=4)` returns 5 points.



(c) (3 marks) A Bézier curve has the three control points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$ of the picture. Find its point at $t = 0.5$ with de Casteljau's algorithm.

(i) Step 1 gives two points:

$\mathbf{Q}_0 = \text{lerp}(\mathbf{P}_0, \mathbf{P}_1, 0.5) = (\ 1\ ,\ 3\ )$

$\mathbf{Q}_1 = \text{lerp}(\mathbf{P}_1, \mathbf{P}_2, 0.5) = (\ 5\ ,\ 5\ )$

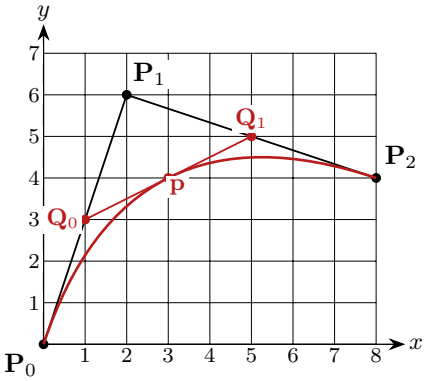
(ii) Step 2 gives the point of the curve:

$\mathbf{p}(0.5) = (\ 3\ ,\ 4\ )$

(iii) `de_casteljau` is given **5** control points.

Lerps in its first step: 4

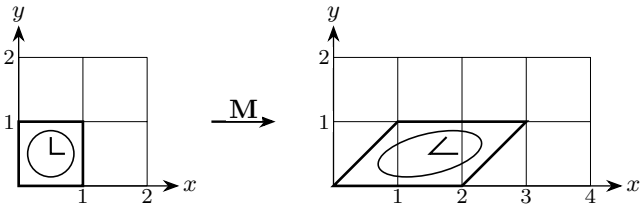
Steps in total: 4



3. (8 marks) **The Clock** *(lectures 7 to 9)*

(a) (2 marks) A matrix **M** moves the clock as shown. The clock starts as a square of side 1.

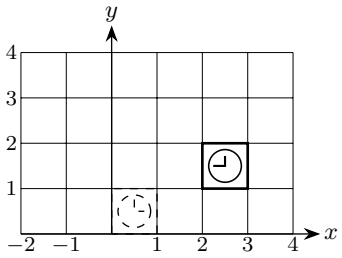
(i) $\mathbf{M} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$



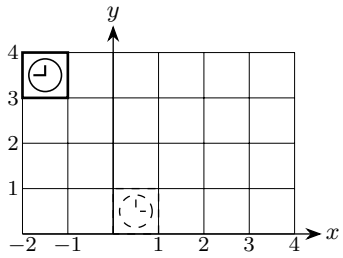
(ii) $|\mathbf{M}| =$ 2

The clock's area is now 2 .

(b) (3 marks) $\mathbf{R}$ = rotate(90°) and $\mathbf{T}$ = translate(3, 1). The clock starts at the origin (dashed) and is moved by $\mathbf{M} = \mathbf{T}\,\mathbf{R}$.



picture I



picture II

(i) The move that acts **first** is ☑ **R** ☐ **T**. The clock ends as in picture ☑ I ☐ II.

(ii) **M** sends the clock's corner (1, 1) to (2 , 2).

(iii) Write the third, homogeneous coordinate.

the **point** (1, 1): (1, 1, 1) the **direction** (1, 1): (1, 1, 0)

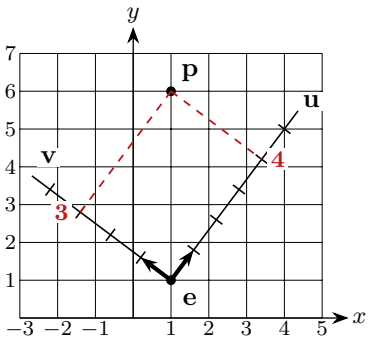
(c) (3 marks) Frames and their matrices.

(i) A frame has its origin at $\mathbf{e} = (1, 1)$ and the axes $\mathbf{u} = (0.6, 0.8)$ and $\mathbf{v} = (-0.8, 0.6)$. The point $\mathbf{p}$ has the world address (1, 6).

$\mathbf{p} - \mathbf{e} =$ (0 , 5)

In the frame, the address of $\mathbf{p}$ is

$(u_p, v_p) =$ (4 , 3). *(2 marks)*



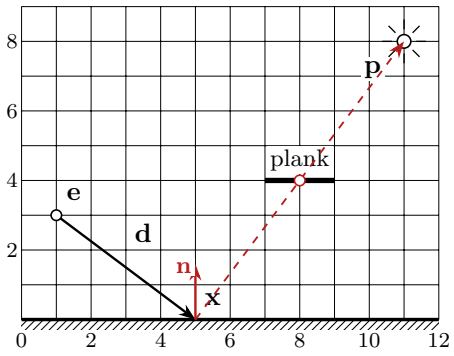
(ii) In a pendulum the bottom piece B hangs on the top piece T, and T hangs in the world W. Write the two-letter subscripts. *(1 mark)*

$\mathbf{M}_{WB} = \mathbf{M}_{\,WT}\ \mathbf{M}_{\,TB}$ $\mathbf{M}_{TB}^{-1} = \mathbf{M}_{\,BT}$

4. (8 marks) **Rays in a Flat World** *(lectures 10 to 12)*

The scene on the right: an eye at $\mathbf{e} = (1, 3)$, a floor along $y = 0$, a point lamp at $\mathbf{p} = (11, 8)$ and a plank from (7, 4) to (9, 4).

The viewing ray is $\mathbf{e} + t\,\mathbf{d}$ with $\mathbf{d} = (0.8, -0.6)$. It meets the floor at the hit point $\mathbf{x} = (5, 0)$.



(a) (2 marks) (i) The viewing ray reaches $\mathbf{x}$ at $t =$ **5** .

(ii) Along another viewing ray three objects report hits at $t = 9$, $t = 4$ and $t = -2$. The pixel shows the object at $t =$ **4** .

(b) (3 marks) At $\mathbf{x}$, write what shading needs. Each vector has length 1.

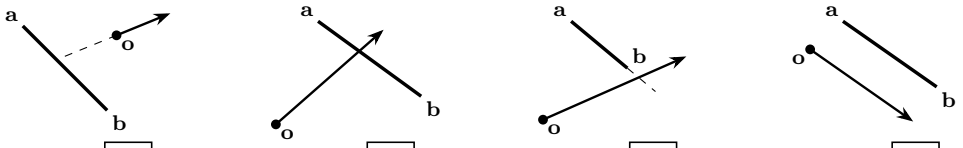
(i) the normal $\mathbf{n} = ($ **0** , **1** $)$ toward the eye, $\mathbf{v} = ($ **−0.8** , **0.6** $)$

(ii) toward the lamp, $\mathbf{l} = ($ **0.6** , **0.8** $)$ distance to the lamp, $r =$ **10**

(iii) The shadow ray $\mathbf{x} + t\,\mathbf{l}$ searches from $t =$ **ϵ** to $t =$ **10** . It meets the plank at $t = 5$, so this lamp's share at $\mathbf{x}$ is ☐ added ☑ zero .

(c) (3 marks) Each small picture shows a ray from $\mathbf{o}$ and a wall from $\mathbf{a}$ to $\mathbf{b}$. Solving for t (on the ray) and u (on the wall) gave these four results, in some order:

A: $|\mathbf{A}| = 0$ **B:** $t = 6,\ u = 0.4$ **C:** $t = -2,\ u = 0.4$ **D:** $t = 3,\ u = 1.2$

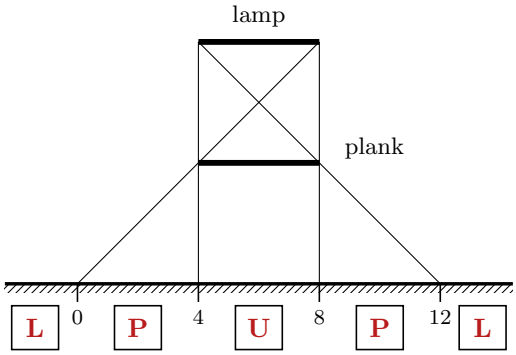


picture 1: **C** picture 2: **B** picture 3: **D** picture 4: **A**

(i) Write the letter of its result in the box under each picture.

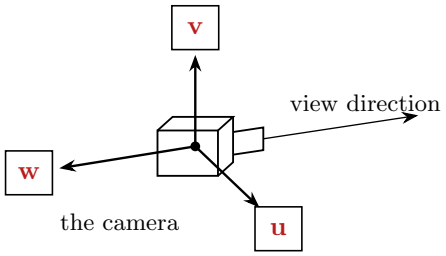
(ii) The lamp is now a segment (picture on the right). In each box on the floor write **U** for umbra, **P** for penumbra or **L** for lit.

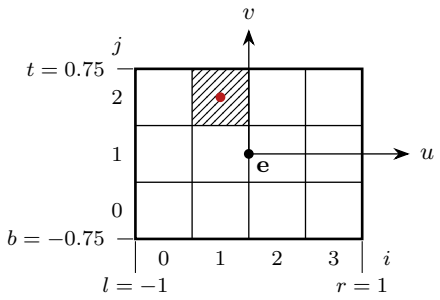
(iii) A floor point fires 8 shadow rays at the lamp and 6 are blocked. Its visibility is **0.25** , so it lies in the **penumbra** .



5. (8 marks) **The Camera and the Sphere** *(lectures 13 to 15)*

(a) (3 marks) Three arrows and a window.





(i) The camera frame (left picture): write **u**, **v** and **w** in the three boxes.

(ii) $\mathbf{a} = (2, 0, 0)$ and $\mathbf{b} = (1, 0, -2)$: $\mathbf{a} \times \mathbf{b} =$ (**0** , **4** , **0**).

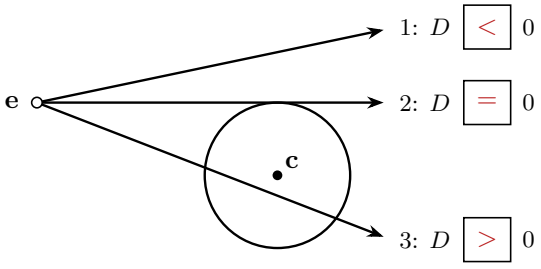
Right hand: thumb along **a**, index finger along **b**. The **middle** finger gives $\mathbf{a} \times \mathbf{b}$.

(iii) The window seen from the eye (right picture) has 4×3 pixels. The centre of the hatched pixel $(i, j) = (1, 2)$ is at $(u, v) =$ (−0.25 , **0.5**).

(b) (3 marks) Three rays leave the eye **e** toward a sphere. Each ray gives a quadratic in t, with $D = B^2 - 4AC$.

(i) In the box of each ray write $<$, $=$ or $>$.

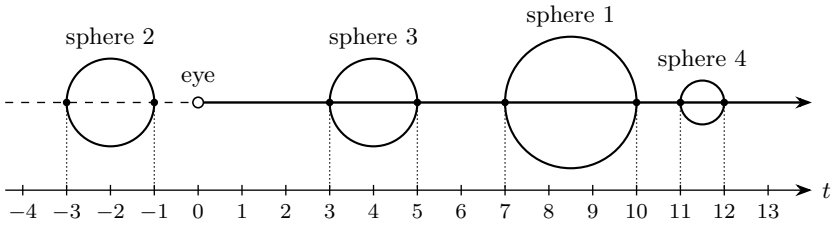
(ii) A ray is searched over t in $[0, \infty)$. Write the t of its hit, or "none".



the two roots	2, 5	−1, 2	−4, −1
hit at $t =$	**2**	**2**	**none**

(iii) A sphere has the centre $\mathbf{c} = (0, 0, -10)$ and the radius 5. A ray hits it at $\mathbf{p} = (0, 3, -6)$. The unit normal there is $\mathbf{n} =$ (**0** , **0.6** , **0.8**).

(c) (2 marks) One ray from the eye. A group asks its spheres in the list order 1, 2, 3, 4, starting with $t_0 = 0$ and $t_1 = \infty$. The picture shows where the ray crosses each sphere.



(i) t_1 after sphere 1: **7** , after sphere 2: **7** , after 3: **3** , after 4: **3** .

The group returns the record of sphere **3** .

(ii) Four sample rays go through one pixel. One hits a ball of colour (1, 0.4, 0); three see the sky, (0.2, 0.4, 0.8). The pixel's colour is (**0.4** , **0.4** , **0.6**).

6. (11 marks) **Light** *(lectures 16 to 18)*

(a) (6 marks) The colour of a pixel, as at the end of lecture 16. Its four terms are numbered.

$$\text{colour} = \underbrace{k_a\, I_a}_{①} + \sum_i \underbrace{\frac{R}{\pi}}_{②} \underbrace{\max(0,\ \mathbf{n}\cdot\mathbf{l}_i)}_{③} \underbrace{\frac{I_i}{r_i^2}}_{④}$$

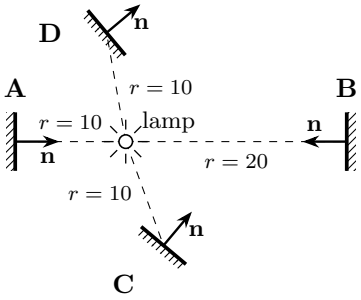
(i) Write the number of the term, 1 to 4, in each blank. *(2 marks)*

the ambient share: **1** the material's factor k: **2**

the tilt of the surface: **3** the lamp and its distance: **4**

The irradiance E_i from lamp i is the product of the terms **3** and **4** .

(ii) One lamp, with $I = 100\pi$, lights the four small patches of the picture. Fill in the irradiance E at each patch. Leave π in your answers. *(2 marks)*



patch	r	$\mathbf{n}\cdot\mathbf{l}$	E
A	10	1	π
B	20	1	$0.25\,\pi$
C	10	0.5	$0.5\,\pi$
D	10	−0.5	**0**

(iii) Patch C is Lambertian, with the reflectance $R = (0.8,\ 0.2,\ 0.2)$. This lamp's share of its colour is (**0.4** , **0.1** , **0.1**). *(1 mark)*

(iv) Patch C also gets ambient light, with $k_a = R$ and $I_a = 0.1$, and a second lamp whose share is $(0.2,\ 0.05,\ 0.05)$. The colour of patch C is (**0.68** , **0.17** , **0.17**). *(1 mark)*

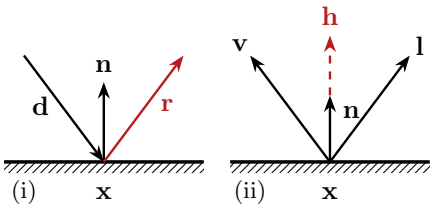
(b) (2 marks) A floor point $\mathbf{x}$ with $\mathbf{n} = (0,\ 1)$. Find the mirror direction $\mathbf{r}$ and the half vector $\mathbf{h}$.

(i) A mirror: the ray arrives with $\mathbf{d} = (0.6,\ -0.8)$.

$\mathbf{d}\cdot\mathbf{n} =$ −0.8 $\mathbf{r} =$ (**0.6** , **0.8**)

(ii) A highlight: $\mathbf{l} = (0.6,\ 0.8)$ and $\mathbf{v} = (-0.6,\ 0.8)$.

$\mathbf{h} =$ (**0** , **1**) $\mathbf{n}\cdot\mathbf{h} =$ **1**



(c) (3 marks) Air has the refractive index $n = 1$ and the glass has $n_t = 1.5$.

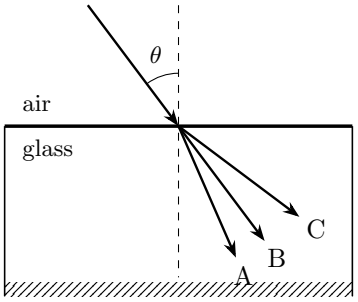
(i) A ray enters the glass (picture). Inside, it travels along ☑ A ☐ B ☐ C .

(ii) It arrives with $\sin\theta = 0.6$.

Inside the glass, $\sin\phi =$ **0.4** .

(iii) Another ray is **inside** the glass and reaches the surface with $\sin\theta = 0.8$. The number to test is $1.5 \times 0.8 =$ **1.2** , so the ray

☐ leaves the glass ☑ is mirrored back inside .



Marking notes

One mark for each line below; no half marks. A follow-on answer that is right for the student's own earlier slip earns its mark.

Part	Marks	One mark each for
1(a)	2	`img[1][6]`, row first (both numbers); (130, 90, 90) (all three)
1(b)	3	(i) all six pixels 2 marks, exactly one pixel wrong 1 mark; (ii) 0.3, 0.5 and 1.0 (all three; accept 3/10, 1/2 and 1)
1(c)	2	120; "no" with 4 neighbours and "yes" with 8 (both)
2(a)	3	one mark a row: the dot product and `facing` both right. Player 3 is exactly sideways: the dot product is 0 and `0 > 0` is `False`
2(b)	2	both dots, at $(3, 2)$ and $(7, 4)$; 5 points
2(c)	3	$\mathbf{Q}_0$ and $\mathbf{Q}_1$ (both); $\mathbf{p}(0.5)$; 4 lerps and 4 steps (both)
3(a)	2	all four entries of $\mathbf{M}$; $\lvert\mathbf{M}\rvert = 2$ and area 2 (both)
3(b)	3	$\mathbf{R}$ first and picture I (both); $(2, 2)$; 1 for the point and 0 for the direction (both)
3(c)	3	$u_p = 4$; $v_p = 3$ (no mark for $\mathbf{p} - \mathbf{e}$ alone); WT, TB and BT (all three)
4(a)	2	$t = 5$; $t = 4$ (the hit at -2 is behind the eye)
4(b)	3	$\mathbf{n}$ and $\mathbf{v}$ (both); $\mathbf{l}$ and r (both); ϵ to 10 and "zero" (all three; accept "a small number" or a value such as 0.001 for ϵ, not 0)
4(c)	3	(i) C, B, D, A (all four); (ii) L, P, U, P, L (all five); (iii) 0.25 (or 2/8) and penumbra (both)
5(a)	3	$\mathbf{v}$ up, $\mathbf{w}$ back, $\mathbf{u}$ right (all three); $(0, 4, 0)$ and the middle finger (both); $(-0.25, 0.5)$
5(b)	3	$<$, $=$, $>$ (all three); 2, 2, none (all three); (0, 0.6, 0.8)
5(c)	2	7, 7, 3, 3 and sphere 3 (all five); (0.4, 0.4, 0.6)
6(a)	6	(i) terms 1, 2, 3, 4 for the four names (all four); E_i is terms 3 and 4 (both); (ii) A and B (both: π and $\pi/4$); C and D (both: $\pi/2$ and 0); (iii) (0.4, 0.1, 0.1), that is $(R/\pi) \times 0.5\,\pi$; (iv) (0.68, 0.17, 0.17): ambient (0.08, 0.02, 0.02) + lamp 1 + lamp 2
6(b)	2	-0.8 and (0.6, 0.8) (both); $(0, 1)$ and 1 (both)
6(c)	3	A; 0.4; 1.2 and "mirrored back" (both)
	50	